Science Schools

Physics

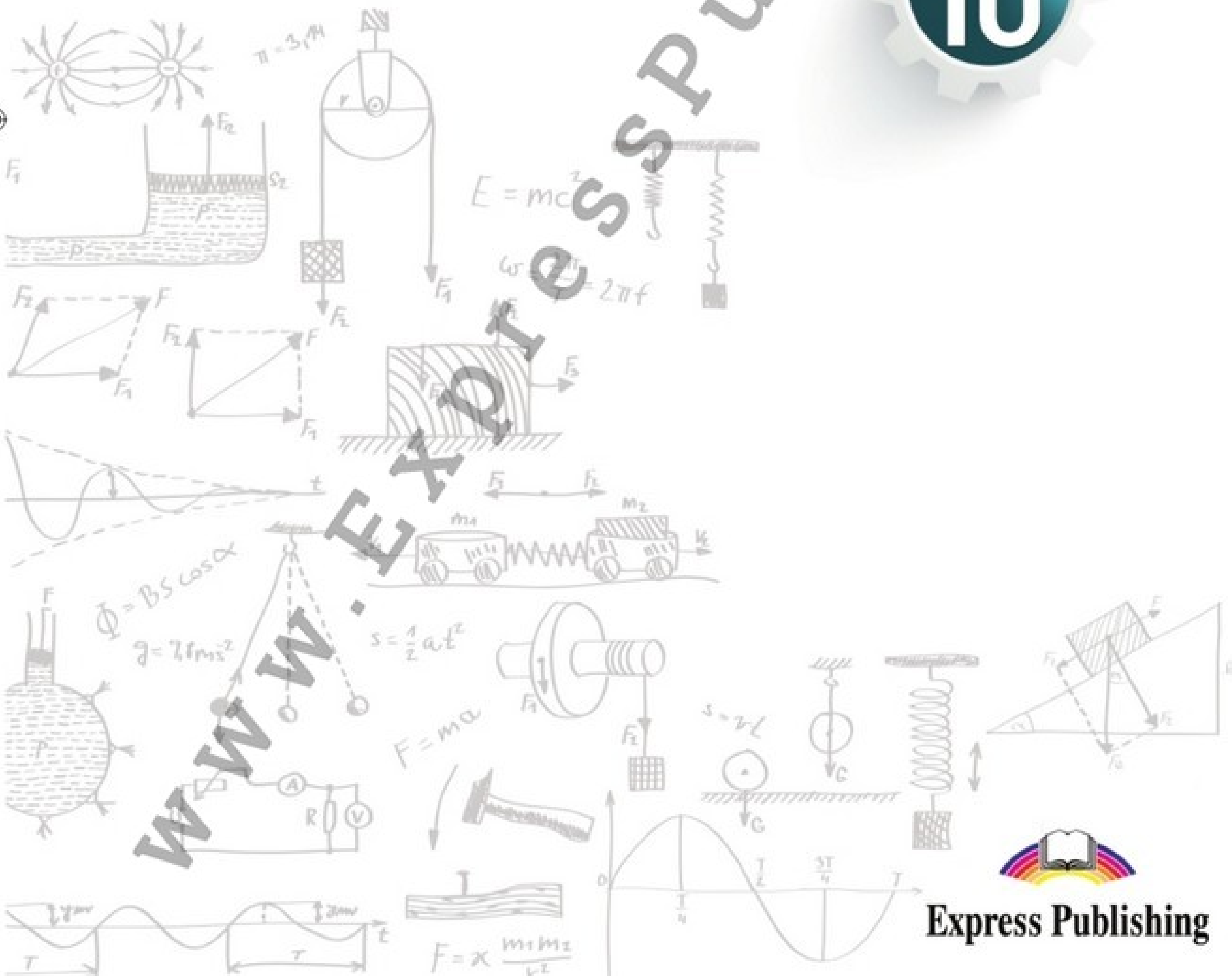
GRADE 10

Express Publishing

Physics

Tom Tierney
Special advisor: Pat Doyle
Zakhidam Julay
Aizat Aimakhanova

GRADE 10



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Tel.: (0044) 1635 817 363
Fax: (0044) 1635 817 463
email: inquiries@expresspublishing.co.uk
www.expresspublishing.co.uk

Exclusive Distributors
LLP 'EDU Stream'
104 Bogenbai batyr, 050002 Almaty, Kazakhstan
Tel: +7(727)293 85 89 – +7(727)293 94 20
Mobile: +7701 720 5916

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Contents

Introduction

For the student

Welcome to your new Physics textbook, *Grade 10 Physics*. Your textbook comes with a **Grade 10 Physics Student's Portfolio** and a range of *digital resources*. As well as deepening your understanding of key areas of Physics, this book aims to develop your learning skills in science. You will develop these skills in class, in laboratory practicals and whilst conducting research within and outside of class with your fellow students. An emphasis will be placed throughout this course on your ability to present core concepts, research and data effectively to others.

Glossary

A comprehensive glossary is included at the back of this book.

For the teacher

Written for the new Grade 10 Physics subject programme in Kazakhstan, *Grade 10 Physics* aims to meet the broad range of learning objectives set out in the Grade 10-11 Physics subject programme document. It focuses on developing learners' knowledge of and about science through the four content and skill strands outlined in the subject programme:

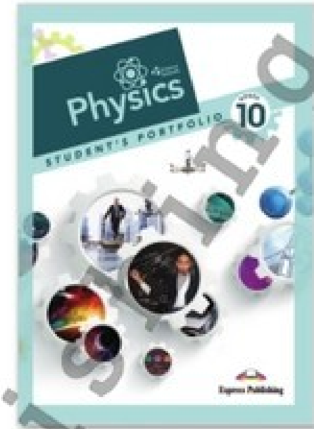
- Understanding of core subject areas in Physics
- Research and experimentation in science
- Communication in science
- Science and society

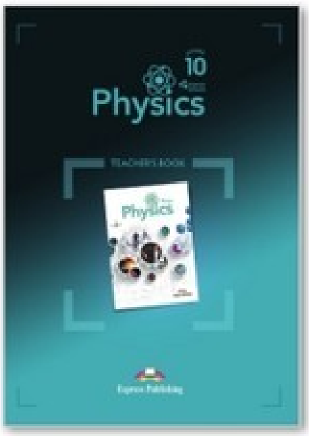
Key features of the textbook

- **Learning outcomes** are clearly stated at the beginning of each module in student-friendly language
- **Activities** and practical demonstrations allow students to build on their knowledge through guided observation, laboratory practicals and research
- **Diagrams** have been fully labelled and are drawn in a simple style so that learners can replicate them easily
- **Questions** are interspersed within sections of the text to offer teachers the opportunity to use a range of teaching strategies. There are regular opportunities for learners to engage in group work and pair work, discussion, giving of presentations and online research.

Student's Portfolio

The Student's Portfolio provides additional revision material and further tasks. The Student's Portfolio enables learners to maintain a detailed record of laboratory practicals, giving them space to reflect on the processes and results of their work. In line with the textbook, it provides detailed sample workings of all calculations they are required to make.





Teacher's Book

A Teacher's Book with **full answers** to all questions in both the Textbook and Student's Portfolio and detailed **worked solutions** of all calculations is provided.

Digital resources

Grade 10 Physics **digital resources** for teachers will further enhance classroom learning. These resources work in conjunction with the Textbook and Student's Portfolio. The resources have been designed to fully integrate with the Textbook to compliment lesson content. Following the principles of the new national Physics subject programme, material is provided to suit a range of learner types and to encourage participation and engagement on the part of the learner.

A series of **videos** allow students to observe science in action across all modules. These videos will reinforce the topic at hand, promote discussion about scientific issues in society and enable teachers to bring a range of perspectives on topics in Physics into the classroom.

Further classroom discussion and participation is opened up through **PowerPoint presentations**, including a thematic presentation of information from the Textbook. **Experiment videos** allow for a visual review of laboratory activities and can be used for demonstration or summative plenary work.

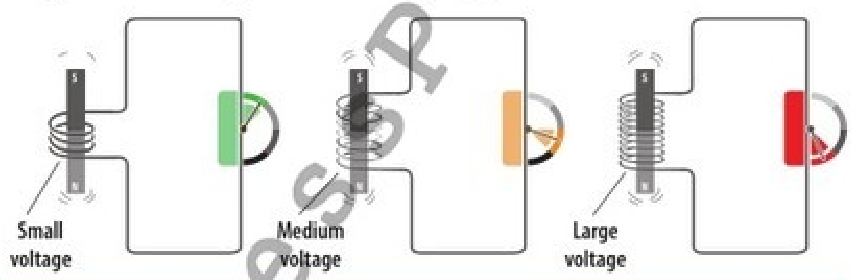
Impact of modern physics on society

Most fields of human endeavour have impacted modern life in some way or other. Wars, commerce, music, fashion and cuisine to mention just a few. All of the arts and humanities have made a mark and so have the sciences. In some instances interesting discoveries were initially little more than curiosities, and no-one knew at the time what practical use they might have. It is not possible to look in detail at how each of the incremental scientific discoveries have brought us to where we are now, but we can look at a few examples.

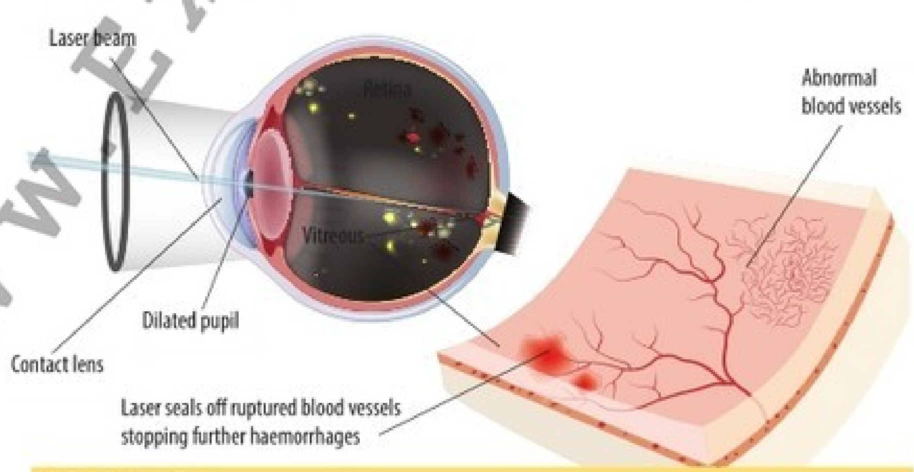
When Michael Faraday discovered how to induce electricity into a conductor using a magnet in the 19th century he could not possibly have imagined that his discovery would make it possible for the entire world to have electrical lighting, not to mention the many other electrically powered appliances in our homes, hospitals, factories, etc.

Faraday's Law

The induced voltage in a coil is proportional to the product of the number of loops and rate at which the magnetic field changes within the loops.

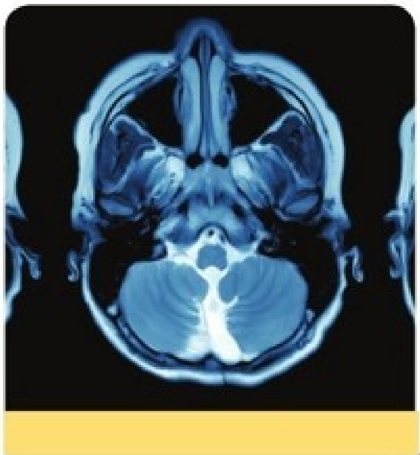


The discovery of the laser in the latter part of the 20th century was initially thought to have potential military applications, and resulted in some of the early work being classified as top secret, but in time lasers have found applications in many different fields: alignment, measurement, computer storage, communications and medicine to list just a few.

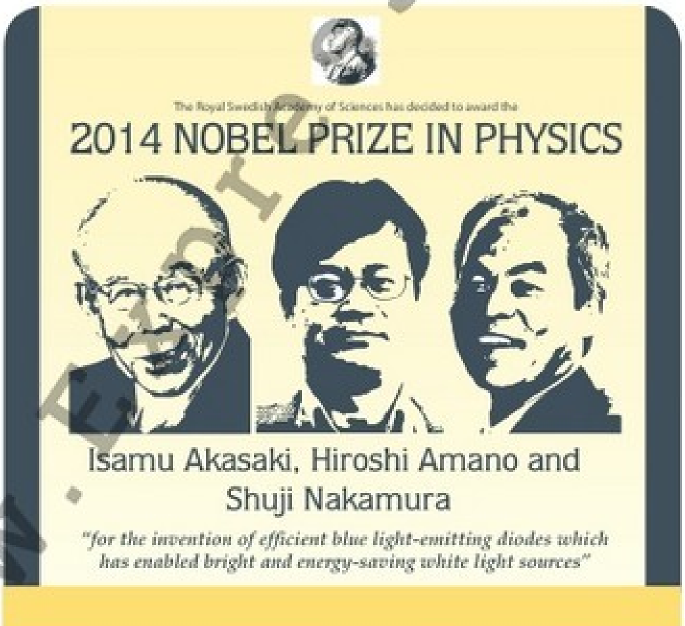


Diabetic Retinopathy

More recently Nuclear Magnetic Resonance Imaging (MRI), a technique that uses the magnetic properties of hydrogen nuclei present in all water molecules has revolutionised the way in which medical doctors can image soft tissue in the human body without causing any damage to the patient. This has enabled them to diagnose and treat conditions which previously could only be accessed through invasive and life-threatening biopsies.



Even something as 'simple' and the blue light emitting diode (LEDs), which initially might have simply looked like an addition to the already available red and green diodes, has had a huge impact on the efficiency of lighting all around the world. With red and green LEDs it was only possible to produce what the eye perceives as very yellow light, but the addition of the blue LED made it possible to produce what the eye perceives as white light. This has revolutionised the lighting industry and has reduced a lot of wasted energy. This was one of the considerations which led the Nobel Foundation committee to issue a prize to the blue LED inventors in 2014.

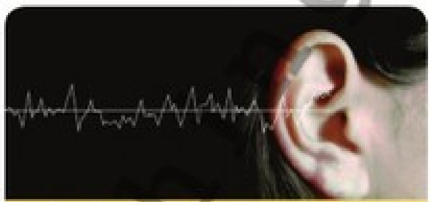


Frequently it is the development of different independent branches of physics which subsequently come together to facilitate totally new areas of technology. Applied branches of physics such as semiconductor, optics, magnetism, and branches of theoretical physics such as sampling theory, digitisation and packet switching have come together to make computing and the internet possible, and so revolutionise so many areas of human life that it is now nearly impossible to imagine how we managed before they were invented!

Each new discovery builds on the successes of previous ones. Isaac Newton famously said "If I have seen further than others, it is by standing upon the shoulders of others." It is the collective scientific human endeavour, and the sharing of discoveries through conferences, journals and private communications and visiting researchers that makes it possible to push the frontiers still further.

What is digital electronics

A diode can control the direction in which a current flows. Based on this idea many other digital electronic devices were developed in the second half of the last century. Indeed, in the last twenty-five years huge improvements have been made in the performance of these devices.

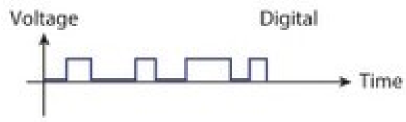


We hear analog signals

Sounds entering our ears can be represented by a wave. This type of wave is an **analog signal**, represented in the diagram here. A digital signal is different, as you can see in the diagram below.

An analog signal

Digital signals, as you can see from the diagrams, are not smooth like analog signals. Digital signals are simple 'true-false' or 'on-off' statements to represent information. Because digital devices recognise only one of two possible signals, they are less affected by unwanted electronic signals.



A digital signal

Digital electronics has had a great impact on society. You might call it a digital revolution.

What is meant by telecommunications?

Keeping in touch by telephone has become easier, cheaper and more reliable in recent years. The popularity of mobile phones has resulted in a greater need for people to be skilled in electronics and computing. Many courses in electronics are offered

Soldering equipment and electronic parts

Optical fibres have replaced old cables for phones and are much more efficient

Antique

by universities and third-level colleges. More people today understand and appreciate the wonders of this area of Physics.

Many modern electronic devices become outdated very quickly. Outdated devices are dumped or recycled and newer devices are manufactured. Earth's natural resources are being used up in the manufacturing process. The problem has also led to a greater awareness of the need to recycle. The copper in old wiring is very often recycled.

Much material is recycled, like these speakers

The internet

The internet is a vast, world-wide network of computers all connected together.

The table shows how advances made in this area of Physics has impacted on people's lives.

Numbers of people using the Internet	
At the end of the year	**Number of people using the internet**
1995	16 million
1999	248 million
2003	719 million
2006	1093 million

The internet is like an infinite library

What do you predict will be the figure in the last row of the table? Research the number of people using the internet worldwide this year. How close was your estimate?

The internet is an amazing library, with an almost endless supply of information. The study of science has been greatly facilitated by the internet: you can find information from others and share information with others so easily. You can study the historical development of science as well as keeping up to date with recent discoveries.

The Large Hadron Collider (LHC) in CERN: the tunnel through which the particles travel has a circumference of 27 km

In groups point to at least one major discovery in Physics that led to each of the following:

- Electricity
- The Transistor
- Flight
- Space flight
- Nuclear energy

Discuss the following statements:

Physics is essential for understanding chemistry. Without physics we cannot understand major chemical principles.

Physics and chemistry are essential for understanding biology. Without physics and chemistry we cannot understand major biology principles.

Name three areas in the other branches of science where you feel this to be particularly true.

Measurements in Physics

SI units

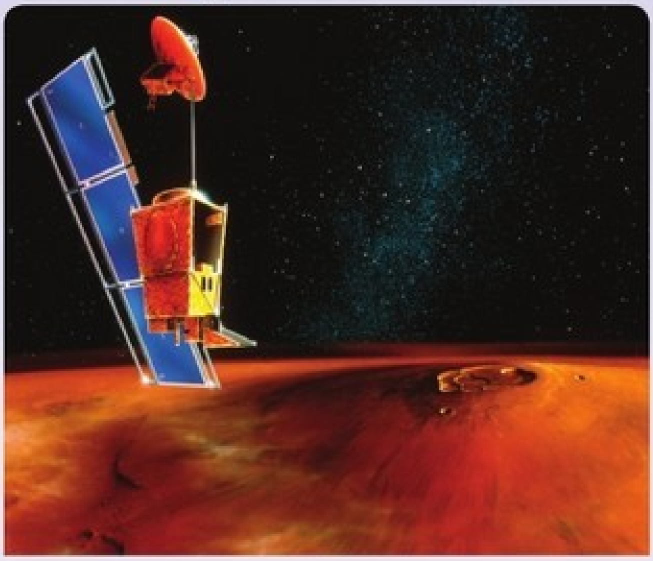
As science developed from the late 1600s onwards, communication between scientists working in different countries was often hampered by variation in the units used. In France in the years after the revolution of 1789, a number of leading scientists – such as Lavoisier and Laplace – came together to develop what became known as the metric system. Almost immediately after seizing power a decade later, Napoleon passed a law formally adopting the new system. Its use quickly spread throughout Europe.

In 1960, the General Conference on Weights and Measures – an international group of academics – revised and extended the metric system to include units for modern areas of study such as electricity. It is now known officially as the **International System of Units**, or ***Le Système international d'unités*** in French, and is usually referred to as the SI system, using the French initials.

The SI system is used throughout the world. Even in the United States and Britain – almost the only countries that still use the older imperial system at a government level – the SI system is used by scientists.

The importance of an international system

This is an artwork showing the Mars Climate Orbiter, which was launched in 1998 to study the Martian climate. Unfortunately, due to poor communication between NASA and the manufacturers, the computer software for one component used old imperial units, whereas all other components used the SI system. This caused the 330 million dollar spacecraft to burn up in the atmosphere and disintegrate.



The SI system has seven base units, which are independent of each other. They are:

- The metre – m (measuring length)
- The kilogram – kg (measuring mass)
- The second – s (measuring time)
- The ampere – A (measuring electric current)
- The kelvin – K (measuring temperature)
- The mole – mol (measuring the amount of a substance)
- The candela – cd(measuring luminous intensity – brightness).

All others are known as derived units and are based either directly or indirectly on the seven base units. For example, the Newton – the unit of force – is defined as the force required to produce an acceleration of 1 metre per second squared on a mass of 1 kilogram. As such, it is based on the units for length, mass and time ($kg\,m\,s^{-2}$).

Proportionality

In physics we make a lot of use of the mathematical concept of proportionality. This is not a difficult concept, but it is not one you will have encountered often in a maths class.

We say that two quantities are directly proportional to each other if the ratio of one to the other is a constant.

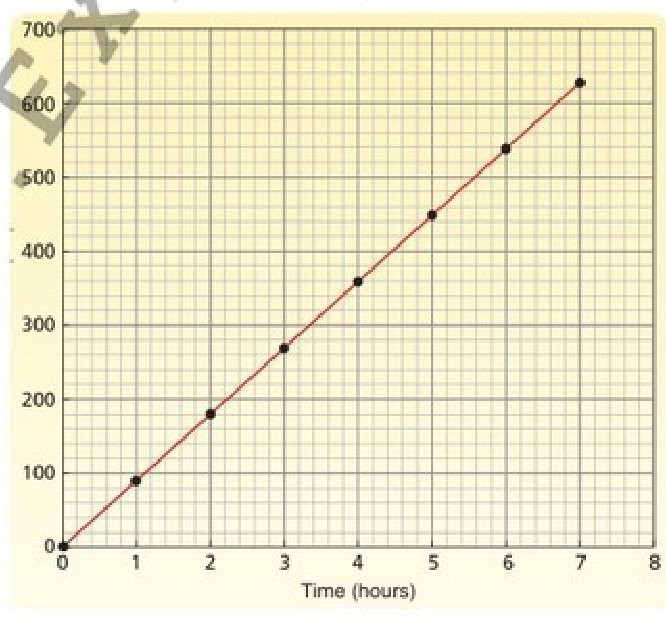
One example of proportionality is the relationship between a car's speed and the distance covered by the car when moving at constant speed. For example, think about a car travelling at $90\,km\,h^{-1}$ on a long straight road. After 1 hour it will have travelled 90 km and after 2 hours it will have travelled 180 km, and so on:

Time / h	1	2	3	4	5	6	7
Distance / km	90	180	270	360	450	540	630

We say that the distance is directly proportional to the time. This can be written as:

$$\textbf{Distance} \propto \textbf{Time}$$

This means that the two measurements are tightly linked without being equal to each other. If we double one, the other one doubles; if we multiply one by ten, the other one is multiplied by ten, and so on. If we represent this information on a graph, it will look like this:



This is not a coincidence: **whenever two measurements are directly proportional, we will find a straight line through the origin when we graph one against the other.**

To show that $\sin i \propto \sin r$, for example, we graph one against the other and show that we get a straight line through the origin.

Inverse proportions

A related concept is that of inverse proportions. This is when one measurement increases as another decreases so that an exact mathematical relationship is maintained: for example, if we double one, the other is exactly halved, and if we multiply one by 10, the other is divided by 10.

The easiest way of showing that two measurements are inversely proportional is to invert one and establish direct proportionality. For example, in Boyle's law we show that for a gas, with some conditions, pressure is inversely proportional to volume.

To do this, though, we draw a graph with pressure (P) on one axis and the inverse of volume $\left(\frac{1}{V}\right)$ on the other. We get a straight line through the origin showing that:

$$P \propto \frac{1}{V}$$

which amounts to the same thing as saying that pressure and volume are inversely proportional.

Percentage error

An important issue in considering the accuracy of any experiment is **percentage error**. All experimental techniques are to some extent flawed, but we can still be happy with a technique if it produces relatively small inaccuracies in our final calculations.

It is vital to do everything we can to make measurement errors as small as possible, but at the same time we have to accept that we cannot eliminate errors completely. This is where percentage error comes into consideration: as a general rule, we try to take large measurements to keep the percentage error low.

The following example shows how this works.

In measuring temperature, a thermometer may only be accurate to within ±0.5°C.

- If we measure a change in temperature of 5°C, the percentage error is:

$$\text{Percentage error} = \frac{0.5}{5} \times 100 = 10\%$$

- If we measured a change in temperature of 20°C, the percentage error is:

$$\text{Percentage error} = \frac{0.5}{20} \times 100 = 2.5\%$$

Clearly, the second measurement is more accurate. This is why we generally attempt to increase the size of measurements, thus decreasing the percentage error.

Module 1 Vectors and Motion

Learning objectives

- Apply equations of motion when solving problems 10.2.1.2
- Define kinematic values for falling bodies at an angle to the horizontal 10.2.1.6
- Investigate the trajectory of falling bodies at an angle to the horizontal 10.2.1.7
- Apply the law of conservation of momentum when solving problems 10.2.4.1
- Derive displacement and velocity formula; plot velocity time graphs 10.2.1.1
- Define the magnitudes of forces empirically and experimentaly test the force composition law 10.2.3.3
- Find centre of mass of a perfectly rigid body and system of material bodies 10.2.3.1
- Infer cause-and-effect relationships when explaining different types of equilibrium 10.2.3.2

Vectors and Scalars

Physics is a natural science. All of our theories and concepts depend on our ability to clearly understand what we mean by terms such as 'acceleration', 'velocity', 'speed' and 'force', and also on our ability to measure them. To do so, first we have to separate all measurements into two categories: vectors and scalars.

- Vectors have both magnitude and direction.
- Scalars have only magnitude.

Most measurements that you come across outside the world of science are scalars. Mass, for example, is a scalar. It is measured in kilograms and has no direction. Other scalar quantities are length, volume, time, energy and electric charge.

Examples of vector quantities are velocity, acceleration, momentum, force and weight. All of these have a direction associated with them. A typical measurement of force, for example, might be 10 N east.

The distinction between scalars and vectors can be confused by the fact that we are sometimes only interested in the magnitude of a vector quantity. For example, you may come across situations in which a force is given as, say, 15 N without any direction being mentioned. It is important to remember that this is not good practice. Even when it is not mentioned, a force always has a direction and it should, strictly speaking, be specified.

It can also be confusing that there are a number of situations in which there are closely related measurements, one of which is a vector and one of which is a scalar. Speed, for example, is a scalar. It is specified only in metres per second. The associated vector quantity is velocity, which is measured in metres per second and a direction, e.g. a speed could be 25 m s^{-1}, whereas a velocity would be something like 25 m s^{-1} east.

Another closely related pair is distance and displacement. Distance is a scalar and displacement is a vector. In figure 1.1, the woman is standing 12 m from her car. Her displacement is 12 m west from her car.

1.1 *Distance and displacement*

One important difference between distance and displacement is that, with displacement, we don't take the path travelled into account. In figure 1.2, a cyclist journeys from school to home along the route shown. When finished, he has travelled a total journey of 3 km. But his displacement is measured in a straight line and is only 2 km east.

1.2 *Velocity equals displacement divided by time*

When first introducing the idea of velocity, we often say that it is speed in a given direction. There are many situations in which this is a reasonable simplification of the situation, but it is not completely true. Strictly, the definitions of velocity and speed are:

$$\textbf{Velocity} = \frac{\textbf{Displacement}}{\textbf{Time}} \qquad \textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}}$$

In the example of the cyclist discussed, if the cyclist completes his journey in 1 h, his average speed is $3\,km\,h^{-1}$, but his average velocity is $2\,km\,h^{-1}$ east.

Vector addition

When adding vectors we are essentially looking to see how two or more vectors could be replaced with a single vector. We want to know how large that single vector would be, and in what direction it would point. It is easiest to picture this by looking at forces.
If two forces, one of 5 N and one of 10 N, push in the same direction, they could be replaced with a single force of 15 N, pushing in the same direction (see figure 1.3).

1.3 *Vector addition*

We say that the total force is given by: FT = 10 + 5 = 15 N, to the right.

If two forces, one of 5 N and one of 10 N, push in opposite directions, they could be replaced with a single force of 5 N, pushing as shown in figure 1.4.

1.4 *Vector subtraction*

We say that: FT = 10 – 5 = 5 N, to the right.

Note that when the two forces were in the same direction, we took them both to be positive, but that when they were in opposite directions, we took one to be negative. This is something we will see a lot of in our study of physics, and motion in particular. It is very important in every situation to be clear about which direction you are thinking of as positive and which direction as negative.

Triangle law

If forces are at an angle to each other, we can add them according to what is known as the triangle law. Let's say an object is being pushed by two forces, of 3 N and 4 N, whose directions are as shown in figure 1.5.

The object will move as if pushed by a single force of 5 N, whose direction is as shown in figure 1.6.

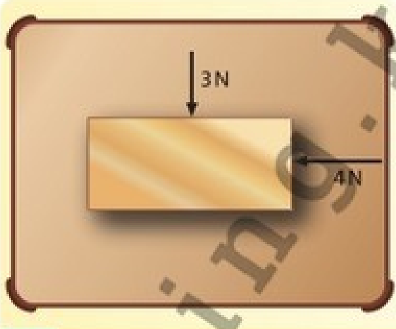
The 5 N is found using a right-angled triangle. The longer side is known as the resultant, and it represents the single vector that would have the same effect as the other two vectors. It is calculated according to Pythagoras' theorem:
$5^2 = 3^2 + 4^2$ (see figure 1.6).

The angle between the different forces can be calculated using trigonometry. Here we can say:

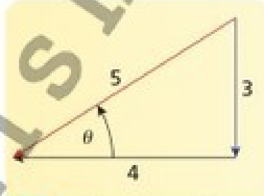
$\tan \theta = \frac{3}{4}$

and therefore the angle θ is 36.86°.

This means that the object will experience a force of 5 N at an angle of 36.86° to the 4 N force, as shown in figure 1.6.



1.5 *Vectors at right angles*



1.6 *The resultant force is 5 N*

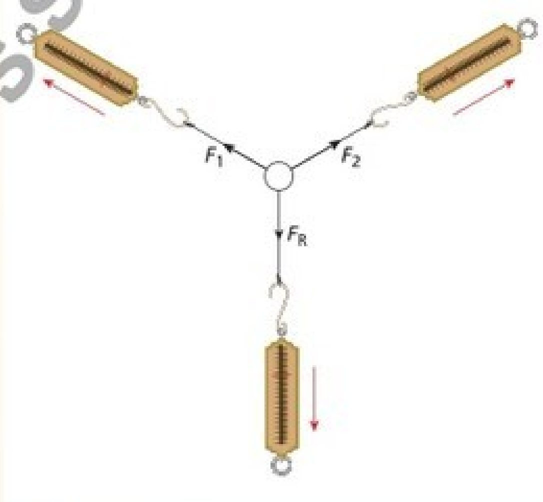
Experiment 1.1: To find the resultant of vectors

Method

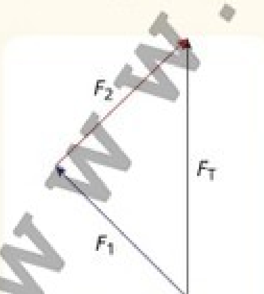
1 Set up the apparatus as shown in figure 1.7.
2 Note the readings on the first two newton meters (spring balances) (F_1 and F_2).
3 Note the reading on the third spring balance (F_R).

Observations

According to Newton's third law, the reading on the third spring balance should be equal in magnitude but opposite in direction to the resultant of the two upper balances.



1.7 *Spring balances*



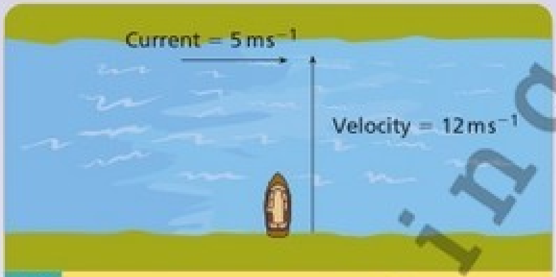
1.8 *Vector addition*

Note

You can also calculate the resultant of the vectors, F_T, using vector addition. The two methods should give identical values.

1.1 Sample Question

A boat moves across a river as shown in figure 1.9, so that the forward velocity is 12 m s^{-1}. The river is flowing with a current of 5 m s^{-1}. In what direction, and with what velocity, would the boat cross the river?



1.9

Sample Answer

Resultant velocity of boat (v_R):

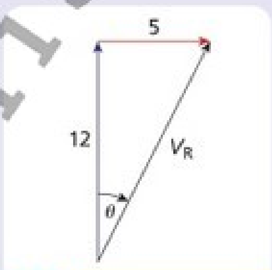
$$v_R^2 = 5^2 + 12^2 = 169$$

$$v_R = 13\,\text{m s}^{-1}$$

$$\tan\theta = \frac{5}{12}$$

$$\theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.6°$$

The boat would cross the river at 13 m s^{-1}, at an angle of 22.6°.



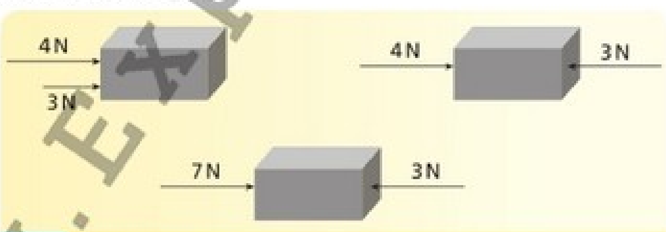
1.10

For you to try

1 Distinguish between the concepts of vector and scalar.

2 Which of these is a vector: mass, weight, distance, speed, velocity, energy, electric charge, acceleration?

3 In the situations shown in figure 1.11, find the magnitude and direction of the resultant force on each block.

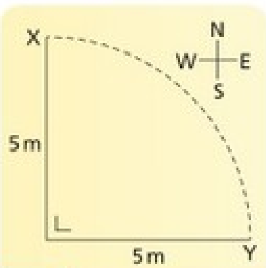


1.11 *Question 3*

4 A woman walks along the curve XY shown in figure 1.12.

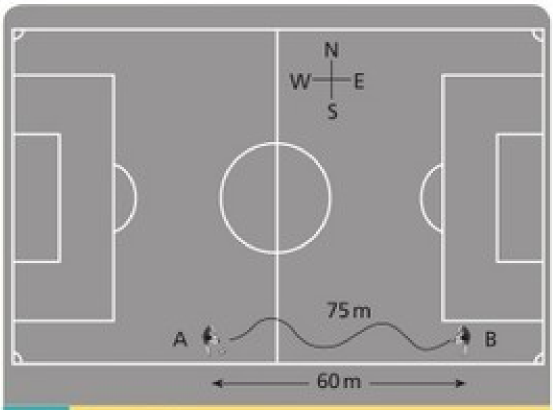
(a) What distance has she travelled?

(b) When she reaches Y, what is her displacement from X?



1.12 *Question 4*

5 A footballer runs from A to B along the path shown in figure 1.13 a total distance of 75 m. He does so in 25 s.

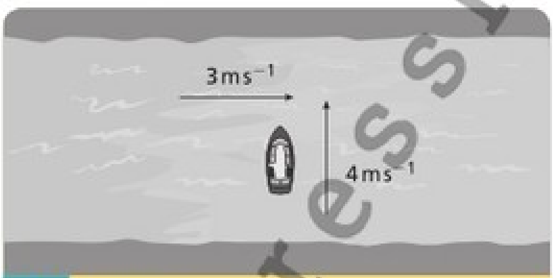


1.13 *Question 5*

(a) What is his average speed on the journey?
(b) What is his average velocity?

6 A square of side 100 m is marked out on grass. If you walk along the lines, starting at a corner and heading north first and then east, what is your displacement from your starting point after you have travelled 175 m?

7 A boat moves across a river so that the forward velocity is $4\,m\,s^{-1}$. The river is flowing with a current of $3\,m\,s^{-1}$, as shown in figure 1.14. Show on a diagram in what direction, and with what velocity, the boat would cross the river.



1.14 *Question 7*

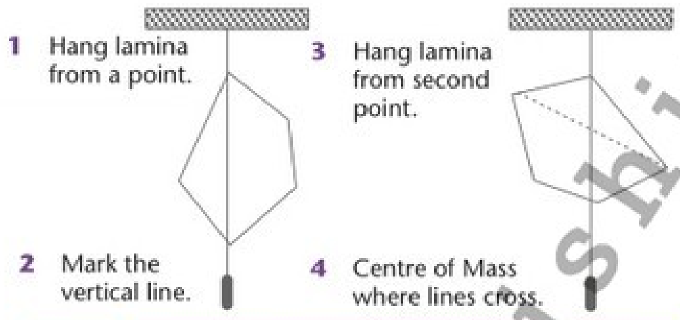
8 A plane is flying east and measures its velocity (with respect to the air) as being $100\,km\,h^{-1}$. The wind speed is in the same direction as the plane and is at $25\,km\,h^{-1}$.
(a) What is the resultant velocity of the plane?
(b) Another plane is flying west and also has an air velocity of $100\,km\,h^{-1}$. What is its resultant velocity?

Centre of Mass

Although gravity affects every little part of an extended body of mass, its effect sometimes appears to act at a single point somewhere near its centre. Consider a book which is resting on its side close to the edge of the table. As it is pushed slowly over the edge, it will reach a point where gravity suddenly seems to cause the book to fall over the edge. It is not that gravity has changed, but that the point at which gravity seems to be acting on the body has gradually moved beyond the edge of the table. This point is called the centre of mass (sometimes the centre of gravity).

It should be noted that the centre of mass will not always be located close to the geometrical centre of an object. This is especially true when the object is a composite of different density materials. Consider a hammer for example. The centre of mass will be close to the hammer head, and not half way down the handle.

A practical way to locate the centre of mass of an irregular object is to suspend it from any given point,, and also hang a string with a bob on the end to mark a vertical line from the same point. Once the object stops swinging, the centre of mass will lie somewhere along the vertical line marked by the string. An example is given in the diagram below, where an irregularly shaped lamina is suspended from different points. The centre of mass will be where the lines intersect.



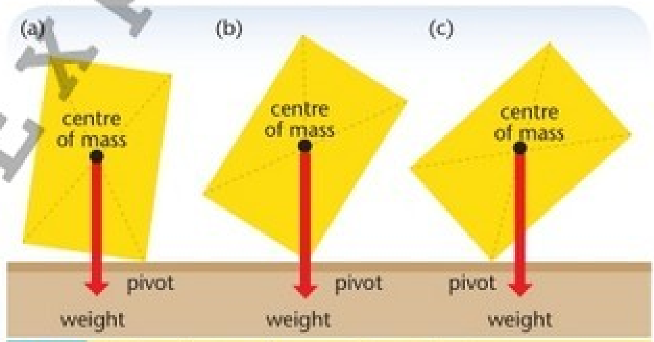
1.15 *Locating the centre of mass of an irregular object*

Method to find the centre of mass of a flat object.

Notice that however the object starts, it will swing back and forth until it settles with the centre of mass directly below the pivot. This is called a **stable equilibrium**.

It is also possible for an object to be in an **unstable equilibrium**. Consider a thin pencil standing upright on a flat table with its point pointing upwards. With sufficient care it is possible to make the pencil stand even though its centre of mass is directly above its support. Experience teaches us that a very slight jolt, or even a slight puff of wind will cause the pencil to fall over onto its side. That is why we call it an unstable equilibrium.

Another type of equilibrium is a **non-determinate equilibrium**. Consider a solid sphere resting on a flat surface. The centre of mass of the sphere is located exactly at the geometrical centre of the sphere, and this will be exactly above the point at which the sphere is supported by the flat surface beneath it. The sphere is in equilibrium, but if it is pushed it will roll without falling because as it rolls, its centre of mass always stays above the point of contact that is supporting it.



1.16 *Position of centre of mass relative to the pivot*

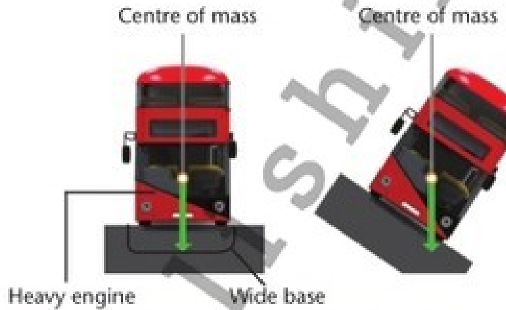
The position of the centre of mass relative to the pivot determines whether an object will fall over or return back to its original position. Provided the centre of mass remains vertically above the width of the base, the object will try to fall back on to its base.

When we stand, our centre of mass is somewhere close to our spine and will be directly above our footprint. If we lean forward we exert additional pressure on our toes, but the centre of mass will still be somewhere inside our footprint. If we lean further forwards, our centre of mass will move to lie in front of our toes (and outside our footprint), and unless we quickly move one foot forwards, we will fall forwards. (Our body will find a position where its centre of mass is as close as possible to the floor!)

When we walk we always maintain the centre of mass of our bodies somewhere above and between the base marked out by the extent of the range of our feet. If we don't, we fall over,

In general objects are at their most stable when their centre of mass has found its lowest position. A book can stand upright on a shelf for a long time, but its most stable position is to lie flat on its side.

The stability of engineering structures and vehicles is an important consideration. Whenever possible, engineers design systems such that the centre of mass is as low as possible.



1.17 *Low centre of mass of a bus and a wide base ensure stability*

The way to ensure an object is as stable as possible it to give it a low centre of mass, and a wide base. The centre of mass of a bus can be lowered by ensuring that all the massive components (engine, gear box, fuel tank, chassis, etc) and put as low as possible.

Notice that there is often a conflict between artistic appeal and engineering considerations. A flower vase normally has a narrow base and a wide opening at the top. This makes it suitable for flowers to be supported with their heads spread apart so that their beauty can be seen, but it also makes the vase very easy to tip over!

Resolving forces and calculating the components of forces

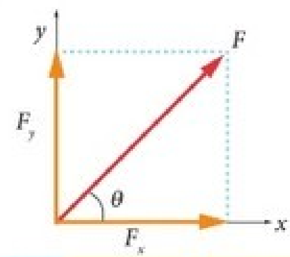
Forces are by their very nature invisible quantities. It requires a trained mind to notice where the forces are acting, in what direction, and what their likely magnitudes must be. We will limit our analysis to systems where the forces are in equilibrium such that the resultant force is always zero.

In the figure 1.18 two people are supporting a weight by means of a rope tensioned between them. It is clear that the string is in tension, and that both persons must be pulling equally, but hot does the magnitude of the tension compare to the size of the weight? In order to find answers to questions like these, we must balance all the forces, resolving their vertical and horizontal components.

1.18 *Two people support a weight by means of a rope*

Before proceeding to some practical examples notice that the force F can be thought of as having two components F_x and F_y. In the diagram 1.19 $F_x = F \cos \theta$, and $F_y = F \sin \theta$.

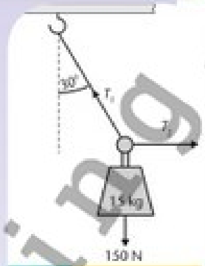
With this understanding we are ready to tackle some practical examples.



1.19 *Resolving a force into two perpendicular components*

1.2 Sample Question

A mass of 15 kilograms is suspended by a string from a hook in the ceiling. It is pulled sidewise by a horizontal force such that the string from the hook makes an angle of 30 degrees relative to the vertical. Calculate the magnitude of the force pulling to the right, and the tension in the string.



1.20

$g = 10\ ms^{-2}$

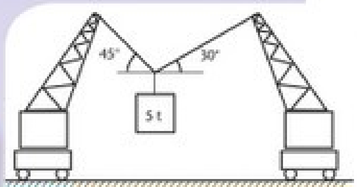
Sample Answer

In order to solve this question we need to resolve and balance the forces both vertically and horizontally.

Vertically: downwards = mg, upwards = $T_1 \cos 30°$, so $mg = T_1 \cos 30°$
Horizontally: to the right = T_2, to the left = $T_1 \sin 30°$, so $T_2 = T_1 \sin 30°$
From the first equation we can immediately calculate the tension.
$T_1 = mg / \cos 30°$
$T_1 = 15 \times 10 / \cos 30° = 173$ N
Once we have calculated T_1, we obtain T_2 using the second equation.
$T_2 = T_1 \sin 30° = 87$ N

1.3 Sample Question

Two cranes are jointly lifting a 5 ton (5000 kg) weight, and their cables are making different angles to the horizontal. Calculate the tensions in the two cables.



1.21

$g = 10\ ms^{-2}$

Sample Answer

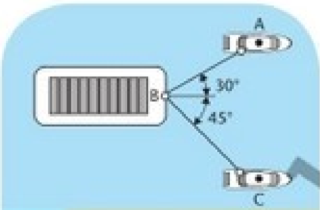
We will follow the same methodology.
Vertically: downwards = mg, upwards = $T_L \sin 45° + T_R \sin 30°$,
where the suffixes L and R denote the left hand crane and the right hand crane.
So $mg = T_L \sin 45° + T_R \sin 30°$
Horizontally: to the right = $T_R \cos 30°$, to the left = $T_L \cos 45°$
So $T_R \cos 30° = T_l\ T_L \cos 45°$
Notice that in this case we cannot solve one equation first, and then use the result to solve the second equation. These are simultaneous equations with two unknowns. We have to eliminate one unknown from both equations first.
Rearranging the second equation we obtain:
$T_R = T_L (\cos 45° / \cos 30°)$, now we replace T_R in the first equation with this.
$mg = T_L \sin 45° + T_L (\cos 45° / \cos 30°) \sin 30°$
This reduces to $mg = T_L (\sin 45° + \cos 45° \tan 30°)$
(remembering that $\sin 30° / \cos 30° = \tan 30°$).
$T_L = mg / (\sin 45° + \cos 45° \tan 30°) = 5000 \times 10 / (\sin 45° + \cos 45° \tan 30°) = 44\,829$ N
$T_R = T_L (\cos 45° / \cos 30°) = 44\,829 (\cos 45° / \cos 30°) = 36\,602$ N

For you to try

1 A heavy landscape picture in an art museum has a mass 50 kg and is supported with a rope attached to both ends as shown in the figure. Calculate the tension in the rope.

1.22 *Question 1*

2 A barge B is pulled along by two boats A and C. The two ropes between the boats and the barge make angles of 30 and 45 degrees respectively with respect to the direction of travel of the barge. Boat A is pulling with a force of 5000 N.



1.23 *Question 2*

(a) Without calculation, try to decide whether Boat C is pulling with a greater, smaller or equal force than boat A.
(b) Calculate the force with which boat C is pulling and see if you were right!
(c) Calculate the drag of the barge B.

Linear motion

The area of physics we call mechanics deals a lot with moving bodies. You will probably recall the relationship between distance, speed and time:

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}}$$

This equation is useful when we are dealing with bodies that are travelling at a constant speed, or where we are only interested in their average speed. However, if we need to take acceleration into account, we require different equations.

The equations of motion

Acceleration and velocity

Acceleration is the rate of change of velocity.

This definition of acceleration is wider than the general meaning of the word in conversation. You will notice, for example, that it refers to a change of velocity rather than speed. Because velocity includes a direction, this means that an object can travel at a constant speed and still be accelerating, if it changes direction. A car turning a corner, for example, is accelerating whether or not it changes its speed.

In terms of the linear motion of a body with uniform acceleration, from the definition of acceleration we can find a way of calculating its value:

Derivation

Acceleration is the rate of change of velocity, i.e.:

$$\text{Acceleration} = \frac{\text{Change of velocity}}{\text{Time}}$$

or

$$a = \frac{v - u}{t}$$

We can rearrange this to give:

$$at = v - u$$

or

$$v = u + at$$

Here we are using a set of symbols with which you will become familiar:

u – initial velocity

v – final velocity (or velocity after a time, t)

a – acceleration

s – displacement

t – time

You might notice that we rarely talk about 'deceleration' in physics. Instead, even when an object is slowing down, we talk about its acceleration – but we take the acceleration to be negative. This is connected to the fact that acceleration is a vector quantity.

- A positive acceleration means an object's velocity is increasing.
- A negative acceleration means an object's velocity is decreasing.

1.4 Sample Question

A car increases its speed from 10 m s^{-1} west to 30 m s^{-1} west over a period of 10 s. What is its acceleration?

Sample Answer

$$v = u + at$$

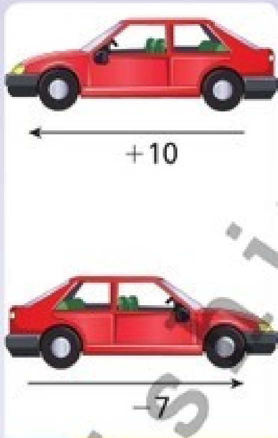
$$30 = 10 + a(10)$$

$$a = \frac{30 - 10}{10}$$

$$= 2\text{ m s}^{-2}\text{ west}$$

1.5 Sample Question

A car is travelling at $10\,m\,s^{-1}$ west and, over a period of 7 s, slows down and turns around so that it is travelling at $7\,m\,s^{-1}$ east. What is its acceleration?



1.24

Sample Answer

$u = 10\,m\,s^{-1}$ west, $v = 7\,m\,s^{-1}$ east

$$v = u + at$$

$$-7 = 10 + a(7)$$

$$a = \frac{-7-10}{7}$$

$$= -2.43\,m\,s^{-1} \text{ west}$$

or

$$= 2.43\,m\,s^{-1} \text{ east}$$

For you to try

1. Define 'acceleration'.
2. A car is travelling east and increases its velocity from $12\,m\,s^{-1}$ to $22\,m\,s^{-1}$ over a period of 4 s. What is its acceleration?
3. A car is travelling at $20\,m\,s^{-1}$ west and, over a period of 10 s, turns around so that it is travelling at $10\,m\,s^{-1}$ east. What is its acceleration?
4. A car begins from rest and with an acceleration of $10\,m\,s^{-2}$ west. What is its velocity after 5 s?
5. A car begins from rest and accelerates at $5\,m\,s^{-2}$. How long does it take to reach a speed of $100\,km\,h^{-1}$?
6. A bird is flying at $8\,m\,s^{-1}$ south and accelerates at $5\,m\,s^{-2}$ north. What is its velocity after 10 s?

So far we have concentrated on the relationship between velocity, acceleration and time. However, for a moving object, the distance travelled is also important. The following two equations show how distance is related to the other key variables for moving objects:

$$s = ut + \frac{1}{2}at^2$$

With constant acceleration, the average velocity in any motion will be given by:

Derivation

$$\text{Average velocity} = \frac{u+v}{2}$$

Also, by definition:

$$\text{Average velocity} = \frac{s}{t}$$

Therefore:

$$s = \text{(average velocity) (time)}$$

$$= \frac{u+v}{2}\,t$$

$$= \frac{u+u+at}{2}\,t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Derivation

$$v = u + at$$

Square both sides:

$$v^2 = u^2 + 2uat + a^2t^2$$

$$= u^2 + 2a(ut + \frac{1}{2}at^2)$$

$$v^2 = u^2 + 2as$$

Another equation that can be useful relates the average velocity of an object, the time involved and the object's displacement:

$$s = \frac{u + v}{2} t$$

These equations are very useful in any situation in which we are looking at moving bodies, and you will see a great deal of them in your studies. Because of the variables used, they are sometimes referred to as the UVAST equations.

1.6 Sample Question

A bicycle is travelling at $2\,m\,s^{-1}$ east and accelerates at $2\,m\,s^{-2}$ for 5 s.

(a) What distance does it travel in that time?

(b) What is its velocity after 5 s?

Sample Answer

$u = 2$, $v = v$, $a = 2$, $s = s$, $t = 5$

(a) $s = ut + \frac{1}{2}at^2$

$= 2 \times 5 + \frac{1}{2} \times 2 \times 5^2$

$= 35\,m$

(b) $v = u + at$

$= 2 + 2 \times 5$

$= 12\,m\,s^{-1}$ east

1.7 Sample Question

A bicycle begins from rest and increases its speed to $18\,m\,s^{-1}$ over a distance of 20 m.

(a) What is the magnitude of its acceleration?

(b) How long does it take to do this?

Sample Answer

$u = 0$, $v = 18$, $a = a$, $s = 20$, $t = t$

(a) $v^2 = u^2 + 2as$

$18^2 = 0^2 + 2a\,(20)$

$a = \frac{18^2}{2(20)} = 8.1\,m\,s^{-2}$

(b) $v = u + at$

$18 = 0 + 8.1(t)$

$t = \frac{18}{8.1} = 2.22\,s$

1.8 Sample Question

A plane lands at a speed of 240 km h^{-1} and travels 2.6 km along the runway before stopping.

(a) What is the average magnitude of its deceleration as it stops?

(b) How long does it take to do this?

1.25

Sample Answer

$u = 240\,\text{km h}^{-1}$

$= 240 \times \frac{10^3}{3600} = 66.67\,\text{m s}^{-1}$

$u = 66.67,\ v = 0,\ a = a,\ s = 2600,\ t = t$

(a) $v^2 = u^2 + 2as$

$0^2 = (66.67)^2 + 2a\,(2600)$

$a = -\frac{66.67^2}{5200} = -0.85\,\text{m s}^{-2}$

(b) $v = u + at$

$0 = 66.67 - 0.85t$

$t = 78.4\,\text{s}$

1.9 Sample Question

An object starts from rest and accelerates at 3 m s^{-2} for 20 s. How far does it travel in this time?

Sample Answer

$u = 0,\ v = v,\ a = 3,\ s = s,\ t = 20$

$s = ut + \frac{1}{2}at^2$

$= 0 \times 20 + \frac{1}{2} \times 3 \times 20^2$

$= 600\,\text{m}$

For you to try

1 A car is travelling at a speed of 20 m s^{-1} west. For a period of 5 s, it accelerates at a rate of 3 m s^{-2} in the same direction. What distance does it travel in this time?

2 The speed of a car increases from 3 m s^{-1} to 26 m s^{-1} over a period of 8 s.

(a) What is its acceleration?

(b) What distance does it travel in this time?

3 Over a distance of 5 m, the speed of a bicycle increases from 2 m s^{-1} to 5 m s^{-1}.
- **(a)** What is the magnitude of its acceleration in this period?
- **(b)** How long does this take?

4 A truck is travelling at 80 km h–1 and decelerates at a rate of 3 m s^{-2}.
- **(a)** How far does it travel before it comes to a rest?
- **(b)** How long does this take?

5 A bird flying at 3 m s^{-1} west is given an acceleration of 1 m s^{-1} east.
- **(a)** After 5 s, what is its velocity?
- **(b)** How far has it travelled in that time?

6 A car is travelling north at 25 m s^{-1} and accelerating at –5 m s^{-2}.
- **(a)** After 7 s, what is its speed and direction?
- **(b)** How far is it from its starting point?

7 A skateboarder starts from rest and accelerates to a speed of 15 m s^{-1} over a distance of 20 m. What is his acceleration?

8 In good weather cars travel on a stretch of motorway at an average speed of 105 km h^{-1}. It takes them 30 min to cover the distance between two exits. On a wet day, the average speed falls to 80 km h^{-1}. How much longer does the journey take?

Measuring velocity and acceleration

You should be familiar with at least one method of measuring velocity and acceleration. The use of both ticker-tape timer and light gates is shown in Experiment 1.2.

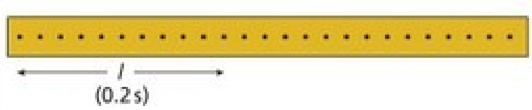
Experiment 1.2: Measurement of velocity and acceleration

Using a ticker-tape timer

Method to measure velocity

1. Set up the apparatus as shown in figure 1.26.
2. Push the vehicle so that it moves along the track at a constant velocity. As this happens the tape is pulled through the timer.
3. After the vehicle has stopped, remove the paper and examine it. It will look something like figure 1.27.

1.26 *A ticker-tape timer*



1.27 *Ticker tape*

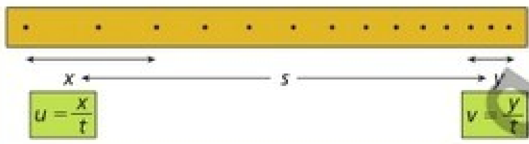
Results

Every 0.02 s a mark is made on the paper by the hammer. To calculate the velocity (or speed), mark out the length covered by, say, 11 marks on the paper (i.e. 10 'spaces'). This corresponds to a time of (10 × 0.02) = 0.2 s. To find the velocity, use the formula:

$$\text{Velocity} = \frac{\text{Distance } (l)}{\text{Time}}$$

Method to measure acceleration

1. Attach a weight to the vehicle and let it fall, so that the vehicle accelerates along the track.
2. After the vehicle has stopped, remove the paper and examine it. It will look something like figure 1.28.



1.28 *Ticker tape, showing acceleration*

Results

The value of the time, t = (the number of spaces between dots) × (0.02). s is the distance travelled. It is measured from the middle of each section, as we are taking the average speed over each of these sections. To find the acceleration, we use the formula:

$$a = \frac{v^2 + u^2}{2s}$$

(derived from $v^2 = u^2 + 2as$).

Using light gates

Method to measure velocity and acceleration

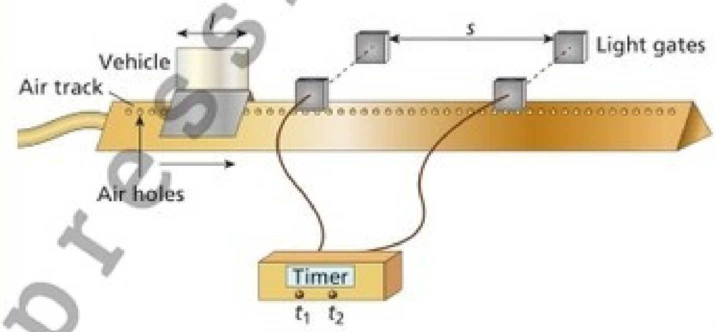
Push the vehicle along the air track, so that it passes through each of the light gates. As it passes through the gates, the light is blocked by the vehicle. The time that it takes to pass is recorded electronically by the timer (see figure 1.29).

Results

As we know the length of the card and the time it takes to pass the first light gate, we can calculate the initial velocity, u, using the formula:

$$u = \frac{l}{t_1}$$

This procedure can be repeated to measure the velocity at the second gate, v.



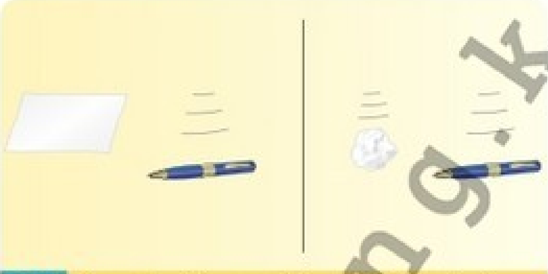
1.29 *Light gates*

The acceleration can be calculated using the formula:

$$a = \frac{v^2 - u^2}{2s}$$

We have looked at two methods of calculating velocity and acceleration in experiment 1.2. There are other methods, many of which make use of modern electronic devices known as data-loggers. These provide very accurate measurements and are a great way to carry out the experiments involved. However, they are difficult to cover in a textbook because the operation of each device is specific to the manufacturer's instructions.

Falling bodies



1.30 *The acceleration caused by gravity is constant*

Exactly how falling bodies behave is something that scientists have studied for many centuries. Greek philosopher Aristotle (384-322 BC) believed that heavier objects will always fall faster than lighter ones. This can often seem to be the case, but the difference is caused by air resistance. If we drop a flat piece of paper to the floor alongside a heavier object such as a pen, the pen will always hit the ground first because the paper experiences more air resistance. However, if you crumple up the paper, so that air resistance is reduced while the weight is not changed, and drop both the pen and paper again, they will hit the ground at the same time.

Italian physicist Galileo Galilei (1564-1642) saw the faults in Aristotle's understanding. He argued that, when we can ignore air resistance, all objects fall at the same rate. He famously demonstrated this by dropping various objects from the Leaning Tower of Pisa.

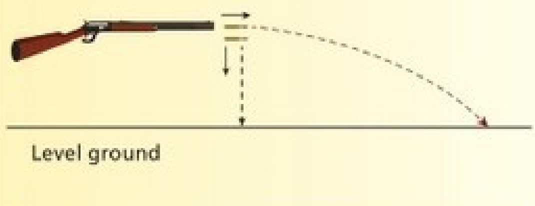
1.31 *The Leaning Tower of Pisa*

Galileo argued that his experiments showed that a hammer and a feather would fall at the same rate in the absence of air. This experiment was famously carried out centuries later by American astronaut David Scott (1932-) on the Apollo 15 mission to the Moon.

English physicist and mathematician Isaac Newton (1642-1727) was born in the same year in which Galileo died, and he carried on with much of Galileo's work and finally developed a theory of gravity. For now we will concentrate on the key part of Galileo's work: that all objects fall at the same rate. This means that they all have the same acceleration. This is usually referred to as the **acceleration caused by gravity** and is denoted by the letter g. Its value is generally taken to be 9.8.

Acceleration caused by gravity, $g = 9.8\,m\,s^{-2}$.

Projectiles



1.32 *All bodies fall at the same rate*

Think about a rifle that is designed to fire a bullet forwards at exactly the same time as it drops a bullet vertically. On level ground, which bullet will hit the ground first?

The answer is that – as long as the bullet is fired horizontally and the person firing the rifle is on level ground – both bullets will hit the ground together. This is another important way of understanding Galileo's work. Just as it is true that all objects will fall at the same rate regardless of their weight, it is also true that all objects will fall at the same rate regardless of whether or not they are moving forwards.

The bullet that is fired forwards will travel a greater distance before hitting the ground. However, while travelling forwards, it also falls with the acceleration caused by gravity.

Falling bodies and the equation of motion

All of the mathematical work covered in the previous section of this module applies to falling bodies, where the acceleration is that caused by gravity. It is important to remember here that acceleration is a vector quantity and so its direction matters. This means that we have to distinguish between the acceleration of those bodies that have been thrown upwards and are slowing down, and those that have been allowed to fall and are gaining speed. Generally, we take the acceleration to be positive when an object is falling (and gaining speed) and negative when it is rising (and slowing down).

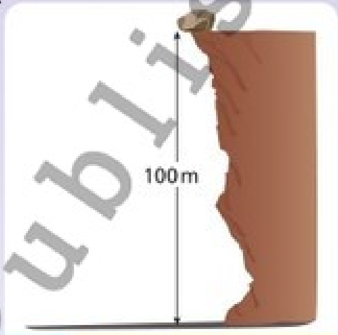
1.10 Sample Question

A stone is dropped from the top of a cliff 100 m high.

(a) With what speed does it hit the ground?

(b) How long does it take to reach the ground?

$u = 0$, $v = v$, $a = g = 9.8$, $s = 100$, $t = t$



1.33

Sample Answer

(a)

$$v^2 = u^2 + 2as$$
$$= 0^2 + 2 \times 9.8 \times 100$$
$$= 1960$$
$$v = 44.3\,\text{m s}^{-1}$$

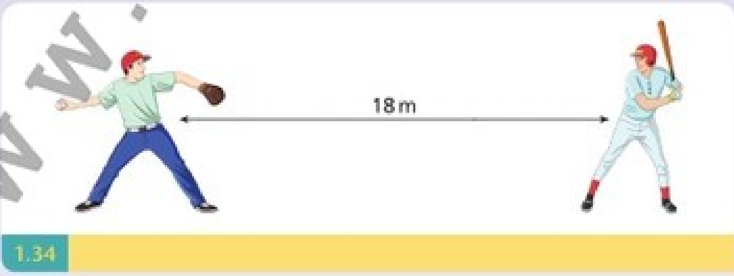
(b)

$$v = u + at$$
$$44.3 = 0 + 9.8(t)$$
$$t = \frac{44.3}{9.8} = 4.52\,\text{s}$$

Take g to equal $9.8\,\text{m s}^{-2}$.

1.11 Sample Question

A baseball player throws a ball horizontally so that it is moving at $35\,\text{m s}^{-1}$. How far has it fallen by the time it reaches the batter, a distance of 18 m away?



1.34

Sample Answer

Horizontal:

$u = 35,\ v = 35,\ a = 0,\ s = 18,\ t = t$

$$s = ut + \frac{1}{2}at^2$$

$$18 = (35)t + \frac{1}{2}(0)t^2$$

$$t = \frac{18}{35} = 0.51\ s$$

Vertical:

$u = 0,\ v = v,\ a = g = 9.8,\ s = s,\ t = 0.51$

$$s = ut + \frac{1}{2}at^2$$

$$= 0 \times 0.51 + \frac{1}{2}(9.8)(0.51)^2$$

$$= 1.27\ m$$

The ball has fallen 1.27 m.

1.12 Sample Question

A bullet is shot vertically upwards with a velocity of 15 m s^{-1}. What is the greatest height reached?

Sample Answer

$u = 15,\ v = 0,\ a = -9.8,\ s = h,\ t = t$

$$v^2 = u^2 + 2as$$

$$0^2 = 15^2 + 2(-9.8)h$$

$$h = \frac{15^2}{11.6} = 11.48\ m$$

1.35

1.13 Sample Question

A bullet is shot forwards at 400 m s^{-1} at a height of 1.7 m and on level ground. At the same time, a similar bullet is dropped to the ground from the same height.

(a) The two bullets hit the ground at the same time, but how long does it take for them to do so?

(b) How far does the bullet fired from the rifle travel forwards in this time?

1.36

Sample Answer

(a) $u = 0,\ v = v,\ a = 9.8,\ s = 1.7,\ t = t$

$$s = ut + \frac{1}{2}at^2$$

$$1.7 = 0(t) + \frac{1}{2}(9.8)t^2$$

$$t = 0.59\ s$$

(b) $u = 400,\ v = 400,\ a = 0,\ s = s,$ $t = 0.59$

$$s = ut + \frac{1}{2}at^2$$

$$= 400(0.59) + 0$$

$$= 236\ m$$

For you to try

1 A stone is dropped from a tall building and takes 4.1 s to hit the ground. What is the height of the building?

2 A stone is dropped from the top of a cliff.
- **(a)** With what speed does it hit the sea, 120 m below?
- **(b)** How long does it take?

3 A stone is thrown upwards with a velocity of $22\,m\,s^{-1}$. What is the greatest height reached?

4 A stone is thrown upwards from a height of 1.8 m with a velocity of $21.4\,m\,s^{-1}$.
- **(a)** How long does it take to reach its greatest height?
- **(b)** What is its greatest height?
- **(c)** How long does it take to fall to the ground?

5 A bullet is fired upwards with a velocity of $400\,m\,s^{-1}$.
- **(a)** What is its greatest height?
- **(b)** How long would you expect it to fall back to the height from which it is fired?

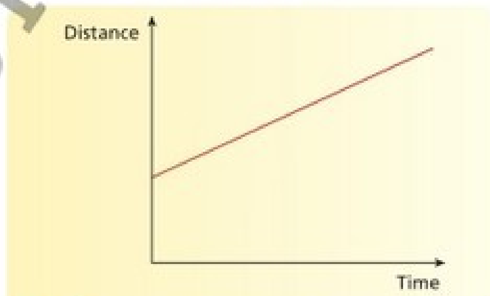
6 A bullet is shot forwards horizontally at the same time as a similar bullet is dropped to the ground from the same height. Which bullet will hit the ground first? Explain your answer.

7 A ball is dropped from a height of 1.5 m. How long does it take to hit the ground?

8 A ball rolls off the edge of a table of height 80 cm.
- **(a)** How long does it take to strike the floor?
- **(b)** If it hits the floor at a distance of 1.5 m, horizontally, from the table, with what speed was it initially rolling?

Distance–time graphs

Graphs are often used in science to help us to study various situations. They are of particular use in physics to represent the motion of a body. One way of doing this is to use the y-axis to represent the distance a body has travelled, while using the x-axis to represent time. The resulting graph lets us see at a glance whether the body is moving forwards or backwards and whether it is speeding up or slowing down.

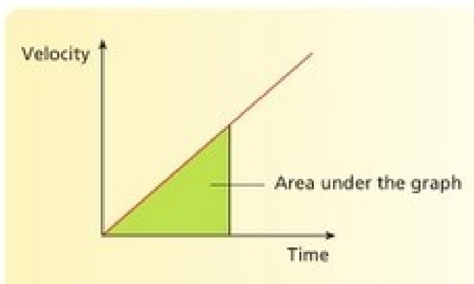


1.37 *This distance–time graph represents the motion of a body travelling at constant speed. The speed is the slope of the graph*

The graph in figure 1.37 represents the distance a car has travelled from a particular point on a road. You can see that it is constantly moving away from that point, and that it is doing so at a constant rate (or speed).

Velocity–time graphs

We can also use the y-axis on a graph to represent the velocity of an object while still using the x-axis to represent time. In the graph shown in figure 1.38, the velocity is constantly increasing. Because the graph has a straight line, we can see that the acceleration is constant. The area between the curve, or line, and the x-axis also holds meaning – it tells us the total distance travelled. This can be explained using the mathematical technique known as integration, which some of you will study in maths class.

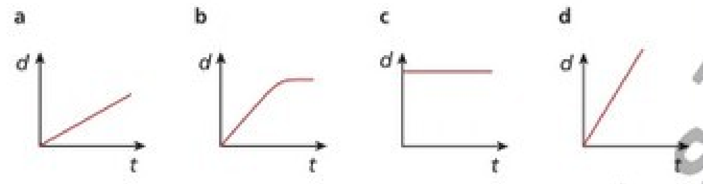


1.38 *In this velocity–time graph, the slope is the acceleration. The area under the graph is the distance travelled*

1.14 Sample Question

The graphs in figure 1.39 represent the motion of a car along a road, with the distance measured from a fixed point on that road.

(a) Which graph represents a car that is motionless?
(b) Which graph represents a car that is slowing down?
(c) Which graph represents the car that is travelling the fastest?



1.39

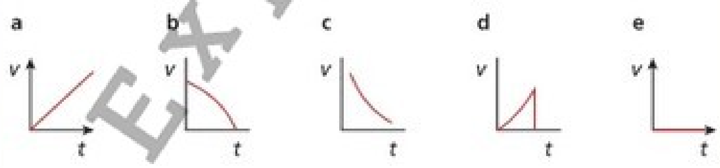
Sample Answer

(a) Graph c represents a car that is motionless.
(b) Graph b represents a car that is slowing down.
(c) Graph d represents the car that is travelling the fastest.

1.15 Sample Question

The graphs in figure 1.40 represent the velocity of various cars along a stretch of road.

(a) Which graphs represent cars that are slowing down?
(b) Which graphs represent cars that have constant acceleration?
(c) Which graph represents a car that has stopped?
(d) Which graph represents a car that initially accelerated before suddenly braking?



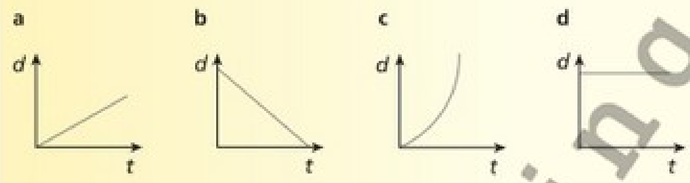
1.40

Sample Answer

(a) Graphs b and c represent cars that are slowing down.
(b) Graphs a and e represent cars that have constant acceleration.
(c) Graph e represents a car that has stopped.
(d) Graph d represents a car that initially accelerated before suddenly braking.

For you to try

1 The graphs in figure 1.41 represent the motion of a car along a road, with the distance measured from a fixed point on that road.

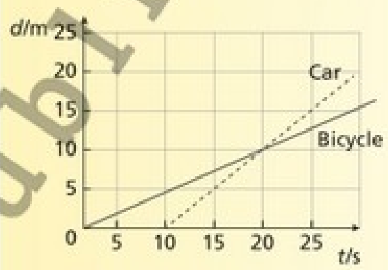


1.41 *Question 1*

(a) Which graph represents a car that is motionless?
(b) Which graph represents a car that is speeding up?
(c) Which graph represents the motion of a car that is towards the fixed point?

2 A cyclist begins a journey and travels as indicated by the graph in figure 1.42. After 10 s, a car passes the same point on the road.

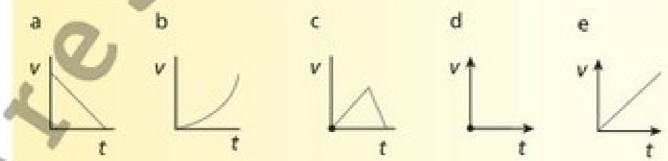
(a) After how many seconds does the car catch up with the bicycle?
(b) What is the speed of each of the vehicles during the journey?



1.42 *Question 2*

3 A skateboarder begins a journey and slowly increases in speed before stopping. She then returns to her starting point at a constant speed. Represent her journey on a sketch of a distance-time graph.

4 The graphs in figure 1.43, represent the velocity of various cars along a stretch of road.



1.43 *Question 4*

(a) Which graphs represent cars that are speeding up?
(b) Which graphs represent cars that have constant acceleration?
(c) Which graph represents a car that has stopped?
(d) Which graph represents a car that initially accelerated before suddenly braking?

5 A bicycle begins from rest and accelerates at $2\,\mathrm{m\,s^{-2}}$ until it is travelling at $20\,\mathrm{m\,s^{-1}}$.

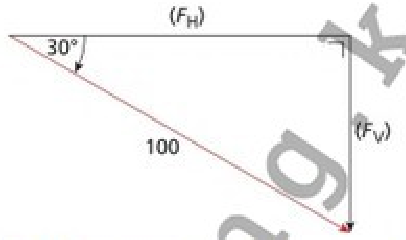
(a) After how many seconds is it travelling at $20\,\mathrm{m\,s^{-1}}$?
(b) Represent its motion on a graph and, from the graph, find the distance travelled in this time.

Vector resolution

If a person pushes a heavy box along the floor they will often, for convenience sake, do so at an angle to the floor. The man in figure 1.44 is pushing the box with a force of 100 N at an angle of 30°. However, the box doesn't move in the direction in which the man is pushing it. Instead, it moves horizontally along the floor.

1.44 *The box moves horizontally along the floorx*

Using trigonometry we can find out how much of the force that the man creates is actually pushing the box forwards along the floor. This is called the **resolution of vectors**. We draw a line to represent the 100 N, and make this the hypotenuse of a right-angled triangle, as shown in figure 1.45.



1.45 *Vector resolution*

The horizontal side of this triangle represents the **horizontal component of the force** (F_H), and the vertical side represents the **vertical component of the force** (F_V). We can find their lengths – and therefore the magnitude of the forces – using trigonometry:

$$\sin 30° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{F_V}{100}$$

$$F_V = 100 \sin 30° = 50\,\text{N}$$

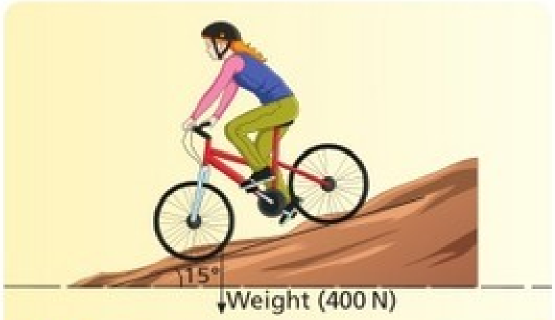
$$\cos 30° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{F_H}{100}$$

$$F_H = 100 \cos 30° = 86.7\,\text{N}$$

This means that the man is creating a single force that is exactly equivalent to creating two separate forces: one of 86.7 N horizontally, and the other of 50 N vertically. The 86.7 N is the force that actually pushes the box forwards. The 50 N also has an effect: it is pushing the box down onto the ground, increasing friction and making it more difficult to push the box forwards.

What is clear is that it would be more efficient to push the box directly forwards along the ground. None of the effort required to create 100 N would then be wasted, and the friction would be minimised. Most of us would do this instinctively when pushing a heavy weight.

Another way of resolving vectors into two components arises when we study objects that are not on level ground. The cyclist in figure 1.46 has a total weight of 400 N and is on ground at an angle of 15° to the horizontal. Her weight pushes vertically down towards the centre of the Earth, as you would expect, but she isn't able to move vertically downwards, so the weight instead causes her to roll down the hill. We can again use trigonometry to calculate what component of her weight is operating along the direction in which she moves.



1.46 *A cyclist moving downhill*

We draw a downwards arrow to represent the direction in which the cyclist's weight operates, the length of which represents the magnitude of her weight, or 400 N. We then draw a right-angled triangle so that this line is the hypotenuse. The smaller angle in the triangle is 15°, the same as the angle between the ground and the horizontal.

The components of the cyclist's weight parallel to the ground ($F_{//}$) and perpendicular to it ($F_\perp$) are again found using trigonometry:

$$\sin 15° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{F_{//}}{400}$$

$$F_{//} = 400 \sin 15° = 103.5\,\text{N}$$

$$\cos 15° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{F_{\perp}}{400}$$

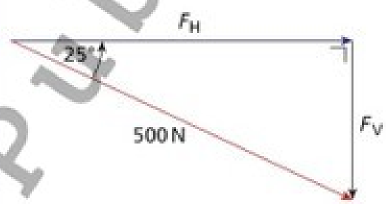
$$F_{\perp} = 400 \cos 15° = 386.4\,\text{N}$$

It is often easiest to see the significance of the resolution of vectors when we are dealing with forces, but it should be remembered that any vector can be resolved into two or more components.

1.47 *Vector resolution*

1.16 Sample Question

A heavy object is pushed along a horizontal floor with a force of 500 N, at an angle of 25° to the horizontal. What are the horizontal and vertical components of this force?



1.48

Sample Answer

$$\sin 25° = \frac{F_V}{500}$$

$$F_V = 500 \sin 25° = 211.3\,\text{N}$$

$$\cos 25° = \frac{F_H}{500}$$

$$F_H = 500 \cos 25° = 453.2\,\text{N}$$

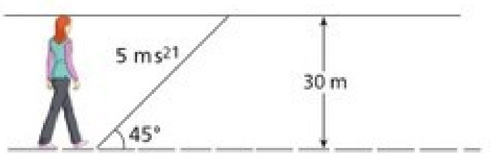
1.17 Sample Question

A woman walking at 4 m s^{-1} crosses a road of width 35 m, at an angle of 45° to the side of the road, as shown in figure 1.49.

(a) What is the component of her velocity parallel to the side of the road?

(b) What is the component perpendicular to the side of the road?

(c) How long will it take her to cross the road?



1.49

Sample Answer

(a) $v_{//} = 4\cos 45° = 2.83$

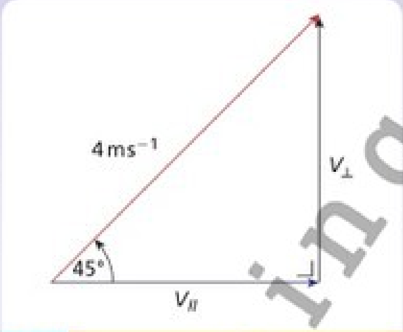
$= 2.83\,m\,s^{-1}$

(b) $v_{\perp} = 4\sin 45° = 2.83\,m\,s^{-1}$

(c) $s = ut + at^2$

$35 = 2.83t + \frac{1}{2}(0)t^2$

$t = \frac{35}{2.83} = 12.37\,s$



1.50

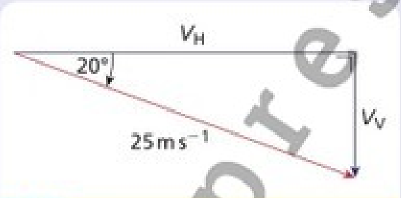
1.18 Sample Question

A hill is at an angle of 20°. A bicycle is moving down the hill with a velocity of $25\,m\,s^{-1}$. Resolve this into horizontal and vertical components.

1.51

Sample Answer

$v_V = 25\sin 20° = 8.55\,m\,s^{-1}$
$v_H = 25\cos 20° = 23.49\,m\,s^{-1}$



1.52

1.19 Sample Question

A BMX bicycle has a weight of 120 N and is on a ramp that is at an angle of 18° to the horizontal.

(a) What is the component of the bicycle's weight parallel to the ramp?
(b) What is the component perpendicular to it?

1.53

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Sample Answer

(a) $W_{//} = 120 \sin 18° = 37.08\,N$
(b) $W_{\perp} = 120 \cos 18° = 114.13\,N$

1.54

For you to try

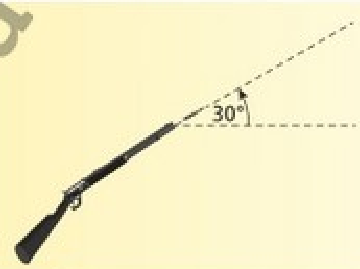
1 Find the horizontal and vertical components of a 450 N force acting at an angle of 60° to the horizontal.

2 A rope is used to pull a sleigh along horizontal, snow-covered ground. The force created by the rope on the sleigh is 1500 N, and the rope is at an angle of 20° to the horizontal.
 (a) What are the horizontal and vertical components of this force?
 (b) Is the vertical component of the force making it easier or harder to pull the sleigh?

3 Find the horizontal and vertical components of the velocity of the bullet shown in figure 1.55 as it leaves the barrel of a rifle at 380 $m\,s^{-1}$.

4 What are the horizontal and vertical components of the acceleration of 5 $m\,s^{-2}$ at an angle of 45° to the horizontal?

5 A bicycle is accelerating down a hill with an acceleration of 5 $m\,s^{-2}$. The hill is at an angle of 25° to the horizontal. Resolve this into horizontal and vertical components.



1.55 *Question 3*

6 A skateboarder has a weight of 650 N and is on a ramp that is at an angle of 22° to the horizontal.
 (a) What is the component of the skateboarder's weight parallel to the ramp?
 (b) What is the component perpendicular to it?

7 A stone on a roof has a weight of 2.4 N. The roof is at an angle of 35° to the horizontal. What are the components of the stone's weight parallel and perpendicular to the roof?

Momentum

The momentum of a body is defined as the product of its mass and velocity:

$$p = mv$$

A 1000 kg car moving in traffic at, say, 5 $m\,s^{-1}$ would have a momentum of $1000 \times 5 = 5000\,kg\,m\,s^{-1}$. An artillery shell of mass 12.5 kg travelling at 400 $m\,s^{-1}$ would also have a momentum of 5000 $kg\,m\,s^{-1}$ (12.5×400). The two are very different objects and very different situations, but they have this in common if nothing else: you wouldn't like to be standing directly in the path of either of them.

Momentum is a vector quantity. Its unit is $kg\,m\,s^{-1}$.

1.20 Sample Question

What is the momentum of a bullet of mass 5 g travelling at 380 m s^{-1} east?

Sample Answer

Momentum, $p = mv$
$= 0.005 \times 380 = 1.9$ kg m s^{-1} east

Conservation of momentum

Momentum is a very useful concept when we are studying the way that two different bodies collide with, or push against, each other. In such situations, as long as there are no external forces affecting the two bodies, we can say that momentum is conserved. This is known as the **principle of conservation of momentum**.

> The principle of conservation of momentum states that the total momentum of two bodies before an interaction is equal to the total momentum after the interaction, provided no external forces are acting on the system.

In many cases, our use of this principle involves a few assumptions about external forces. When two balls on a snooker table collide, we usually assume that momentum is conserved and we ignore the fact that friction between the balls and the table must have some effect, and that this would be an external force. This is a reasonable compromise as long as we look at the total momentum immediately before the collision and immediately afterwards. In the very short time in between, friction would have a very small effect.

In the case of somebody kicking a football, however, we wouldn't attempt to use the principle of conservation of momentum. We could measure the momentum of the kicker's foot immediately before the collision and that of the ball immediately afterwards, but it is impossible to ignore the external forces involved: the kicker's legs and whole body are playing some sort of role, and they are having far too great an effect for us to ignore them.

The principle of conservation of momentum can be treated mathematically using this equation:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where:

- m – mass
- u – velocity before the interaction
- v – velocity after the interaction

1.21 Sample Question

A bullet of mass 25 g travels at a speed of 200 m s^{-1} and strikes a wooden block of mass 1 kg that is free to move. The bullet is embedded in the block, and after the collision the two move together. What is their combined velocity?

1.56

Sample Answer

Momentum before = Momentum after OR

$$p_{before} = p_{after}$$

$$(mu)_{bullet} + (mu)_{block} = (mv)_{bullet+block}$$

$$(0.025 \times 200)_{bullet} + (1 \times 0)_{block} = (1.025v)_{bullet+block}$$

$$1.025v = 5$$

$$v = \frac{5}{1.025} = 4.88\,\text{m s}^{-1}$$

1.57

It is important to realise that conservation of momentum is not only useful when we are looking at collisions. It also applies when two bodies push against each other and move off in opposite directions. This is the case when a bullet is fired from a pistol, for example. Beforehand, the total momentum is zero. Afterwards the momentum of the bullet – with low mass and high velocity – and the gun – with high mass and low velocity – will be equal in magnitude but opposite in direction. The total of the two is still zero.

1.22 Sample Question

A pistol of mass 360 g fires a bullet of mass 7.5 g at 400 m s^{-1}. What is the recoil velocity of the pistol?

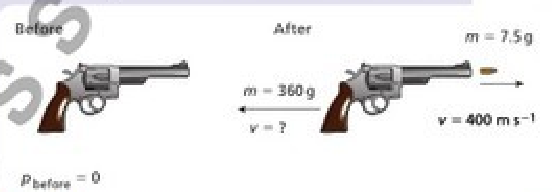
Sample Answer

$$p_{before} = p_{after}$$

$$0 = 0.360v + (0.0075)(400)$$

$$v = \frac{-3}{0.360} = -8.33\,\text{m s}^{-1}$$

The pistol moves at 8.33 m s^{-1}, in the opposite direction to that in which the bullet moves.



1.58

As seen in sample question 1.22, recoil velocity is the term we use to describe the speed of the pistol immediately after firing. This movement has to be controlled by the user, and if they are not properly prepared, it can easily injure them.

1.59 *The recoil velocity is small where there is no actual bullet*

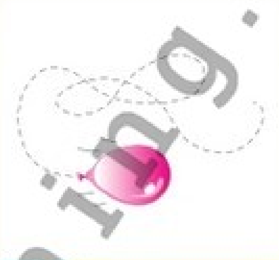
Bigger guns that fire larger bullets or shells can have a very large recoil velocity, and it can be very hard to control them. The recoil of cannons on ships in the 1700s could be big enough to tear the cannon from its bearings and to cause very serious damage. Modern artillery guns often use powerful springs built into the gun itself to absorb the recoil to avoid damage.

When actors fire pistols in movies, they are usually firing blank cartridges. These contain gunpowder, but no actual bullet. Because of this they have a much lower mass than a regular cartridge, and the recoil velocity is much less than it would usually be. This allows actors to treat the firing of a pistol rather casually.

Experiment 1.3: To demonstrate recoil

Method

1. Inflate a balloon without tying the end and hold it between your fingers at arm's length.
2. Let go of the balloon.
3. Note how the balloon flies around the room.



1.60 *The total momentum is conserved*

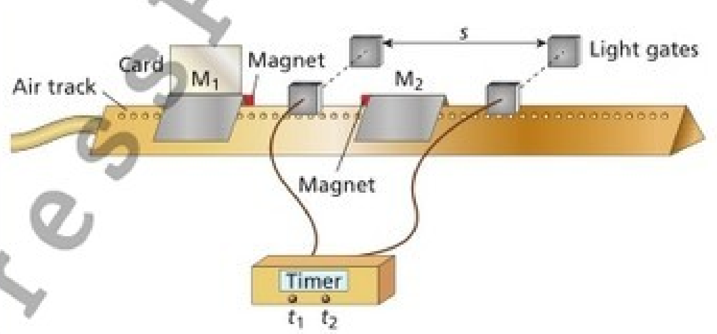
Observations

The initial momentum of the balloon is zero. After release, the air pushed out of the balloon in one direction will have the same total momentum as the balloon pushed in the other direction. As momentum is a vector quantity, the total momentum immediately before and after release remains at zero.

Experiment 1.4: Verification of the principle of conservation of momentum

Method

1. Set up the apparatus shown in figure 1.61.
2. With the second vehicle stationary, give the first a gentle push. After collision the two vehicles combine, using magnets, and move off together.
3. Note the times t_1 and t_2. From these find the initial velocity, u, and the final velocity, v.



1.61 *Experimental apparatus*

4. Calculate the momentum before the collision, $p_{before} = m_1u$, and the momentum after the collision, $p_{after} = (m_1 + m_2)\,v$.
5. Repeat several times, with different velocities and different masses. Record the results.

Results and Conclusions

You should find that the results consistently show that $p_{before} = p_{after}$, thus confirming the conservation of momentum.

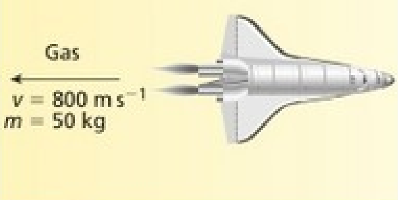
Accuracy

- The air track reduces the effect of friction.
- It is important to measure the velocities immediately before and after the collision, to offset the slowing down due to friction.

A ticker-tape timer could also be used for this experiment.

For you to try

1 What is meant by the term 'momentum'?
2 What is the unit of momentum?
3 What is the momentum of a car of mass 1050 kg travelling at 80 km h^{-1}?
4 Which has a higher momentum, a car of mass 1100 kg travelling at 22 m s^{-1}, or a bullet of mass 30 g travelling at 190 m s^{-1}?
5 Two cars are travelling in the same direction on a straight road. One has mass of 1070 kg and is travelling at 18 m s^{-1} north. The other has a mass of 950 kg and is travelling at 22 m s^{-1} north. What is the total momentum of the two cars?
6 A cyclist, whose total mass including her bike is 68 kg, is travelling at 13 m s^{-1} but applies her brakes and slows down to 5 m s^{-1}. What is the change in her momentum?
7 What is the principle of conservation of momentum?
8 A bullet of mass 45 g is travelling horizontally at 400 m s^{-1} and strikes a block of wood of mass 5 kg, which is at rest. If the bullet becomes embedded in the block, what is its initial velocity immediately after impact?
9 A car of mass 900 kg travelling at 20 m s^{-1} collides with another car of mass 1050 kg that is initially at rest. If the two cars stick together and travel in the same direction as the first car was moving, what is the initial velocity of the wreckage?
10 A pistol of mass 600 g is at rest. It fires a bullet of mass 8 g horizontally at velocity 390 m s^{-1}. What is the recoil velocity of the pistol?
11 A red snooker ball is at rest when it is struck by the yellow ball, of identical mass, moving at 5 cm s^{-1}. The two balls then move along the same line in which the yellow ball was moving. The red ball has an initial velocity of 3 cm s^{-1}. What is the initial velocity of the yellow ball?
12 A rocket of mass 10 000 kg is travelling forwards in space at 3 km s^{-1}. It fires a mass of 50 kg of gas as shown in figure 1.62, with a velocity of 800 m s^{-1}.
   (a) What is the total magnitude of the momentum of the gas?
   (b) What is the magnitude of the momentum of the rocket after the gas is ejected? (Ignore the loss in mass of the rocket.)
   (c) What is the magnitude of the velocity of the rocket after the gas is emitted?



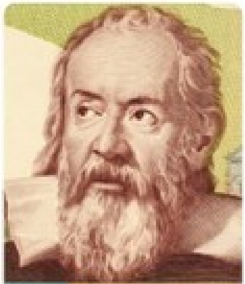
1.62 *Question 12*

Module 2 Forces

Learning objectives

- Create possible algorithms for solving problems for bodies in motion under the action of forces 10.2.2.1
- Explain the meaning of inertial and gravitational mass 10.2.2.2
- Explain the curve of gravitational intensity and potential of the material point and the distance 10.2.2.3
- Use the law of universal gravitation in problem-solving 10.2.2.4

Galileo and Newton



2.1 *Galileo Galilei (1564-1642)*

When Galileo Galilei attended university as a young man in Italy in the 1600s, he initially studied medicine, which then would have included philosophy and ethics as well as what we now think of as science and mathematics. When he learnt about physics, much of the current knowledge was based on the work of the ancient Greek philosophers such as Aristotle (384–322 BC) and Archimedes (287–212 BC). Some of this was of great value and is still studied today, but much of it was flawed: Aristotle, for example, believed that heavier objects will always fall faster than lighter bodies.

Galileo saw that this was untrue and argued that – in the absence of air resistance – all bodies fall at the same rate. You will already have learnt about this. However, a key part of his work was not just this new knowledge, it was his whole approach to how we should learn about the world and deepen our understanding. He believed that we should always test our ideas out with experiments and that we should be open to changing our ideas once we saw the results of those experiments.

So when Galileo dropped a cannonball and a musket ball from the Leaning Tower of Pisa, he wasn't just establishing the truth of how these objects fall. He was also helping to establish the scientific method: that all of our ideas should be subject to test and verification. This approach gradually led to the splitting of science from the world of philosophy. This process continued after he died and was greatly advanced by the work of Isaac Newton (1642–1727).

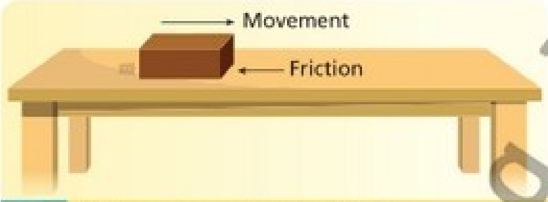
Newton built on what Galileo and others had done, and much of what are called 'Newton's laws' were known and understood before him. However, he developed the mathematical techniques that allowed these ideas to be clarified and, crucially, verified.

Newton's laws of motion

Newton's laws of motion were laid down in his great work, *Philosophiae Naturalis Principia Mathematica* (often called simply the Principia), which was first published in 1687.

1. A body will continue in a state of rest or of uniform velocity unless an unbalanced external force acts upon it
2. The rate of change of a body's momentum is proportional to the force that causes it and takes place in the direction of that force
3. If body A exerts a force on body B, then body B exerts an equal but opposite force on body A.

Newton's first law, in particular, owes a lot to Galileo.



2.2 *Friction causes moving objects to stop*

Many of us are happy to accept that moving objects slow down and stop because of friction. And similarly, we know that when we reduce friction objects can travel further. However, we often struggle to accept the obvious conclusion of this, which is **Newton's first law**: that in the absence of friction a moving object would travel forever. In many ways, the genius of Galileo and Newton was to recognise the obvious.

Put in other words, the first law tells us that, just as it is true that an object at rest will remain at rest unless a force makes it move, it is equally true that once an object is moving, it will keep moving unless a force makes it stop.

You may notice this in a car. If the car stops suddenly, you feel yourself being apparently pushed forward. In fact, what is happening is that you are continuing to travel at a constant speed, until the seat belt forces you to stop, as described by the first law.


2.3 *Apollo 13 travelled over 500 000 km with almost no fuel in 1970. In the absence of air resistance, no fuel was needed*

The first law is central to space travel. Spacecraft require large quantities of fuel to take off from Earth and to effectively escape the Earth's gravitational pull, but once they have done so they can switch off their engines. Typically they travel at speeds of several kilometres per second, and will continue to do so indefinitely: in the absence of friction and air resistance, there is no force in space to cause them to slow down and stop.

The **second law** is crucial to making sense of all of Newton's work. In it, he defined what exactly he meant by 'force'. This allowed force to be measured, which meant that all of his other theories could be tested and verified.

The second law can be used in combination with the law of gravitation to predict exactly how long it should take Earth – and each of the planets – to travel around the Sun. Because these 'predictions' match the reality with great precision, it means that we can trust the laws of motion, and gravitation, to be true.

F = ma

From the second law, we can see that force is proportional to the rate of change in a body's momentum. That is, for a body of mass m, changing from a velocity u to one of v:

Derivation

$$F \propto \frac{mv - mu}{t}$$

$$\text{so } F \propto \frac{m(v - u)}{t}$$

$$\text{so } F \propto ma \qquad \left(\text{as } a = \frac{v - u}{t}\right)$$

$$\text{so } F = kma$$

As this is the formula that is used to define the unit of force, the Newton, we can choose a value for k, and we choose the value of 1, so:

F = ma

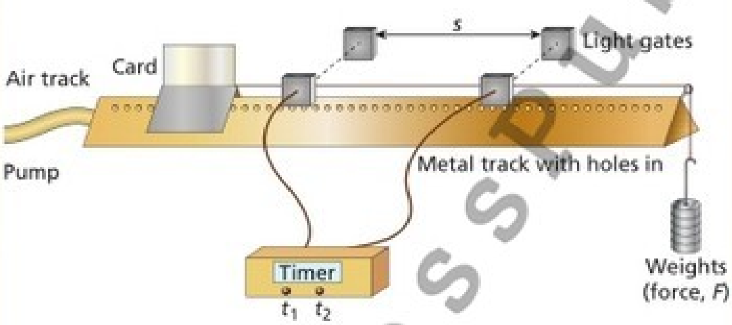
The third law is seen in effect in the use of seat belts. When a person is thrown forwards, the belt expands slightly, which extends the time over which the person slows down. This decreases the value of a, the acceleration, and with it reduces the value of F, the force on the person.

From the second law, we can take a definition of force:

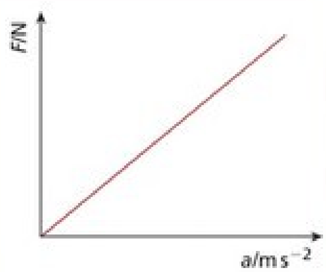
> A force is anything that causes or tends to cause an acceleration.

2.4 *Air bags and seat belts reduce the forces experienced during a crash*

Experiment 2.1: Verification that acceleration is proportional to force (i.e. $a \propto F$)



2.5 *Experimental apparatus*



2.6 *A straight line through the origin verifies that acceleration is proportional to force*

Method

1. Set up the apparatus as shown in figure 2.5.
2. Set the weights (F) and release the vehicle from rest.
3. Calculate the initial velocity, *u*, and the final velocity, *v*.
4. Find the acceleration, using $a = \frac{v^2 - u^2}{2s}$.
5. Remove one 1-N disc from the slotted weight, attach it to the vehicle, and repeat.
6. Continue for a number of values of F, and record the results.
7. Draw a graph of F in N against a in $m s^{-2}$.

Results and conclusions

A straight line through the origin shows that, for a constant mass, the acceleration is proportional to the applied force. (The mass of the system is given by the slope of the line.)

Accuracy

- The weights are transferred between the string and the vehicle to keep the total mass constant. They must be stuck onto the vehicle so that they will move with it.
- The air track reduces friction, improving accuracy.

2.1 Sample Question

A mass of 12kg is made to accelerate at $3ms^{-2}$. What is the magnitude of the force acting on it?

Sample Answer

$F = ma$

$= 12 \times 3$

$= 36N$

2.2 Sample Question

A force of 150 N acts on a body of mass 18 kg. What is the acceleration?

Sample Answer

$F = ma$

$a = \frac{F}{m}$

$= \frac{150}{18} = 8.33\ m\ s^{-2}$

2.3 Sample Question

A car of mass 1050 kg accelerates from rest to a speed of 80 km h^{-1} over 9 s. What is the force acting on it?

Sample Answer

$v = u + at$

$v = \frac{80 \times 1000}{(60 \times 60)} = 22.22\ m\ s^{-1}$

$22.22 = 0 + 9a$

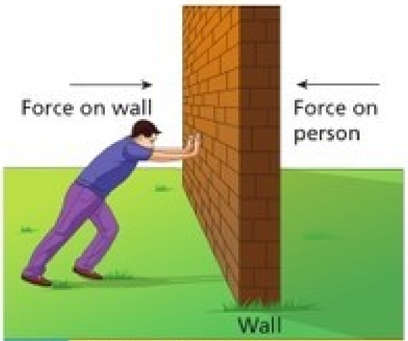
$a = \frac{22.22}{9} = 2.47\ m\ s^{-2}$

$F = ma$

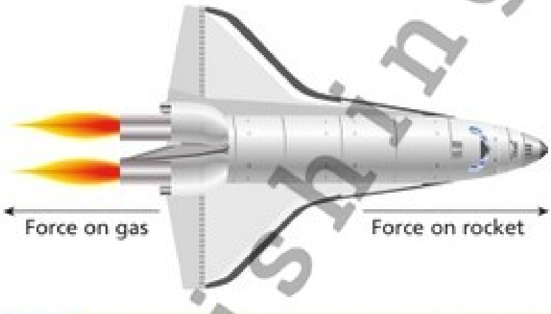
$= 1050 \times 2.47 = 2593.5\ N$

Newton's **third law** is both widely known and widely misunderstood. The common statement that 'every action has an equal but opposite reaction' is a valid way of expressing this law, but it is important to know when using it that the word 'action' here has a very specific meaning: that one body is creating a force on another. Because this is easily misinterpreted, we tend to avoid the use of the word 'action' altogether in modern books.

We encounter the third law all the time. Indeed, it is such an integral part of our daily lives that it is easy to miss it. If you push against a wall, you clearly create a force (see figure 2.7). The wall will create an equal force in the opposite direction, pushing back on you. If you push very hard, it is unlikely that the wall will fall, but it is very likely that you will be pushed into an upright position. This is the effect of the 'reaction', the force created by the wall.



2.7 *Newton's third law*



2.8 *The third law is seen in rocket propulsion. The large force created towards the rear of the rocket creates an equal, but opposite, force forwards. The rocket moves forwards as a result*

Experiment 2.2: To demonstrate Newton's third law

Method 1

1 Connect two newton meters (spring balances), as shown in figure 2.9.
2 Pull on one of the balances and note the readings on both.

2.9 *Demonstrating Newton's third law*

Observations

The two spring balances will always show identical readings, as the forces are exactly equal in magnitude, although opposite in direction.

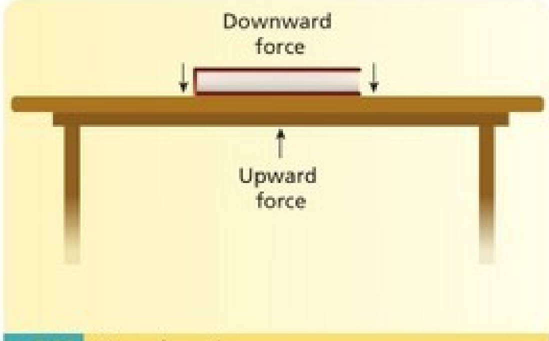
Method 2

1 Inflate a balloon and hold the end between your fingers, before letting go.
2 Observe the balloon as it flies around the room (see figure 2.10).

2.10 *Newton's third law*

Observations

The force created by the balloon on the air, forcing it through the narrow opening, is equal in magnitude to the force created on the balloon by the air, propelling it forward. Note that this is similar to the forces created during rocket propulsion in space travel.



2.11 *Normal reaction*

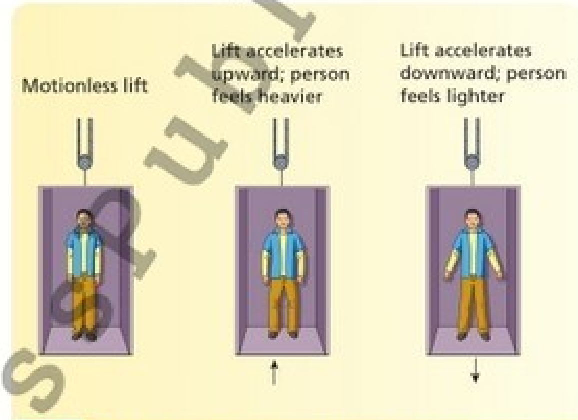
Normal reaction

When one object rests on another it is important to remember that each is creating a force on the other. If a book is sitting on a table, as shown in the figure 2.11, it creates a downward force on the table, and the table creates an equal, but opposite, upward force on the book. This is the **normal reaction**.

If you stand on the ground, your weight is acting downwards, and this causes you to create a downward force on the ground. At the same time, the Earth is creating an upward force on you.

Again, this is the normal reaction. The force that we are aware of when we think about our own weight is in fact the normal reaction, not the weight itself. This is obvious when you think about it: if you jump off a height, your weight doesn't disappear while you are falling, but you are unaware of any force acting on your body until you hit the ground, when the normal reaction makes itself felt.

It is also the normal reaction that creates the reading on a weighing scale and allows us to, indirectly, measure our own weight.



2.12 *The effects of the normal reaction in a lift*

The normal reaction has an effect in lifts (see figure 2.12). If a lift is motionless, or moving at constant velocity, your weight creates the force acting on the floor, which is equal to the normal reaction. If you were standing on a weighing scales, the reading would be equal to your weight. If the lift accelerates upwards, however, the normal reaction is greater than your weight, and this causes you to feel temporarily heavier, a sensation that would be matched by an increased reading on the scales. Similarly, if the lift accelerates downwards, the normal reaction is reduced, the reading on the scales is reduced, and you, briefly, feel lighter.

When astronauts are in orbit aboard the International Space Station (ISS), they are about 400 km above the surface of the Earth. This height is enough to reduce their weight, but only by a very small amount. They feel weightless, however, because of the absence of the normal reaction.

2.13 *Astronauts on the ISS experience a feeling something similar to being in a freefalling lift. Orbit is often described as being in 'perpetual freefall'*

Their situation is a little like being in a lift that is falling freely at the acceleration caused by gravity. If you can imagine yourself in such a situation, you would be able to float about inside the lift as if you were weightless. Remember that when a lift accelerates downwards, you feel lighter for a moment because the normal reaction is reduced. So, when the lift is in freefall, the normal reaction is reduced to zero.

2.4 Sample Question

A man of mass 70 kg is standing on a weighing scales in a lift. When the lift is motionless, the reading on the scales is 686 N. What is the reading when the lift is:

(a) Travelling upwards with a constant velocity of 4 m s^{-1}

(b) Accelerating upwards with an acceleration of 2 m s^{-2}

(c) Travelling downwards and slowing down with a deceleration equal to 1.5ms^{-2}?

Sample Answer

(a) Constant velocity, so no acceleration. The reading on the scales:
Weight = 70 × 9.8 = 686 N

(b) Accelerating upwards, so the reading on the scales is increased (he feels heavier):
Weight = 686 + (70)(2) = 826 N

(c) Accelerating downwards, so the reading on the scales is decreased (he feels lighter):
Weight = 686 – (70)(1.5) = 581 N

Friction is a force that tends to oppose relative motion. It is encountered whenever one body slides, or attempts to slide, across the surface of another. It is both beneficial and problematic: one the one hand, without friction we could not walk or drive but would instead slide about the world; on the other hand, friction increases wear in machinery and reduces efficiency.

For you to try

1 State Newton's first law.

2 If a cyclist is travelling at a constant speed (say, 6 m s^{-1}), is there any net force acting on him?

3 A parachutist is falling with a constant speed of 2 m s^{-1}. Her weight is 90 kg. What is the total upward force acting on her?

4 State Newton's second law.

5 State Newton's third law.

6 A mass of 25 kg is made to accelerate at 8 m s^{-2}. What is the magnitude of the force acting on it?

7 A force of 150 N acts on a body of mass 18 kg. What is the acceleration?

8 A car of mass 900 kg is simultaneously experiencing two forces, as shown in figure 2.14. What is its acceleration?

2.14 *Question 9*

9 A car accelerates from rest to 27 m s^{-1} due north in 6 s. Its mass is 1250 kg. What is the net force acting on it?

10 A bullet of mass 30 g is travelling at 280 m s^{-1} when it strikes a tree. It travels 9 cm into the tree before coming to rest. What is the average force created by the tree on the bullet?

2.15 *Question 11*

2.16 *Question 12*

11 An arrow has a mass of about 65 g. If a bow could create forces of 400 N on this arrow when the string was pulled back by 70 cm, with what speed would an arrow leave the bow?

12 A catapult-type device is used on aircraft carriers to accelerate aircraft from $0\,m\,s^{-1}$ to $60\,m\,s^{-1}$ in a distance of 80 m. What force does this create on an aircraft of mass 14 tonnes?

13 A woman of mass 60 kg is standing on a weighing scales in a lift. When the lift is motionless, the reading on the scales is 588 N. What is the reading when the lift is:

(a) Accelerating upwards with an acceleration of $3\,m\,s^{-2}$

(b) Travelling upwards with a constant velocity of $5\,m\,s^{-1}$

(c) Travelling downwards and slowing down with a deceleration equal to $2\,m\,s^{-2}$

(d) Falling with an acceleration equal to $9.8\,m\,s^{-2}$?

Gravity

Remember that in his third law of motion Newton said that whenever one body creates a force on another, the second body will also create an equal but opposite force. Hold a pen in front of you and think about the forces acting on it. The pen has weight, and you know already that this is a force created by the Earth. But if the Earth is creating a force on the pen, where is the other force, predicted by Newton's third law, that must match it?

2.17 *Both the Earth and the pen experience a force*

The answer, surprising in some ways, is that the pen is also creating a force on the Earth. Just as the pen is pulled down to the Earth, the Earth is pulled upwards towards the pen, and the two forces are exactly equal in magnitude, although opposite in direction.

This can be hard to accept, but remember that while the two forces are equal in size, they need not be equal in effect. The pen, with a mass of a few grams, is clearly going to fall downwards due to this force. The Earth, by contrast, with a mass of 6×10^{24} kg, is not going to move much as the result of such a tiny force.

The idea that forces occur in pairs, and that this idea applies even to gravity, is the essence of Newton's law of gravitation:

Newton's law of gravitation states that the force of attraction between any two point masses is directly proportional to the product of the masses, and inversely proportional to the square of the distance between them:

$$F = \frac{Gm_1m_2}{d^2}$$

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This law tells us that all objects with mass create gravity, and the size of the gravitational force is determined by the masses of the bodies and by the distance between them.

Two masses, each of 1 kg, sitting on a table in front of you are attracted to each other by gravity, just as they are attracted downwards towards the centre of the Earth. However, we have discovered that the force between them is very, very small.

Gravitational forces are generally small unless very large objects – such as the Earth or another planet – are involved. The gravitational force created by the Earth – or other planets – on an object is what we call its 'weight'. This is due to the size of G, the gravitational constant, which is usually given as $6.7 \times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2}$.

We use the concept of centre of gravity when finding the distance between two objects and calculating gravitational forces. For large spheres such as the Earth, this means that we measure all distances from the centre of the Earth.

Isaac Newton

Isaac Newton lived from 1642 to 1727. He showed no aptitude for the farming life into which he had been born and was sent to school and to university instead, where it was hoped he might take religious orders and manage to provide for himself. Instead he dedicated himself to his studies and became the central figure in the scientific revolution, making contributions to many areas of science and mathematics that are covered throughout this course.
Newton was an intense and difficult figure. He had few friends and never married. He was deeply uncomfortable with any form of criticism and tended to hold a grudge against anybody who questioned his work. At the same time, he was jealous of other scientists and often quarrelled with his contemporaries. It is even possible that he was only driven to publish his great work on gravity by the fear that others would make similar discoveries and would take the credit.

He spent many years away from his more successful scientific studies working on such matters as alchemy and studying the Bible with great intensity. He seems to have been a devout but unorthodox Christian, who never took holy orders. He also served in Parliament for a period and in later life he was appointed to run the Royal Mint – the British institution responsible for the printing of money.

Inverse square laws

Newton's law of gravitation is an example of an inverse square law. This means that the forces created are inversely proportional to the square of the distance between them. Mathematically:

$$F \propto \frac{1}{d^2}$$

There are other laws that follow a similar pattern, describing for example the forces created through magnetism and electricity. Finding a link between them all is one of the great quests of modern physics. Einstein spent the last few years of his career attempting to find such a link, and others have tried since, but no one has so far succeeded in finding one.

2.5 Sample Question

What is the gravitational force created by two masses of 1 kg, placed 50 cm apart?

Sample Answer

2.18

$$F = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6.7 \times 10^{-11})(1)(1)}{(0.5)^2}$$

$$= 2.68 \times 10^{-10}\text{N}$$

2.6 Sample Question

What is the weight of a man of mass 80 kg, when he is standing on the surface of the Earth?

Radius of Earth = 6.4×10^6 m
Mass of Earth = 6×10^{24} kg

Sample Answer

$$F = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6.7 \times 10^{-11})(80)(6 \times 10^{24})}{(6.4 \times 10^6)^2}$$

$$= 785.16\text{N}$$

2.7 Sample Question

A woman has a weight of 600 N on the surface of the Earth. What weight would she have on a planet with a radius three times that of the Earth, and a mass twice that of the Earth?

m_e = mass of Earth
m_w = mass of woman
r_e = radius of Earth

Sample Answer

On Earth:

$$F = \frac{Gm_em_w}{r_e^2} = 600\text{N}$$

On other planet:

$$F = \frac{Gm_{planet}m_w}{(r_{planet})^2}$$

$$F = \frac{G2m_em_w}{(3r_e)^2}$$

$$= \frac{2}{9}\left(\frac{Gm_em_w}{r_e^2}\right)$$

$$= \frac{2}{9} \times 600$$

$$= 133.33\text{N}$$

Henry Cavendish

2.19 Part of Cavendish's apparatus

Greek philosophers found a way of measuring the circumference of the Earth, using astronomy, as far back as 300 BC. Over time, that measurement has become more and more accurate. But how do we know the mass of the Earth?

British scientist Henry Cavendish (1731-1810) devised an experiment in 1798 to establish the mass of the Earth. He used a balance in which two lead balls hung from a 2 m-long arm. Gravitational attraction caused the balance to swing towards another pair of heavier lead balls. The strength of this force could be measured by measuring how far the balance rotated.

In modern terms, this allowed us to find a value for G, the gravitational constant.

For you to try

> Radius of Earth = 6.4×10^6 m
> Mass of Earth = 6×10^{24} kg

1 State Newton's law of gravitation.

2 If a ball is kicked into the air, the Earth creates a force on the ball that acts downwards. According to Newton's third law, an equal but opposite force is also created. What is this force, and on what object does it act?

3 What is an inverse square law?

4 A can of coffee has a mass of 454 g. What is its weight?

5 A person has a weight of 735 N on Earth. What is their weight on another planet, with a mass three times that of the Earth, and a radius twice that of the Earth? (Try to do the question without using the given values for the mass and radius of the Earth.)

6 Two objects, each of mass 10 kg, are situated 2 m apart in space.
   - **(a)** What is the gravitational force on each of them?
   - **(b)** What acceleration would this create on either one of the masses?
   - **(c)** How long would it take until the objects collided?

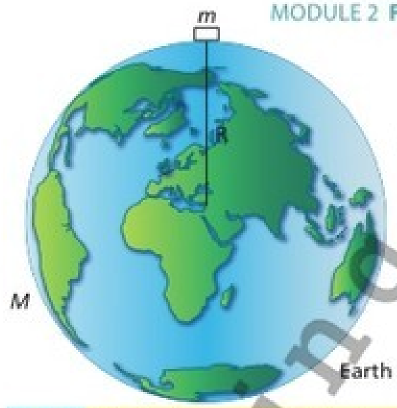
> Radius of Moon = 1.7×10^6 m
> Mass of Moon = 7×10^{22} kg
> Radius of Mars = 3.39×10^6 m
> Mass of Mars = 6.46×10^{23} kg

7 A neutron star has a mass about the same as that of our Sun, 2×10^{30} kg, and a radius of 5×10^3 m. If an object of mass 10 kg were on the surface of the star, what gravitational force would it experience?

8 The Hubble Space Telescope has a mass of 11 600 kg. What is its weight when it is:
   - **(a)** On Earth
   - **(b)** In orbit, at a height of 600 km above the Earth?

Weight

Newton's law of gravitation offered mathematical support to theories that Galileo had developed years earlier. In particular, Galileo had argued that, when we can ignore air resistance, all objects will fall at the same rate. He had done experiments to show that this was true, but Newton's work offered further verification. We can see how Newton did this if we think about a body of mass, m, on the surface of the Earth (with mass *M*) (see figure 2.20).



2.20 *A body of mass* m *on the Earth*

The weight of the body is how we describe the force acting on the body and pulling down towards the centre of the Earth. There are two separate ways that we can look at this force mathematically.

Firstly, we can use Newton's law of gravitation, which gives us:

Derivation

$$F = \frac{GMm}{R^2}$$

However, the weight is also a force, so we can also calculate the value of the weight using Newton's formula, $F = ma$. This gives us:

$$F = mg$$

Equating these two, we can see that:

$$mg = \frac{GMm}{R^2}$$

If we simplify this, we get:

$$g = \frac{GM}{R^2}$$

This equation shows us that, as Galileo had predicted, the acceleration of a falling body does not depend on the mass of the body, but only on the mass and the radius of the Earth.

We usually take the value of g to be $9.8\,m\,s^{-2}$, but this figure is not a constant, as its value is dependent on the value of R, the radius of the Earth, which is itself not a constant.

This means that if you are on top of a high mountain, for example, you are further from the centre of the Earth than you would be at sea level, and the value of g is therefore a little smaller. Also, remember that the Earth is not a perfect sphere, but that it is slightly flattened at the poles, and bulges a little at the equator. This means that the value of g is lower on the equator, where it is $9.78\,m\,s^{-2}$, than it is at the north or south pole, where it is $9.83\,m\,s^{-2}$.

It is also important to note that the weight of a body is given by:

$$W = mg$$

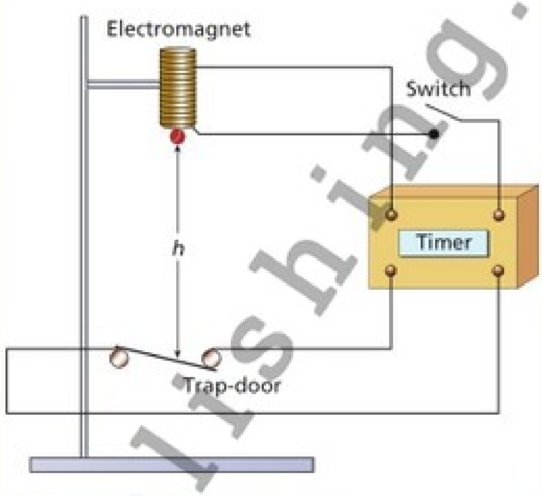
This is really a rewriting of $F = ma$, but because weight is such a key measurement, we often use this version of the formula.

Also, you can see from the equation that a body is almost never truly weightless. Although when it is far from any planet the value of g – and therefore the weight – may be very small, it will never be zero. In a spacecraft in orbit, both astronauts and shuttle are caught in what is essentially perpetual freefall. The astronauts appear weightless, but the acceleration due to gravity in those orbits is actually close to its value on the surface of the Earth (generally, it is about $8.7\,m\,s^{-2}$).

Experiment 2.3: Measurement of g (by freefall)

Method

1 Set up the apparatus as shown in figure 2.21. The millisecond timer starts when the ball is released and stops when the ball hits the trap-door.
2 Measure the height, h, as shown, using a metre stick.
3 Release the ball and record the time, t, from the millisecond timer.
4 Repeat three times for this height, h. Take the smallest time as the correct value for t.
5 Repeat for different values of h.



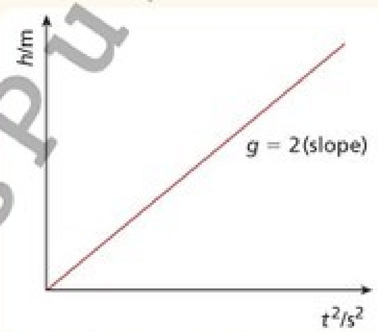
2.21 *Experimental apparatus*

Results and Conclusions

Calculate the values for g using the equation:

$$h = \frac{1}{2} g t^2$$

Obtain an average value for g.
Alternatively, draw a graph of h against t^2 (figure 2.22).



2.22 *Height against t^2*

Accuracy

- The shortest of the three times is taken as t, as various factors can delay the falling of the ball, but nothing should speed it up. A piece of paper between the ball bearing and the electromagnet will help ensure a quick release.
- Larger values of h will decrease the percentage error.

2.8 Sample Question

What is the acceleration due to gravity on the surface of the Earth?

Sample Answer

Radius of Earth = 6.4×10^6 m
Mass of Earth = 6×10^{24} kg
$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}

$$g = \frac{Gm}{d^2}$$

$$= \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(6.4 \times 10^6)^2}$$

$$= 9.81 \text{ ms}^{-2}$$

2.9 Sample Question

What is the acceleration due to gravity on the International Space Station, about 400 km above the Earth?

Sample Answer

$$g = \frac{Gm}{d^2}$$

$$= \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(6.4 \times 10^6 + 4 \times 10^5)^2}$$

$$= 8.69 \text{ ms}^{-2}$$

For you to try

1 Calculate the acceleration due to gravity on the Moon's surface, given that the mass of the Moon is 7×10^{22} kg and the radius of the Moon is 1.7×10^6 m.

2 Show how to derive the formula $g = \frac{Gm}{d^2}$

3 Geostationary orbits are all positioned at a height of 35 900 km above the surface of the Earth. What is the acceleration due to gravity at that height?

4 The Andromeda Galaxy has a mass of 6×10^{41} kg.

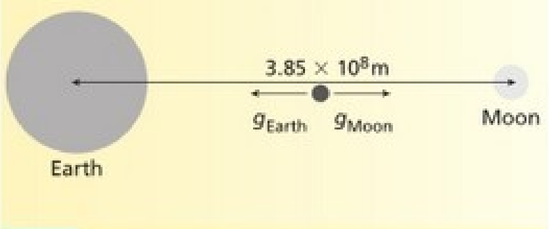
(a) What is the acceleration due to gravity created by that mass on our galaxy, the Milky Way, which is 2×10^{22} m away?

(b) What is the total force on the Milky Way, which has a mass of 7×10^{41} kg, towards the Andromeda Galaxy?

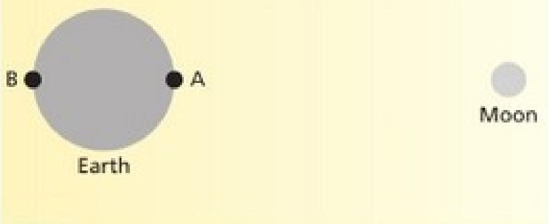
5 How far from the centre of the Earth would you have to be for the acceleration due to gravity to be equal to 1 m s^{-2}? How high above the surface of the Earth is this?

6 Figure 2.23 shows the Earth and the Moon, a distance of 3.85×10^8 m apart. At what distance from the Earth does the acceleration due to gravity created by the Earth equal that created by the Moon?

7 The distance from the centre of the Moon to the centre of the Earth is 3.85×10^8 m. The Earth has a radius of 6.4×10^6 m. The Moon creates a gravitational force on Earth and hence an acceleration caused by gravity, g_m. What is the difference in the values of g_m at the two points shown in figure 2.24?



2.23 *Question 6*



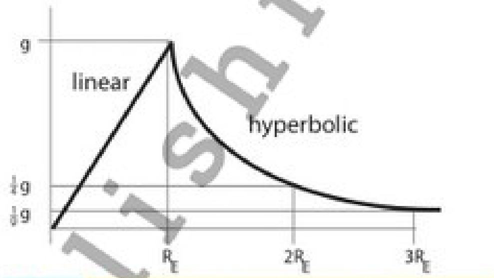
2.24 *Question 7*

Gravitational Field Strength away from the Earth's surface

The gravitational field strength decreases as the inverse square of the distance from the centre of gravity of the mass that is causing the gravitational field around it. Normally we only consider the gravitational field around the largest object (the Earth or the Sun), but every mass creates a gravitational field around it.

The strength of the gravitational field also decreases as we tunnel deep underground because, once underground, some of the mass is attracting upwards. At the centre of the Earth, there is as much mass attracting downwards as there is upwards. So the gravitational field strength drops to zero.

The figure shows how the gravitational field strength varies both below and above the radius of the Earth. Only very close to the radius of the Earth the gravitational field strength is equal to g (~9.81 m s^{-2}). At twice the Earth's radius the gravitational field strength drops to $g/4$, and decreases more and more as the distance is increased.



2.25 *Variation of the gravitational field strength inside and outside the Earth*

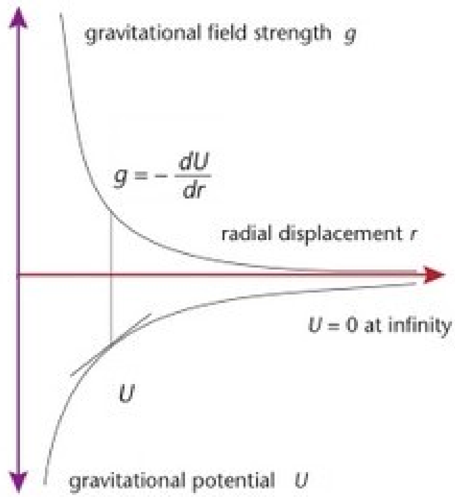
This shows clearly why the most difficult part of launching a rocket into space is the first part of the journey. As the journey progresses it becomes easier and easier for the rocket to accelerate away from the attraction of the Earth.

Close to the Earth's surface we say that gravitational potential energy $E_{pg} = m\,g\,h$ where m is the mass, g is 9.81 ms^{-2} and h is equal to the height above sea level. However, when we consider much greater differences we can no longer use this definition. Instead we define gravitational potential energy as being zero at infinity! This may seem like a strange thing to do, but large distances away from anywhere (approximately infinity) are what is outer space has most of; so most of outer space has a potential of zero.

Compared to the potential a long way away from the Earth, the gravitational potential closer to Earth is negative. That means that a mass that is inside the Earth's gravitational field has an 'energy debt'. By that we mean that it needs to be given additional energy if it is ever going to escape from the Earth's gravitational field.

Another way of looking at this is to say that the potential at any point inside a gravitational field is the amount of work that the gravitational force has done in order to bring an object all the way from infinity up to the point where it currently sits.

Mathematically the gravitational potential is equal to the negative of the slope of the gravitational field strength. This is shown clearly in the figure below.



2.26 *Relationship between the gravitational field strength and the gravitational potential*

Mathematically we say:

$$F_g = \frac{GMm}{r^2} \qquad U_g = \frac{GMm}{r}$$

The force is directed in the opposite direction to which the distances are measured. The potential is always negative, it is only zero at infinity.

Module 3 Circular and Simple harmonic motion

Learning objectives

- To apply the basic equation of gyrodynamics in different forms in problem-solving 10.2.2.6
- To define a radius of path curvature, tangental, centripetal and complex acceleration of a body when in curvilinear motion 10.2.1.5
- To investigate physical measurements characterizing translation and rotational motions 10.2.2.7
- To determine the moment of body inertia through the experimental method 10.2.2.8
- To use Steiner's theorem to calculate the moment of inertia of material bodies 10.2.2.5
- To apply conservation laws when solving calculation and experimental problems 10.2.4.1

Circular motion

There are many situations in which we see a moving object follow a circular path. Examples include an object tied to the end of a string, a planet in orbit, the rotating drum of a washing machine or the wheel of a car. These are all very different, but there are very simple mathematical rules that they all obey, regardless of the exact process that causes them to follow a circular path. In this part of the module, we look at the maths governing circular motion.

3.1 *Wind turbines, one of many examples of objects in circular motion*

Linear and angular velocity

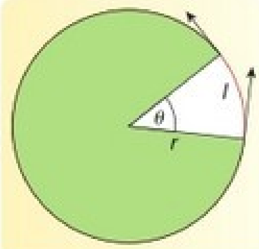
We can measure the velocity of an object travelling in circular motion in two very different ways. The **linear velocity** is measured in the same way as it would be for an object following a straight-line path. Essentially, linear velocity (and speed), v, is found using the formula:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

or

$$v = \frac{l}{t}$$

For an object following circular motion, the distance travelled, *l*, forms the arc of a circle, as shown in figure 3.2.

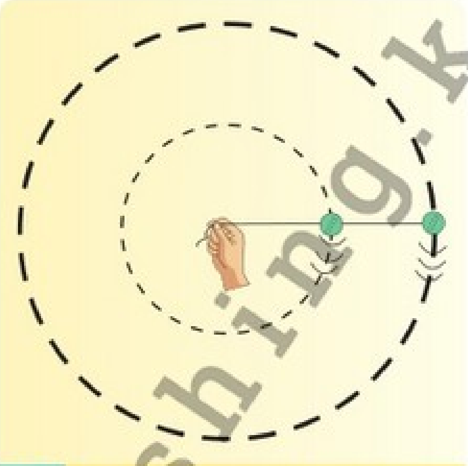


3.2 *Circular motion*

Instead of measuring the distance the object moves every second, however, we could also measure the angle, θ, through which it moves every second. This is known as the **angular velocity**, usually denoted by the letter ω and measured in radians per second, rad s^{-1}. To calculate it, we use this formula:

$$\omega = \frac{\theta}{t}$$

Linear velocity (v) and angular velocity (ω) are connected. Think of a string being rotated in a circle onto which two objects have been attached at different points, as shown in figure 3.3. Both objects are moving through the same angle each second, and therefore have the same angular velocity. However, the object at the end of the string is travelling through a greater distance every second and, therefore, has a larger linear velocity than the object closer to the centre of the string. The greater the radius of the motion, the greater the linear velocity tends to be.



3.3 *Two objects attached to a single string*

Connecting v and ω

The connection between linear and angular velocity can also be seen mathematically:
We know that linear velocity follows this formula:

$$v = \frac{l}{t}$$

However, from your studies of maths, you also know the arc of a circle, l, is connected to the angle in radians at its centre by this formula:

$$l = r\theta$$

Substituting this into the above formula:

Derivation

$$v = \frac{r\theta}{t}$$

$$v = r\frac{\theta}{t}$$

But as $\omega = \frac{\theta}{t}$ we can therefore say:

$$v = r\omega$$

Period

Objects travelling in circular motion repeat the same motion over and over again. With any situation like that, it can be useful to measure how long exactly it will take for one complete motion, or cycle, to be completed. This is time is known as the **period** of the motion.

We can do this by rearranging the formula for angular velocity, which gives us:

$$t = \frac{\theta}{\omega}$$

When an object travels through one complete circle, the angle through which it has moved is 2π radians. This means that the period, T, of a particle in circular motion is given by:

$$T = \frac{2\pi}{\omega}$$

We can also talk about the frequency of the motion. The frequency of a circular motion measures the number of full rotations, or circles, travelled in one second. It is measured in hertz.

Just as is the case when we analyse wave motions, the frequency and period of circular motion are related to each other:

$$f = \frac{1}{T}$$

3.1 Sample Question

An object is in circular motion with an angular velocity of $9\,\text{rad}\,\text{s}^{-1}$ and a radius of 20 cm. What is its linear velocity?

Sample Answer

$$v = r\omega$$
$$= 9 \times 0.2$$
$$= 1.8\,\text{m}\,\text{s}^{-1}$$

3.2 Sample Question

A bicycle wheel of radius 45 cm spins so that it completes three complete rotations each second.

(a) What is the frequency of the motion?

(b) What is the period of the motion?

(c) What is the linear velocity of the object?

Sample Answer

(a) Three rotations per second is the frequency, i.e. $f = 3\,\text{Hz}$.

(b) $T = \frac{1}{f} = \frac{1}{3} = 0.33\text{ s}$

(c) $T = \frac{2\pi}{\omega}$

so $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.33} = 19.04\,\text{rad}\,\text{s}^{-1}$

$$v = r\omega$$
$$= (0.45)(19.04)$$
$$= 8.57\,\text{m}\,\text{s}^{-1}$$

3.3 Sample Question

When vinyl records were the main method by which people listened to music, a 'single' had a radius of 8.9 cm and spun at 45 rpm (revolutions per minute).

(a) What was the period of its motion?

(b) What was the linear velocity of a point on its edge?

Sample Answer

(a) f = 45 revolutions per minute, so revolutions per second:

$$= \frac{45}{60} = 0.75\,\text{Hz}$$

$$T = \frac{1}{f} = \frac{1}{0.75} = 1.33\,\text{s}$$

(b) $T = \frac{2\pi}{\omega}$

so $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.33} = 4.72\,\text{rad s}^{-1}$

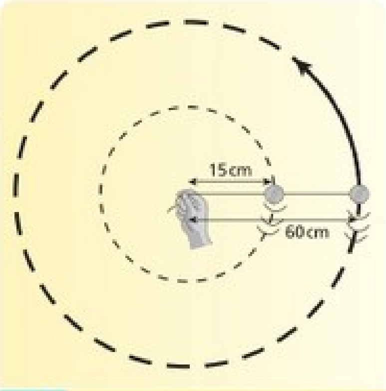
$v = r\omega$

$= (0.089)(4.72)$

$= 0.42$ ms-1

For you to try

1. What do we mean by the term 'angular velocity'?
2. What is the relationship between angular velocity and linear velocity?
3. An object is in circular motion with an angular velocity of $5\,\text{rad s}^{-1}$ and a radius of 15 cm. What is its linear velocity?
4. A wheel of radius 50 cm rotates so that a point on its surface is travelling at $4\,\text{m s}^{-1}$. What is the angular velocity of the wheel?
5. A string of length 60 cm is being spun in a circle with an angular velocity of $6\,\text{rad s}^{-1}$. It has two weights attached to it, one at a distance of 15 cm from the centre and one at the edge, as shown in figure 3.4. What is the linear speed of each of the weights?



3.4 Question 5

6. What do we mean by the 'period' of a circular motion?
7. A standard vinyl record has a radius of 15 cm and spins at 33 rpm (see figure 3.5). What is its angular speed?
8. **(a)** The Earth spins about its axis once each day. Given that the radius of the Earth is 6400 km, what is the linear speed of an object placed on the equator due to this motion?
   (b) The Earth also rotates about the Sun once each year. The distance to the Sun is 150 million kilometres. What is the speed of the Earth due to this motion?

3.5 Question 7

Centripetal acceleration

At every moment while an object is travelling in a circle, its direction is at a tangent to that circle: for example, if a weight is tied to the end of a piece of string and is rotated in a circle and then released, the weight will fly off at a tangent to the circle. However, as the object travels in the circle, its direction constantly changes. Remember that velocity is a measure of both speed and direction. This means that an object travelling in a circle may have a constant speed but, because its direction is always changing, it cannot have a constant velocity.

Remember too that acceleration is the rate of change of velocity, and this means that any object with a changing velocity (including an object travelling in a circle) has an acceleration. We refer to the acceleration of an object travelling in a circle as **centripetal acceleration**.

Acceleration is a vector quantity and, therefore, must always have a direction. The direction of centripetal accelerations is always towards the centre of the circle.

Centripetal acceleration is the acceleration of an object in circular motion. Its direction is towards the centre of the circle.

These formulae are derived using vector addition and subtraction.

The value of the centripetal acceleration is related to both the linear and angular velocity of an object, and to the radius of motion, using the formulae:

$$a = \frac{v^2}{r} \text{ and } a = r\omega^2$$

Centripetal force

We know from Newton's second law, and the formula $F = ma$, that whenever there is an acceleration, there must also be a force. The force experienced by an object in circular motion is known as the **centripetal force**.

Centripetal force is the force on a body in circular motion. Its direction is towards the centre of the circle.

As $F = ma$, we can say that the centripetal force is given by:

$$F = m\frac{v^2}{r} \text{ and } F = mr\omega^2$$

It is important to remember that any force that ensures an object travels in a circle is a centripetal force. This means that when a weight is tied to the end of a string and is rotated at speed, the tension in the string is the centripetal force. And when a disk drive spins in a computer, the centripetal force is created by the electric motor. And when a planet rotates around the Sun, or a satellite rotates around the Earth, the centripetal force is created by gravity.

These situations differ from each other in many details, but the mathematical formulae we have looked at in this module apply to all of them.

3.4 Sample Question

A bicycle travels around a circular bend with a radius of 12 m. It has a constant speed of 15 m s^{-1}. What is its centripetal acceleration?

Sample Answer

$$a = \frac{v^2}{r}$$
$$= \frac{15^2}{12} = 18.75\,\text{m}\,\text{s}^{-2}$$

3.5 Sample Question

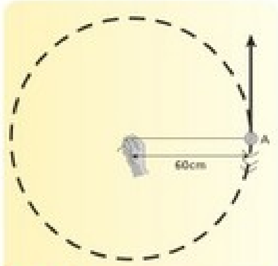
One section of a roller-coaster involves a circle of radius 9 m. A car of mass 300 kg travels at 10 m s^{-1} on this section. What centripetal force does it experience?

Sample Answer

$$F = m\frac{v^2}{r}$$
$$= 300 \times \frac{10^2}{9} = 3333\ \text{N}$$

For you to try

1 What is meant by 'centripetal acceleration'?
2 What is the direction of centripetal acceleration?
3 What is meant by 'centripetal force'?
4 If an object travels in a circle with a constant speed, does it have an acceleration? Explain your answer.
5 A car is travelling around a circular bend with a radius of 20 m. It has a constant speed of 22 m s^{-1}. What is its acceleration?
6 The drum of a washing machine has a radius of 25 cm and spins at 1200 rpm during its spin cycle.
(a) What is its angular velocity?
(b) If it has a mass of 1.5 kg, what is the centripetal force involved?
7 A mass of 500 g is tied to the end of a string and is spun in a circle of radius 60 cm, as shown in figure 3.6.
(a) If its angular velocity is 9 rad s^{-1}, what is its linear velocity?
(b) The mass is released at point A, when it is 1.25 m above ground level and travelling vertically upwards. On a diagram, show what path it then follows. What is the greatest height that it reaches?
8 A model plane is being flown attached to a string that can withstand up to 180 N of tension. The mass of the plane is 750 g, and it travels at 15 m s^{-1}. Assuming the string is horizontal, what is the radius of the smallest circle in which the plane can be flown?



3.6 Question 7

Satellites

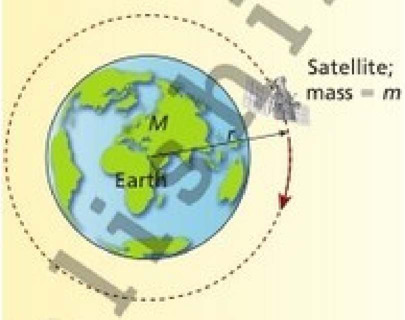
As we have seen, there are many different processes by which an object can be made to travel in a circular path. A satellite is any object travelling in orbit around another object. Examples are the motion of the Earth and the other planets around the Sun, the movement of the Moon around the Earth, and the movement of the many thousands of artificial satellites that have been placed in orbit around the Earth over the last few decades. All of these are maintained in their circular paths because of the effect of gravity.

This means that for satellites, the centripetal force is created by gravity. And it also means that the formulae we have to describe gravitational forces, from Newton's law, must agree with the formulae we derived for centripetal forces.

Period of satellites

Think of an artificial satellite of mass *m* travelling in an orbit of radius *r* around the Earth (which has mass, *M*), as shown in figure 3.7.

When an object is in orbit, the centripetal force it experiences is created by gravity. This means that the force can be described either by our formula for the centripetal force, or our formula for the gravitational force. The two must be equal.



3.7 *A satellite in orbit around the Earth*

Derivation

$$\frac{GMm}{r^2} = mr\omega^2$$

Also, remember that in terms of the period, *T*, of a motion:

$$\omega = \frac{2\pi}{T}$$

$$\frac{GMm}{r^2} = mr\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow T^2 = \frac{4\pi^2 mr^3}{GMm}$$

Or, simplifying:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

This is a very interesting result. If we arrange it to just look at the variables, it can be further simplified as:

$$\boldsymbol{T^2 \propto r^3}$$

This gives us the relationship between the period of a satellite and the radius of its motion. It is very important historically, because it fits in with Kepler's law.

Kepler had studied the motion of the planets for many years (see the box on Kepler opposite) and he had finally arrived at exactly this mathematical relationship between the period and radius of their motions.

When Newton could then do the maths shown here and prove that his gravitational law fitted in exactly with what Kepler had already established, and with the observed motion of all of the planets in the solar system, it meant that his equations could be trusted. They had survived the crucial scientific test of being verifiable by experience.

Another way of looking at the motion of a satellite is to think of its speed rather than its period. As we have argued above, the centripetal force is equal to the gravitational force:

$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Simplifying this equation yields:

$$v^2 = \frac{GM}{r}$$

The values of G and M will not change, so this equation tells us that the velocity of an object travelling in circular motion about the Earth is controlled only by the radius of its motion. This is very important for space engineering.

Think about a situation in which astronauts in a space shuttle are given the job of making repairs to a satellite situated a few hundred metres ahead. In order to catch up with the satellite, most of us would instinctively fire up the rockets to make the shuttle go a little faster. But the problem with this is that it will push the shuttle out to a further orbit, away from the satellite.

Both equations for the motion of satellites show that the velocity of a satellite can be controlled only by the height of its orbit: to increase velocity, the satellite must reduce r – that is, it must move closer to the Earth – and to slow down it must move further away.

What the astronauts need to do is to move downwards towards the Earth a little, where the reduced value of r will yield a larger value for v. The shuttle will then quickly catch up with the satellite, and then it can return to its original orbit and carry out the repairs.

Johannes Kepler

3.8 *Johannes Kepler (1571–1630)*

The names of Nicolaus Copernicus (1473–1543) and Galileo Galilei (1564–1642) are famous throughout the world. In the popular imagination they are the men who established that the Earth is not the centre of the universe, but instead that it is rotating about the Sun. Both deserve credit, but in many ways the central figure in the story should be Johannes Kepler.

Copernicus was a Polish mathematician and astronomer who, just before his death, published a book called *On the Revolutions of the Celestial Spheres*, in which he outlined a theory that all of the planets are in orbit around the Sun. His ideas slowly spread around Europe and became the basis of a new understanding of astronomy. However, his theory did have flaws: because he believed the orbits had to be perfect circles, his maths did not quite fit the reality of the motion of the planets.

Galileo deserves credit for the development of the telescope and for his astronomical observations, particularly the discovery of the moons of Jupiter. His famous battles with the Church of the day, though, have perhaps helped to obscure the work of Kepler.

Kepler was a German scientist and mathematician. He studied the wonderfully precise observations of the planets made by Danish astronomer Tycho Brahe (1546–1601), and combined them with the work of Copernicus to produce mathematical formulae that described with great precision the motions of the planets. This work then provided Newton with a basis on which to build his famous theory of gravity a generation later.

Geostationery satellites

The first artificial satellite was launched by the Soviet Union in 1957. It stayed in orbit for three months and did nothing but constantly broadcast a series of beeps, which could be picked up from Earth.

Since then, many thousands of satellites have been placed in orbit, with many purposes. They can help with weather forecasting, spying missions, space exploration and communications.

The most obvious connection most of us have with communications satellites is the satellite dishes used on many homes to provide TV programmes. These dishes all point to the particular satellite from which they receive a signal. For this reason, it is important that the satellite stays in the same place from our perspective here on Earth.

To do this, the satellite has to move with the Earth, constantly staying above one point on the Earth's surface and moving through one complete orbit every day. Such satellites are known as **geostationary satellites**.

To match the spin of the Earth, these satellites must be above the equator and must have a period of exactly one day. Following on from the maths we've already seen, this means that they must all have the same radius of motion and travel at the same speed: they are all at a height of approximately 36 000 km above the Earth's surface, and they travel at close to $3\,\mathrm{km\,s^{-1}}$.

The need for these satellites to share an orbit can lead to disputes, which are generally dealt with through an agency that is part of the United Nations.

3.9 *Geostationary orbit*

3.6 Sample Question

(a) What is the radius of orbit of a geostationary satellite?

(b) How high above the Earth's surface is this?

(c) At what speed is it travelling?

Mass of the Earth = $6 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth = $6.4 \times 10^{6}\,\mathrm{m}$

Sample Answer

(a) $T^2 = \dfrac{4\pi^2 r^3}{GM}$

$T = 24\,\mathrm{h} = 86\,400\,\mathrm{s}$

$r^3 = \dfrac{GMT^2}{4\pi^2}$

$= \dfrac{(6.7 \times 10^{-11})(6 \times 10^{24})(86\,400^2)}{4\pi^2}$

$r = 4.23 \times 10^{7}\,\mathrm{m}$

(b) $4.23 \times 10^{7} - 6.4 \times 10^{6}$

$= 3.59 \times 10^{7}\,\mathrm{m}$

$\approx 36\,000\,\mathrm{km}$

(c) $v^2 = \dfrac{GM}{R} = \dfrac{(6.7 \times 10^{-11})(6 \times 10^{24})}{4.23 \times 10^{7}} = 9\,503\,546$

$v = 3083\,\mathrm{m\,s^{-1}}$

For you to try

1 Name three scientists who contributed to our understanding of planetary motion.
2 What is the period of a satellite that is in orbit around the Earth, with a radius of motion of 6.6×10^6 km?
3 **(a)** What is the period, in seconds, of a satellite that completes seven complete orbits of the Earth every 24 hours?
   (b) What is the radius of its motion?
4 A satellite is placed in orbit 600 km above the surface of Jupiter. Jupiter has a radius of 7.1×10^7 m and a mass of 1.9×10^{27} kg. What is the period of the satellite?
5 The International Space Station (ISS) is in orbit at a height of 400 km above the Earth's surface.
   (a) What is the period of the space station's motion?
   (b) How many orbits will it make of the Earth in a 24-hour period?
6 **(a)** What is the period of a satellite that is in orbit around the Earth and whose radius of motion is twice the radius of the Earth?
   (b) What is its speed?
7 Venus has a mass of 5×10^{24} kg. It rotates very slowly about its axis with a period of 243 days.
   (a) What is the period of an object in geostationary orbit around Venus?
   (b) What is the radius of its motion?

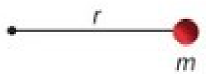
Moments of inertia in rotational motion

Just as mass has inertia, and a force is required to cause a mass to accelerate (and we say $F = ma$), mass also has a moment of inertia, and a torque is required to cause a mass to increase its angular speed. In fact there are many other similarities between linear motion and rotational motion; many of the equations looks similar, except that the symbols and meanings are different. Look at the table below:

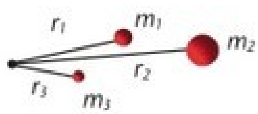
Table 1.1 Similarities between the equations of linear motion and rotational motion

	straight-line motion	rotational motion
displacement	Δx	$\Delta\theta$
velocity	v	ω
acceleration	a	β
inertia	m	I
inertia x acceleration	F (force)	τ (torque)
Newton's second law	$\Sigma F = ma$	$\Sigma\tau = Ia$
work	$W = F\,\Delta x$	$W = \tau\,\Delta\theta$
kinetic energy	$K_E = 1/2\ mv^2$	$K_E = 1/2\ I\omega^2$
momentum	$p = mv$	$L = I\omega$
impulse	$F\,\Delta t = \Delta p$	$\tau\,\Delta t = \Delta L$

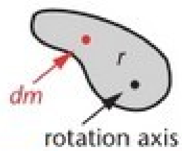
Notice that the inertia for linear motion is simply the mass, but the moment of inertia is given the symbol I. This is to reflect the fact that the ease or difficulty with which something can be made to spin round depends not only on the mass, but on the distribution of that mass. The further away the mass is situated from the axis about which it is to rotate, the harder it is to cause it to spin.



$$I = mr^2$$



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$



3.10 *Distance from rotational axis*

Distribution of masses about the axis of rotation determine the moment of inertia

Steiner's Theorem

The moment of inertia of an object about an axis going through its centre of gravity is equal to the sum of each mass element multiplied by the square of its perpendicular distance to the axis about which it will rotate.

The moment of inertia of a rigid body with respect to a specific axis is defined as :

$$I = \int r^2 dm$$

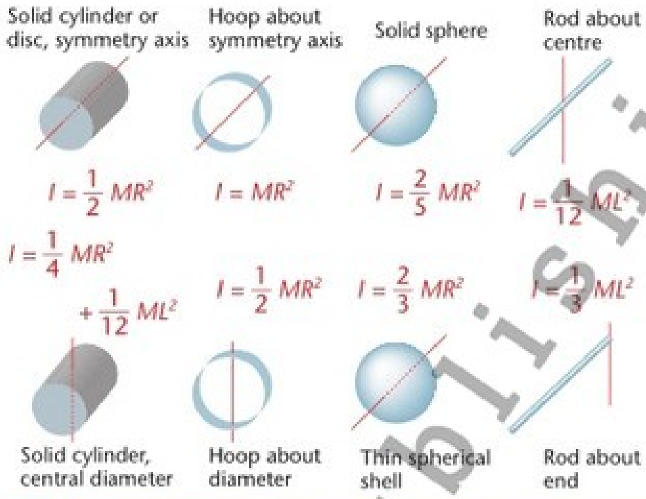
where r is the perpendicular distance of the element of mass dm to the axis.

To calculate the moment of inertia, we put the origin of the coordinate system at the location of the axis as shown in figure 3.11:

3.11 *Moment of inertia of an object about an axis*

$$I = I_0 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

The diagram above represents this in visual form. The calculation of the moment of inertia for various mathematical shapes is generally quite mathematical, and beyond the scope of this book. Some of the more common shapes are shown in the figure below. Notice that the object with the largest moment of inertia (for its mass) is the hoop rotating about its symmetry axis. That is because the hoop has all of its mass at the largest possible distance from the axis of rotation.



3.12 *Moments of inertia for various common shapes when made to rotate about different axes*

For you to try

1. A solid cylinder and a sphere both have a mass of 2 kg and a radius of 5 cm. Calculate their moment of inertia in units of kg m².
2. A constant torque is applied to an axis running perpendicularly through the midpoint of a solid rod, and it causes the rotation to accelerate at a rate of 5 rad/s. At what rate would the rotation accelerate if the axis of rotation was about one end of the rod?
3. A circular flat disc of constant thickness has a mass of 2.0 kg and a diameter of 60 cm. What is its moment of inertia? If a disc 30 cm diameter were to be cut out concentrically from the original disc, what would the moment of the inertia of the annular disc be? (Hint: The moment of inertia of the original disc is equal to the sum of the moments of inertia of the smaller disc and the larger annular disc).

Use information from the diagram above.

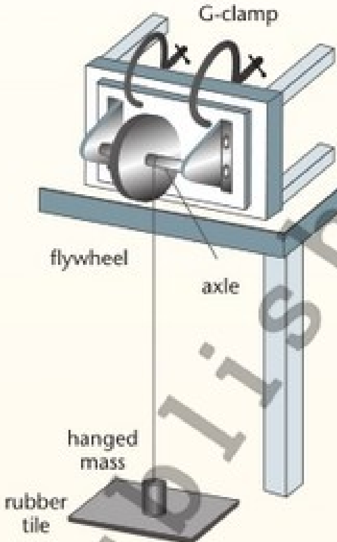
The moment of body inertia

A convenient way to measure the moment of inertia of a flywheel experimentally, is by causing it to spin by means of a weight attached by a string wrapped round its axle as shown in the figure below.

Experiment 3.1: To measure the moment of inertia of a flywheel

Method

1 Set up the equipment as shown in the diagram.
2 We will assume that the bearings of the flywheel are of such good quality that the frictional losses are negligible. However, it is possible to take that into account later.
3 Measure the diameter of the spindle around which the string will be wrapped, so that the linear velocity v of descent of the mass is related to the angular velocity ω of the flywheel by the expression $v = \omega r$, where r is the radius of the spindle.



3.13 *Experimental apparatus to measure the moment of inertia of a flywheel*

Calculation

We will assume that the bearings of the flywheel are of such good quality that the frictional losses are negligible. However, it is possible to take that into account later.

It is necessary to measure the diameter of the spindle around which the string will be wrapped, so that the linear velocity v of descent of the mass is related to the angular velocity ω of the flywheel by the expression $v = \omega r$, where r is the radius of the spindle.

Rather than calculate torque and the acceleration (which is a little more difficult to do), it is easier to use the principle of the conservation of energy. Equate the potential energy lost with the linear and angular kinetic energy.

$$m_1 g h = \frac{m_1 v^2}{2} + \frac{I \omega^2}{2}$$

Where m_1 is the mass of the weight hanging from the spindle, h is the total height the mass will fall by, v is the final velocity of the falling mass, I is the moment of inertia that we are trying to find, and ω is the final angular velocity of the flywheel.

The final velocity can be estimated assuming that the acceleration has been approximately constant during the fall of the mass on the hanger. This would mean that the distance h is equal to the average velocity v multiplied by the time taken for the mass to reach the ground. This time is fairly

easy to measure using a stopwatch. So we say $\frac{v + U}{2} t = h$. U is the initial velocity, and that is zero.

In this way it is possible to calculate all the terms needed to calculate the moment of inertia of the flywheel.

A better estimate of the moment of inertia of the flywheel can be calculated by adding a constant K to the right hand side of the main equation, to represent the work done against friction of the bearings while the string travels the entire distance. This frictional loss will remain fairly constant even if the mass on the hanger is doubled.

The experiment can be repeated a number of times with different masses on the hanger. Each descent will take a different time. In this way it should be possible to eliminate K.

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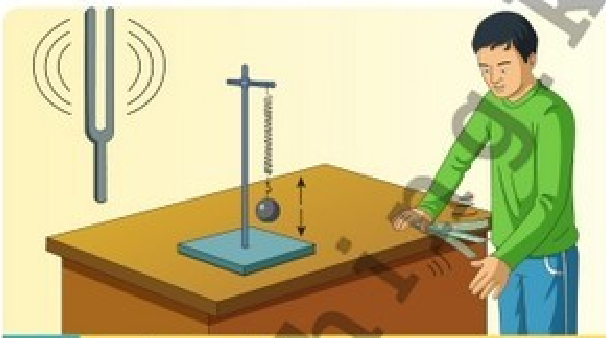
Simple harmonic motion

Look at figure 3.14. The weight attached to the end of the spring is constantly oscillating up and down. The end of the metre stick has been pushed down and released so that it also repeats the same motion over and over. In a comparable way, the prongs of a tuning fork also vibrate when struck, repeating the same motion many times.



3.14 *Examples of simple harmonic motion*

These are all examples of what we call **simple harmonic motion (SHM)**. Other examples are the swinging of a pendulum, including a child's swing, and – to an extent – the rise and fall of the tides in a harbour.

3.15 *A child's swing demonstrates simple harmonic motion*

Any linear motion that is repeated over and over again is likely to be SHM, but to qualify as such it must satisfy a simple mathematical test: at any point the acceleration, *a*, of the object must be proportional to its displacement, s, from an equilibrium position.

Mathematically, we say that:

$$a \propto -s \qquad \text{OR} \qquad a = -\omega^2 s,$$

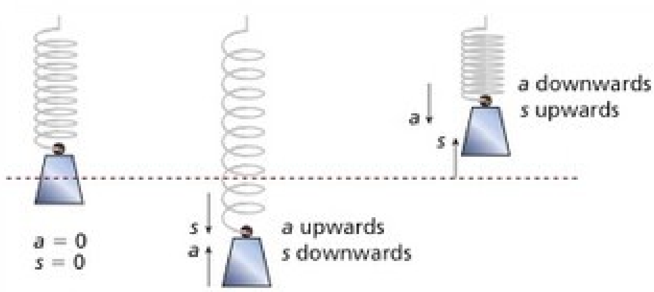
where ω^2 is a constant.

> Any motion in which the acceleration (a) of a particle is proportional to its distance (s) from an equilibrium position, and towards that position, is simple harmonic motion (SHM).

Look at the example of an oscillating spring shown in figure 3.16. When the weight is initially pulled down and released, it always moves back towards the central, or equilibrium, position. The further the spring is from the equilibrium position, the greater its acceleration. This is why we can say that for SHM the acceleration is always proportional to the displacement.

3.16 *A spring exhibits simple harmonic motion*

The reason for the minus sign in the formula is that the acceleration and displacement are both vector quantities and will both therefore have a direction. For example, when the displacement of the weight is upwards, the acceleration is downwards, and when the displacement is downwards, the acceleration is upwards.



3.17 *At any point, acceleration (a) is proportional to displacement (s)*

Period of simple harmonic motion

Any repetitive motion has a period, *T*. We have seen this concept already with waves and with circular motion, and there is a connection between the period of circular and simple harmonic motions.

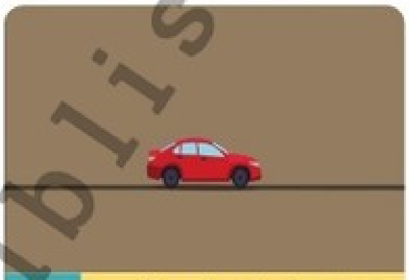
Look at figure 3.18. In it we are looking from above and can see a car driving along a circular path. As this is circular motion, the period of its motion is given by:

$$T = \frac{2\pi}{\omega}$$

where ω represents the angular velocity of the car.

3.18 *A car in circular motion*

Now look at figure 3.19, in which we are looking at the same motion from the side. Now all we see is the car moving backwards and forwards along a straight line. The car, or at least what we can see of it, would now be following SHM. However, the period has not changed.

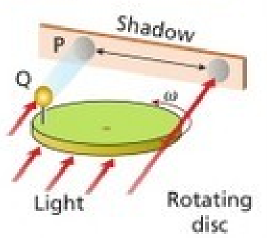


3.19 *Viewed from the side, this is SHM*

The period of SHM therefore follows exactly the same formula as that for circular motion:

$$T = \frac{2\pi}{\omega}$$

The car is only one example, but there is always a connection between the two types of motion. Figure 3.20 indicates a handle rotating in circular motion on a wooden disc; you can also see its shadow on the screen created by the light source. The shadow is following SHM and has the same period as the handle on the disc.



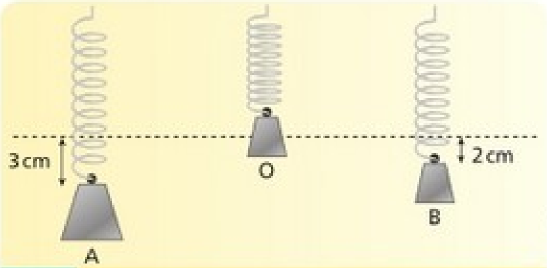
3.20 *The shadow undergoes SHM*

The reason ω^2 is used as the constant in the equation $a = -\omega^2 s$ is related to this connection between circular and simple harmonic motions.

For you to try

1. What is 'simple harmonic motion' (SHM)?
2. What is meant by the 'period' of SHM?
3. What is meant by the term 'equilibrium position' when we are talking about SHM?
4. Give two examples of bodies that move with SHM.

5 The spring in figure 3.21 is exhibiting SHM. At position A, the value of s is 3 cm, which is its maximum value, and the acceleration is $15\,\text{cm}\,\text{s}^{-2}$.



3.21 *Question 5*

(a) What is the value of ω^2?
(b) What is the acceleration at point B?
(c) What is the acceleration at point O, the equilibrium position?

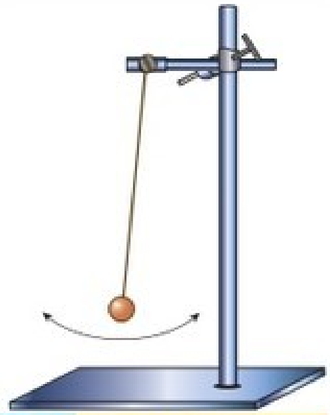
6 A swing is oscillating with a period of 2.8 s. What is the acceleration when the displacement of the swing's seat is 20 cm from equilibrium?

7 The weight on the end of a spring is demonstrating SHM.
(a) If the acceleration of the weight is $3\,\text{m}\,\text{s}^{-2}$ when its displacement is 12 cm, what is the period of the motion?
(b) What is its frequency?

Simple pendulum

A simple pendulum is an arrangement like that shown in figure 3.22, where the weight (or 'bob') repeatedly swings from left to right and back again.

If you move the string through a small angle and release it, as indicated, it will move back towards the centre position (which is the equilibrium position). The greater the displacement from the equilibrium position, the greater the acceleration will be when the bob is released. When the bob moves through the equilibrium position, its velocity will be at a maximum but its acceleration will be zero.



3.22 *A simple pendulum*

These are all characteristic of SHM. But remember that for the motion of an object to be considered SHM, it must satisfy the equation $a = -\omega^2 s$. A detailed analysis of this derivation is beyond the scope of this book, but it can be shown that as long as the angle through which the string moves is small (less than 10°, say,) the formula does apply, and the simple pendulum can be considered an example of SHM.

The period of a pendulum will follow the formula we've already seen:

$$T = \frac{2\pi}{\omega}$$

However, we can also carry out a more detailed analysis of the period of the pendulum. Indeed, this analysis is almost as old as science itself.

Galileo used to attend mass in the Cathedral of Pisa. On Sunday mornings in the summer, it could become uncomfortably hot and the two large doors of the cathedral were often left open to create a slight breeze. This meant that a large lamp hanging over the altar would begin to swing backwards and forwards like a pendulum. Galileo began to wonder about its motion and noticed, over time, that the period was a constant.

3.23 *An engraving of Galileo observing the lamp in Pisa Cathedral. The lamp is still there today, though it is now electric*

If the pendulum was swinging through a very small angle, it was also moving very slowly. If it moved through a larger angle, it also moved with greater speed. In both instances, the period (the time for one full swing) was the same. This later became the basis of the grandfather clock, and of metronomes, which are used by musicians to keep a steady rhythm in music.

We now know that the period of a pendulum is not absolutely constant. It depends on two things: the length of the pendulum, l, and the value of g, the acceleration caused by gravity:

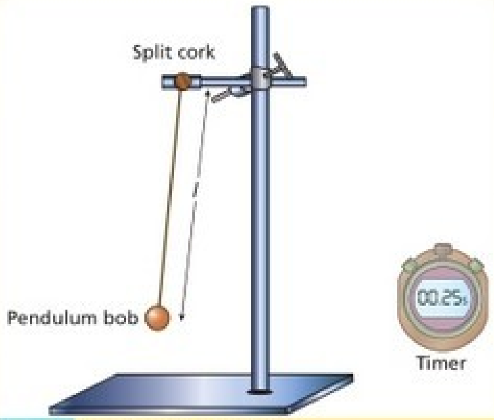
$$T = 2\pi\sqrt{\frac{l}{g}}$$

This allows us to devise an experiment in which we can measure the acceleration due to gravity (see Experiment 3.2).

Experiment 3.2: Investigation of the relationship between period and length for a simple pendulum and calculation of g

Method

1 Place the thread of a pendulum between two halves of a split cork, as shown in figure 3.24.
2 Set the pendulum swinging through a small angle (<10°). Measure the time t for 30 complete oscillations. Divide this time t by 30 to get the periodic time, T.
3 Record both l and T (the length is from the centre of gravity of the bob to the bottom of the cork).
4 Repeat for different lengths, l, of the pendulum.



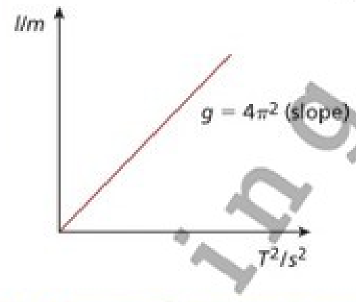
3.24 *Experimental apparatus*

Results and Conclusions

Draw a graph of l/m against T^2/s^2 (see figure 3.25).

Accuracy

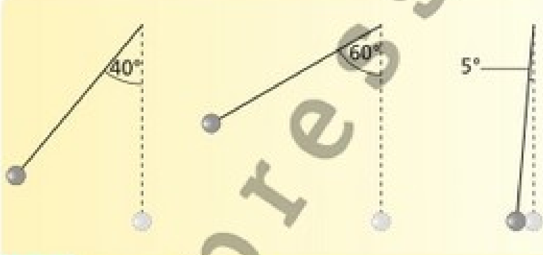
- The split cork ensures that the pendulum swings evenly in all directions.
- Longer lengths will reduce the percentage error in measurements of the length.
- Take acceleration due to gravity on Earth, g, to equal 9.8 m s^{-2}.



3.25 A graph of l/m against T^2/s^2

For you to try

1 The oscillation of a pendulum cannot always be considered simple harmonic motion. Which of the situations in figure 3.26 would qualify as SHM?



3.26 Question 1

2 If a pendulum has a period of 0.8 s when it moves with SHM, what is its length?

3 A pendulum has a period of 1.1 s on Earth. What would its period be on the Moon, where the acceleration due to gravity is one-sixth that of Earth?

4 The swing in figure 3.27 is hung from the sharply rising branch of a tree. One of its ropes is 2.5 m in length, and the other is 2.8 m.
  (a) Explain why the two sides will never swing evenly.
  (b) What is the difference between the periods for the two ropes of the swing?

5 A pendulum of length 30 cm is swinging with a period of 0.8 s. Is it on Earth?

Take acceleration due to gravity on Earth, g, to equal 9.8 m s^{-2}.

3.27 Question 4

Hooke's law and Elasticity

Any material that can return to its original shape after being stretched or deformed is said to be **elastic**. We tend to think of very stretchy things such as rubber bands when talking about elasticity, but there many different materials that are elastic. It is possible to slightly bend a metre stick, for example, but when you let it go, it will return to its original shape. That means that it is, to some extent, elastic. There are many plastics with similar properties. A spring is a device that is designed to maximise elasticity, and the most common type of spring is that made from coiled metal, as shown in figure 3.28.

3.28 *A metal spring*

An English scientist, Robert Hooke, studied elasticity in general, and springs in particular, and devised a law that describes how they operate. When the weight at the end of a spring is pulled down and then released, it will move upwards – towards the equilibrium position. Hooke noticed that the greater the extension on the spring, the greater the restoring force that brings the spring back towards its original length. These two measurements are proportional.

Robert Hooke

Robert Hooke (1635–1703) was a scientist, mathematician and architect. He was such a skilled experimentalist that he was appointed as first curator of the Royal Society in London in 1661, just as that society was becoming the centre of world science, bringing together such scientists as Newton and Boyle and publicising their work.

3.29 *Robert Hooke's drawing of a 'Blue Fly'*

In 1665 he published the *Micrographia*. This was a book in which he used his skills as an artist to draw pictures of what he had observed using microscopes and telescopes. The book was a phenomenal success, allowing people for the first time to see that small insects, for example, had body parts in some way comparable to those of larger creatures.

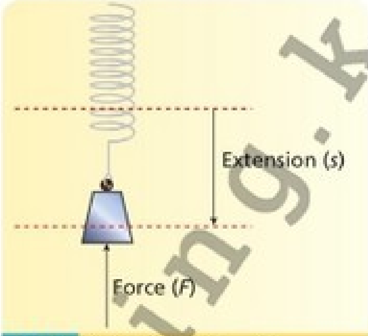
Hooke's work on springs and pendulums came as he tried to design a more accurate watch, or timepiece, than was then possible. This was not only a matter of convenience but a key part in allowing sailors to measure how far east or west they had travelled at sea.

As an architect, Hooke played a key role in rebuilding London after the Great Fire of 1666. He also wrote a number of important articles about gravity, and had clearly come close to developing a full theory similar to that later published by Newton. It seems likely, in fact, that Newton published his work mainly to deny Hooke the credit that would go with the discovery.

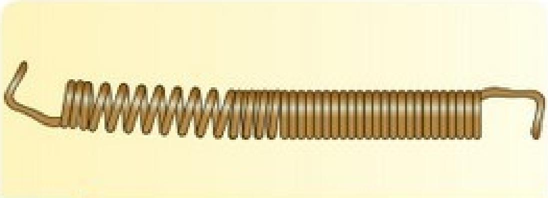
Hooke's law states that the restoring force, F, on a spring is proportional to the extension, s, of the spring, provided that the elastic limit is not breached:

$$F = -ks$$

The **elastic limit** is the point at which a spring loses its elasticity. With a small spring it is fairly easy to extend it to the point at which the coils will not pull the spring back to its original length when it is released, like the spring shown in figure 3.31. This spring has passed its elastic limit.



3.30 *Hooke's law: F = −ks*



3.31 *A broken spring has exceeded its elastic limit*

The constant, k, is sometimes referred to as the spring constant. It has units of N m^{-1}.

A spring with a large value for k indicates a stiff spring, one that is difficult to extend or compress, whereas a spring with a small value for k indicates that it is very weak, and can be easily extended or compressed.

3.32 *Many pieces of playground equipment use springs that are based on Hooke's law*

Hooke's law and simple harmonic motion

Remember that for an object to have SHM, it must be moving so that the formula $a \propto s$ is being followed. We can show that a spring that obeys Hooke's law must also exhibit simple harmonic motion:

Derivation

If: $F = -ks$ (Hooke)

it follows that: $ma = -ks$

so: $a = -\frac{ks}{m}$

and: $a \propto s$

or: $a = -\omega^2 s$

i.e. the spring is exhibiting simple harmonic motion.

From the above we can also derive a useful relationship between ω, k and m:

$$\omega^2 = \frac{k}{m}$$

3.7 Sample Question

If a force of 12 N is applied to a spring, it creates an extension of 3 cm.

(a) What is the value of the spring constant?

(b) What would the restoring force on the spring be if the extension were 8 cm?

Sample Answer

(a) $F = -ks$

$k = \frac{12}{0.03} = 400\,\text{N m}^{-1}$

(b) $F = -ks$

$= 400 \times 0.08 = -32\,\text{N}$

3.8 Sample Question

The force would be 32 N, and the direction would be opposite to that of the displacement. A spring has length 20 cm and when a mass of 125 g is attached to it, its length increases to 22 cm.

(a) What is the spring constant?

(b) The spring is then stretched to a length of 24 cm and released. What is the maximum acceleration of the mass?

$F = ma$

$m = 0.125\,\text{kg}$

$F = (0.125)(9.8) = 1.225\,\text{N}$

Sample Answer

(a) $F = -ks$

$k = \frac{F}{s}$

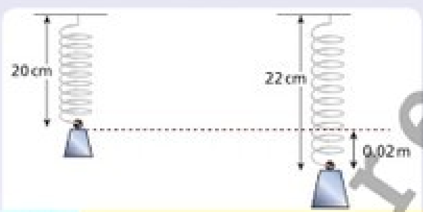
$k = \frac{1.225}{0.02} = 61.25\,\text{N m}^{-1}$

(b) $\omega^2 = \frac{k}{m}$

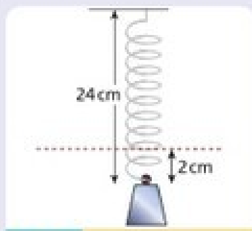
$= \frac{61.25}{0.125} = 490\,\text{N m}^{-1}\,\text{kg}^{-1}$

$a = -\omega^2 s$

$= (490)(0.02) = 9.8\,\text{m s}^{-2}$



3.33



3.34

For you to try

Take g to equal $9.8\,\text{m s}^{-2}$.

1. State Hooke's law.
2. What is meant by the term 'elastic limit'?
3. Would you expect the spring constant for the springs in a car to be greater or less than the spring constant for the springs used on a typical bicycle?
4. A spring experiences an extension of 7 cm and has a spring constant of $285\,\text{N m}^{-1}$. What is the restoring force on the spring at that point?
5. A spring experiences a restoring force of 25 N and an extension of 10 cm, when a weight of 98 N is attached to it.
   (a) What is the value of the spring constant?
   (b) What is the acceleration of the spring when it is stretched by another 5 cm and released?
6. A machine for strengthening the muscles in one's hand uses a coiled spring. If a force of 90 N is required to compress the spring by 2 cm, what force is required to compress it by 5 cm?
7. A car is hauling a 100 kg trailer, to which it is attached by a spring. The spring constant is $2100\,\text{N m}^{-1}$. If the car accelerates at $0.25\,\text{m s}^{-2}$, by how much will the spring stretch?

Module 4 Temperature

Learning objectives

- To describe the relationship between temperature and average kinetic energy of translation motion of molecules 10.3.1.1
- To apply the model of ideal gas 10.3.1.5
- To apply the basic equation of the molecular-kinetic theory in problem solving 10.3.1.3

Heat and Temperature

Most people would agree that boiling water is hot and freezing water is cold. However, the concept of what is hot and what is cold is not as clear as it might seem. If we were to be served a cup of coffee at, say 60°C, we would be annoyed that it was so cold, whereas if we were to order a soft drink and find it served at 30°C, we would find it unpleasantly warm.

Clearly, the everyday concept of hot and cold is far too vague to be of any use in science. We need to define the terms much more precisely.

4.1 *When we say that summer is hot and winter is cold, what do we mean?*

To understand exactly what we mean when we talk about heat, we have to also consider another closely connected concept – that of temperature. Temperature is a concept so tightly linked to heat that it is difficult to separate the two, but it is very important to do so.

Heat energy

Every object consists of a large number of atoms, or molecules, which are constantly in motion. In a solid, the particles are locked in place but still vibrate; in a liquid the particles move about, jostling against each other; and in a gas the particles fly about, colliding with each other and with the walls of their container (see figure 4.2). In all of these cases the particles are endlessly in motion, and therefore they always have kinetic energy.

4.2 *The three states of matter*

Think about a kettle full of water that has been brought to the boil. If we could see the individual molecules of water inside, we would see that they were all moving much faster than when the water was at a lower temperature. This movement corresponds to the increase in temperature, and it is always the case that when the temperature of a body increases, the particles within it are moving or vibrating more quickly.

Greater speed means greater kinetic energy, and as there has been an increase in kinetic energy it follows that there has been in increase in the **total energy** within the water. We often refer to this increase as the **heat energy** involved.

Heat is the addition or removal of energy from a body in such a way that it either increases or decreases the temperature of the body.

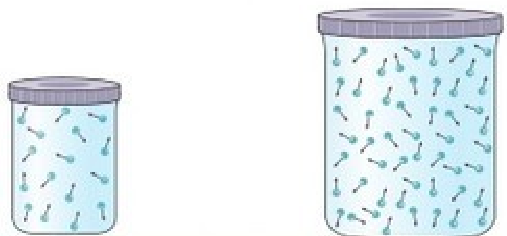
Temperature

The kettle that we talked about above would contain a very large number of molecules, or particles, and the hotter the water, the faster those particles would be moving. Not all of the particles would be travelling at the same speed, however. An individual particle might be involved in a head-on collision, which stops it moving almost entirely, for example, but a moment later could be hit by another particle in such a way that it suddenly travels at high speed again. It is still the case, though, that the hotter an object becomes, the faster the particles within it tend to travel.

Basically, **temperature is a measure of the average kinetic energy** of the particles within a body.

A simple, but valid, way of putting this is to say:

The temperature of a body is a measure of how hot it is.



4.3 *The container on the right holds more particles and therefore more heat energy*

To summarise this, we can say that heat is related to the **total energy** of the particles in a body, whereas temperature relates to the **average kinetic energy** of the particles within a body.

The two containers of gas shown in figure 4.3 are at the same temperature: the average kinetic energy of the particles in each is the same. However, the container to the right is larger and contains more particles. It therefore contains more energy, or more heat.

Measuring temperature

To measure temperature directly we would need to look at a very large number of individual atoms or molecules within an object and measure their kinetic energy. This is impossible, so to measure temperature we must do so indirectly. This involves the use of what we call **thermometric properties**.

> A thermometric property is any property of a body that changes measurably with temperature.

Examples of thermometric properties are:

- The length of a column of liquid in a tube
- The resistance of a wire or a thermistor
- The electromotive force (emf) of a thermocouple
- The colour of some substances
- The pressure and/or volume of a fixed mass of gas.

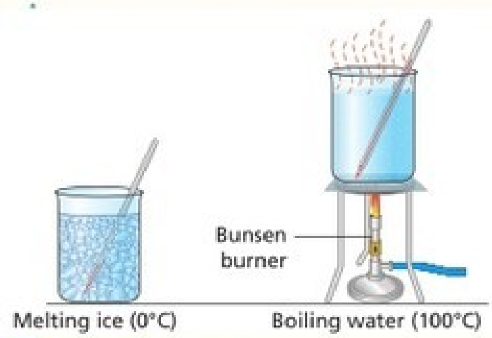
To establish a temperature scale we need two fixed temperature points, and a device with which we can measure a thermometric property (i.e. a thermometer). The length of a column of mercury in a glass tube is often used. Once a thermometer has been graduated (marked), the temperature can be measured by checking the length of the column of mercury.

The most commonly used temperature scale is the Celsius, or centigrade, scale. It is based on two easily replicated and fixed points of reference, the boiling point and melting point of water, and gives these a value 100°C and 0°C, respectively. All other temperatures are compared to these two.

Experiment 4.1: To graduate and use a thermometer

Method

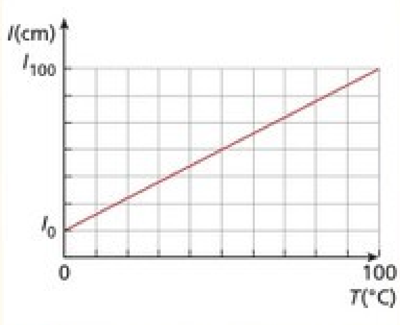
1. Take a mercury thermometer and place it in a container of melting ice. Mark the level of the liquid, l_o, inside the thermometer.
2. Take the same thermometer and place it in boiling water. Mark the level of the liquid, l_{100}.
3. Place the thermometer in a beaker of water from the tap. Note the level of the liquid, l_x.



4.4 *Experimental apparatus*

Results

You can then find the temperature of the water from the tap using a graph (see figure 4.5). Alternatively, you can mark the space between the l_o and l_{100} marks into 100 separate points, and then read the temperature from the scale.



4.5 *Graph to find the temperature of the water*

A standard thermometer

Experiment 4.1 is based on the assumption that the liquid inside the thermometer will rise at a constant rate as the temperature rises. This may not in fact be the case, but no matter what thermometric property you use, and regardless of what process you follow, there will always be some similar assumption involved. **Because of these variations within thermometric properties, different thermometers do not necessarily give the same reading at the same temperature.** This is not just a case of inaccuracy. It is inherent in the design of the Celsius scale and is a problem even with highly accurate thermometers. Because of this we must designate one thermometer to be a **standard**, i.e. a thermometer against which others are measured. Whatever temperature our standard thermometer reads is taken to be the correct temperature.

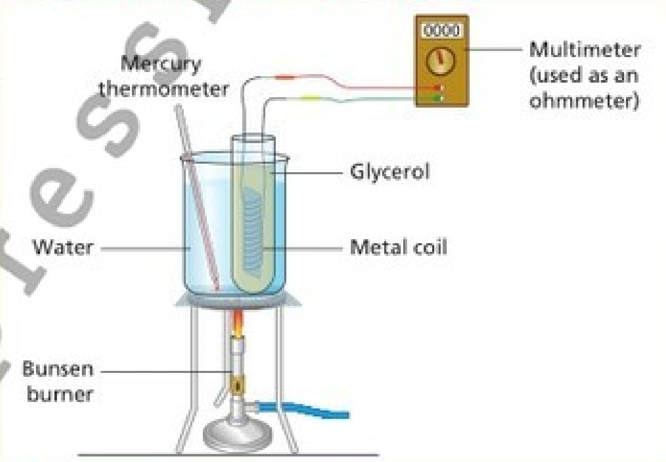
For reasons of simplicity, in schools the standard thermometer is usually taken to be the mercury thermometer.

Another way of calibrating a thermometer is simply to compare it to another thermometer. This has the obvious flaw that the new thermometer can never be more accurate than the original, but in many practical situations, great accuracy is not required and the method has the benefit of simplicity.

Experiment 4.2: Calibration curve of a thermometer using the laboratory mercury thermometer as a standard

Method

1. Set up the apparatus as shown in figure 4.6. Start with the water in the beaker cooled to about 10°C.
2. Record the temperature *T*, in °C, from the mercury thermometer and the corresponding resistance *R*, in ohms, from the ohmmeter.
3. Allow the temperature of the glycerol to increase by about 10°C and again record the temperature and the corresponding resistance.
4. Repeat the procedure several times.



4.6 *Experimental apparatus*

Results and conclusions

Plot a graph of resistance, *R*, against temperature, *T*. To measure temperature using the metal coil, measure the resistance and find the corresponding temperature from the calibration curve.

Accuracy

The glycerol has the benefit of creating a good thermal contact with the metal: bubbles do not tend to form on its surface as the liquid heats up, which would happen with water.

The same procedure can be followed to calibrate any thermometer.

Absolute zero and the Kelvin scale

Most people struggle with the concept of absolute zero when they first encounter it. Why should there be a coldest temperature? Why is it possible to be at –260°C but not at –280°C? It can seem at times as if an artificial barrier is being created.

When you consider the scientific meaning of temperature, however, it is obvious that it must have a lowest possible value. Remember that temperature is essentially a measure of the kinetic energy of the particles inside a body: the faster the particles move, the higher the temperature, and the slower they move, the lower the temperature. It follows from this that when the particles within a body stop moving, and therefore, have no kinetic energy, that the temperature must then be zero. This is what we mean by **absolute zero**.

Absolute zero is –273.15°C.

In reality, this temperature is never reached, and when we get close to it, matter behaves very strangely indeed – solids, liquids and gases no longer exist.

Some of the confusion caused by the concept of absolute zero is really due to the nature of the Celsius scale. It is convenient to use the freezing point of water as a reference point and to label this as 0°C, but it does mean that we have to be happy then to talk about minus temperatures. There are few comparisons in science. We don't often talk about negative lengths, for example, or negative masses. It would be very confusing if we did.

However, it is necessary to have another scale for measuring temperature, which takes zero to be the lowest possible temperature and compares all higher temperatures to that. We do this with the **kelvin scale** of temperature, which is the scale that scientists usually use.

To avoid the extra confusion that would ensue if we had two entirely different systems, it has been agreed that the size of one kelvin (1 K) should be the same as 1 degree Celsius (1°C). This means that although the actual temperature will be different on the two scales, a change of, say, 20 K is equal to a change of 20°C.

For example, imagine a body of water at 60°C: this is at 333 K. Now imagine the water heated up to 90°C: this is now at 363 K. The water has been heated by 30°C (90 – 60). On the kelvin scale, it has also been heated by 30 K (363 – 333).

The SI unit of temperature is the kelvin (K). In practice, the Celsius scale is used. Conveniently, a change of 1 K = a change of 1°C, and the two are connected by the formula:

$$T(\text{K}) = T(°\text{C}) + 273.15$$

In practice, when great accuracy is not essential, the '0.15' is often omitted and the calculation is given as:

$$T(\text{K}) = T(°\text{C}) + 273$$

4.1 Sample Question

Translate these temperatures to the kelvin scale: 10°C, –100°C, 153°C, 300°C.

Sample Answer

10°C = 10 + 273 = 283 K
–100°C = –100 + 273 = 173 K
153°C = 153 + 273 = 426 K
300°C = 300 + 273 = 573 K

4.2 Sample Question

Translate these temperatures to the Celsius scale: 273 K, 215 K, 100 K, 10 K.

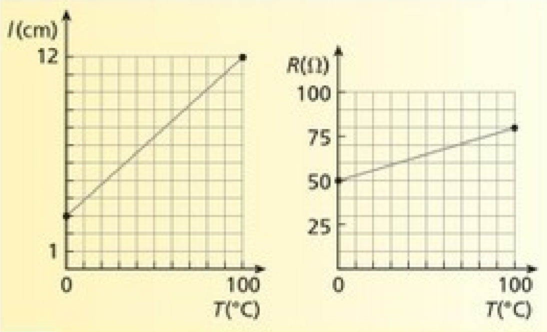
Sample Answer

273 K = 273 – 273 = 0°C
215 K = 215 – 273 = –58°C
100 K = 100 – 273 = –173°C
10 K = 10 – 273 = –263°C

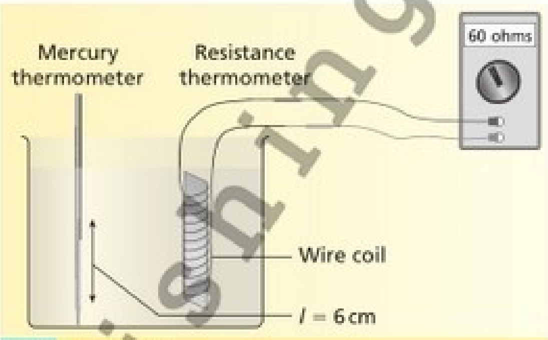
For you to try

1 What is meant by the term 'heat'?
2 What is a thermometric property? Give three examples.
3 What is 'temperature'?
4 What are the two fixed points used to create the Celsius scale?
5 What is the Celsius value of absolute zero?
6 Translate each of these temperatures to the kelvin scale: 100°C, 0°C, –173°C, 357°C.
7 Translate each of these temperatures to the Celsius scale: 250 K, 10 K, 273 K, 373 K.
8 Why do we need to have a standard thermometer?

9 The two graphs in figure 4.7 show measurements taken with a mercury thermometer and a resistance thermometer. Figure 4.8 represents both thermometers in a container of water at an unknown temperature. Use the figures shown to find the temperature indicated by each thermometer. If we rule out inaccuracy as an issue, why do you think the two thermometers might still disagree like this? What can we do to resolve the problem?



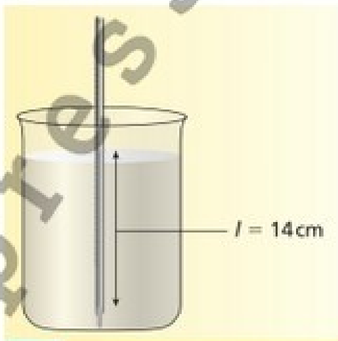
4.7 *Question 9*



4.8 *Question 9*

10 The table below shows the measurements for the length of a column of alcohol in a thermometer, compared to the temperature readings taken with a mercury thermometer. Draw a graph to represent these readings and use the graph to say what the temperature of the liquid is in figure 4.9.

l / cm (alcohol)	7	9	11	12	13	15
Temp / °C	10	20	30	40	50	60



4.9 *Question 10*

Module 5 Liquids, Solids and Gases

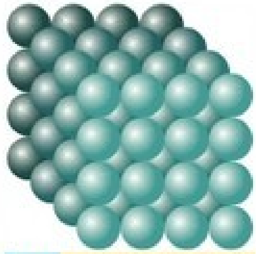
Learning objectives

- Describe and apply the model of an ideal gas 10.3.1.5
- Apply the ideal gas law in solving problems 10.3.2.1
- Investigate pressure and gas volume relationship at constant temperature 10.3.2.2
- Investigate gas volume and temperature relationship at constant pressure 10.3.2.3
- Investigate pressure and temperature relationship at constant volume 10.3.2.4
- Apply gas laws when solving calculation problems 10.3.2.5
- Understand that air contains invisible water vapour and that the maximum amount of water vapour that air can contain depends on temperature 10.3.4.1
- Understand the underlying cause that gives rise to surface tension and the definition of surface tension in mathematical form 10.3.4.2
- Distinguish between crystalline and amorphous and understand some of the visual clues that may indicate which type a particular solid is likely to be 10.3.1.2
- Understand the definition of Young's modulus 10.3.4.4
- Describe laminar and turbulent flow of liquids and gases 10.2.5.1
- Apply the continuity equation and Bernoulli's relation when solving experiment, calculation and qualitative problems 10.2.5.2
- Apply Torricelli formula when solving experiment, calculation and qualitative problems 10.2.5.3
- Define factors that influence the results of experiment and suggest ways to improve it 10.2.5.4

Particles in Solids, Liquids and Gases

Particles in a solid:

- are closely packed, and tightly bound to each other. As a result, solids have a fixed volume and a fixed shape.
- constantly vibrate; this is the only movement that they are capable of, because they are tightly bound.
- vibrate, on heating, more and more until, at the melting point, they break free from each other and a liquid is formed.



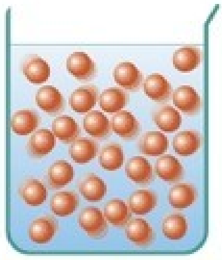
5.1 *Arrangement of particles in a solid*

When a solid melts, there is usually an increase in volume of between 5% and 30%. This is because, in a liquid, the particles are usually more loosely arranged than in a solid.

Particles in a liquid:

- are still close together
- can slip by one another easily – as a result, liquids can flow, and do not have a fixed shape.

The volume of a liquid at a particular temperature is fixed, because of the forces that hold its particles together. On heating, the particles move with greater speed and, at the boiling point, they escape completely from the other particles, and a gas is formed.



5.2 *Arrangement of particles in a liquid*

5.3 *Arrangement of particles in a gas*

When a liquid changes to a gas, there is a very large increase in volume. For example, when 10 cm^3 of water boils at 100°C and at a pressure of 101,325 Nm^{-2}, about 16,000 cm^3 of water vapour are formed. This happens because, in a gas, the particles are separated by relatively large distances. The particles of a gas:

- are relatively free of each other, and so a gas has no fixed shape or volume at a particular temperature
- move very rapidly and in a random manner, colliding with each other and with the walls of their container.

Because gas particles are separated by relatively large distances, gases are easily compressed, unlike solids and liquids. For example, if 100 cm^3 of air is subjected to a tenfold increase in pressure, its volume is reduced to 10 cm^3 (approximately).

For you to try

1. Three types of particle can make up solids, liquids and gases. What are they?
2. Why does a solid usually expand on melting?
3. Why does a liquid have a fixed volume at a particular temperature?
4. Why does a liquid not have a fixed shape?
5. Why does the volume of a gas get much smaller when it changes to a liquid?
6. Why are gases easily compressible?
7. Why are solids not easily compressible?
8. What effect has increased pressure on the volume of a liquid?

Diffusion

There is strong experimental evidence for the existence of particles. Diffusion experiments provide some of this evidence. **Diffusion is the spontaneous spreading out of a substance, and is due to the natural movement of its particles.**

Diffusion of gases

Diffusion of gases may be easily observed. For example, if a gas tap is turned on in a laboratory, and left on for a little while, the smell of the gas is soon noticeable throughout the room, even in the absence of draughts. This happens because the particles of the gas move rapidly and randomly throughout the room.

Demonstration

Diffusion of smoke in air

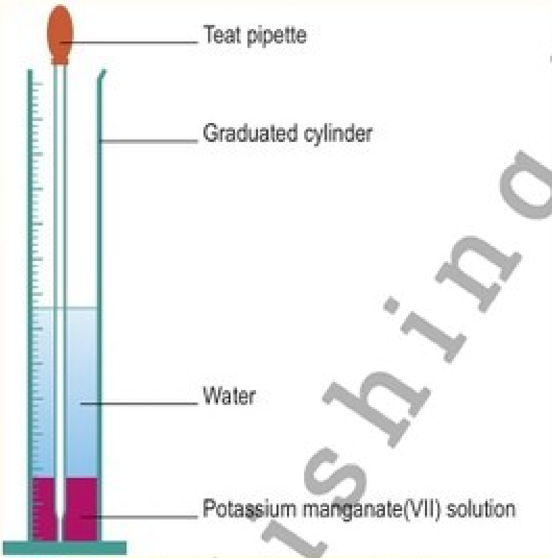
Equipment needed

Brown paper
Matches

Procedure

NB: Wear your safety glasses.

1. Get a smouldering piece of brown paper.
2. Observe and describe the movement of the smoke.
3. Explain why the smoke moves in this way.



5.4 *Diffusion of potassium manganate(VII) in water*

Diffusion in liquids

Diffusion in liquids is much slower than diffusion in gases. Nevertheless, diffusion in liquids can be easily observed. For example, if a layer of potassium manganate(VII) solution, which is purple, is carefully placed under water in a graduated cylinder (Figure 5.4), the purple colour slowly spreads throughout the liquid, due to diffusion of the potassium manganate(VII) particles.

Ink diffuses in a manner similar to potassium manganate(VII) solution.

Demonstration

Diffusion of ink in water

In this demonstration, diffusion in a liquid is observed.

Chemicals needed

Blue ink

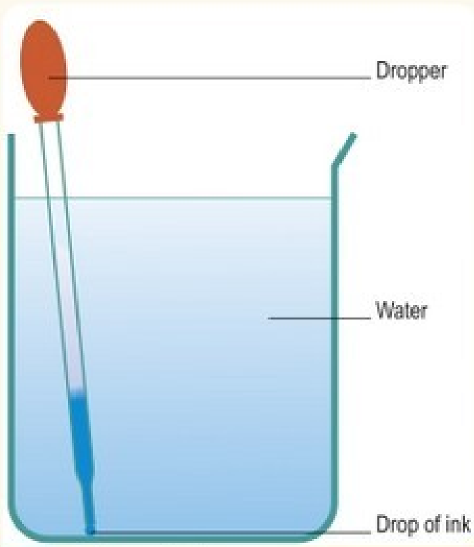
Equipment needed

Beaker
Pipette
White paper

Procedure

NB: Wear your safety glasses.

1. Three-quarters fill a beaker with water.
2. Using a pipette, carefully place some blue ink under the water at the bottom of the beaker.
3. Using a background of white paper, describe and draw what you see.
4. Allow to stand overnight.
5. Using a background of white paper, describe and draw what you see.
6. Explain in terms of particles what has happened.



5.5 *Diffusion of ink in water*

Gas Laws: Boyle's Law and Charles' Law

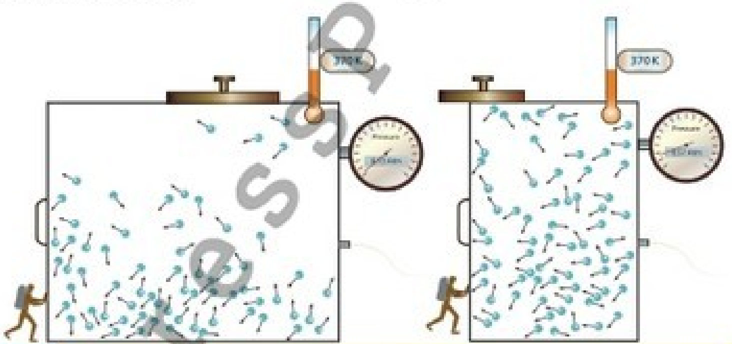
Boyle's law

Boyle's law states that, at constant temperature, the pressure on a fixed mass of gas is inversely proportional to its volume.

When you picture a mass of gas as being made of many thousands of millions of tiny particles moving at speed and colliding with each other and with the walls of a container, Boyle's law can seem a statement of the obvious. To appreciate the historical significance of the law, and to appreciate Boyle's achievement, it is important to realise that he was operating at a time when the presence of atoms, as we now know them, was not at all clear. Beyond that, air itself had hardly been studied and the discovery of oxygen was still decades in the future. In fact, it is largely due to Boyle and his contemporaries that we now know so much about the nature of matter.

If you look at figure 5.6, each diagram represents a situation in which the same mass of gas is held in a container. As the particles move about, they collide with each other and with the walls of the container, which creates pressure on the walls.

When the left-hand wall is pushed in (shown in the diagram on the right), the same number of particles, moving at the same speed, is now trapped in a smaller space. Therefore, those particles collide with the walls more often, and the pressure increases. A decrease in volume leads to an increase in pressure.



5.6 *A decrease in volume will lead to an increase in pressure*

Boyle's law goes a little beyond this. Boyle did not just say that a decrease in volume would cause an increase in pressure. He said that the two were inversely proportional. This means that if we were to halve the volume, the pressure would double, that if we were to multiply the volume by three, the pressure would be divided by three, and vice versa: the relationship between the two should be mathematically precise.

The easiest way to write this in mathematical form is to say that the pressure is proportional to the inverse of the volume:

$$P \propto \frac{1}{V}$$

where:

P – pressure

V – volume

Alternatively, we can say that the product of pressure and volume must be constant:

$$PV = constant$$

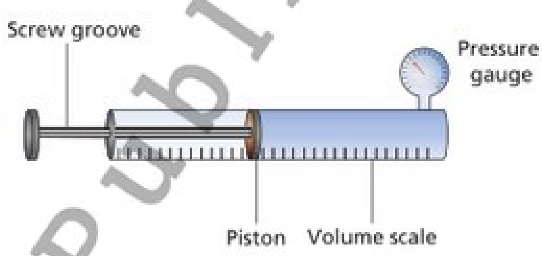
Effects and applications

- Boyle's law is seen in effect in a bicycle pump. If you hold your finger over the end of the pump as you push in the handle, you can feel the pressure increase as the volume decreases.
- The carbon dioxide bubbles that appear in soft drinks are usually formed low down in the drink and rise to the surface. As they rise, they increase in volume, due to the diminishing pressure.
- Helium balloons released into the atmosphere to study effects in the higher parts of the atmosphere are only partially inflated when they are released. This is because the volume grows as the balloon rises and the atmospheric pressure decreases. If fully inflated, the balloons would burst before they reached a high altitude.

Experiment 5.1: Verification of Boyle's law

Method

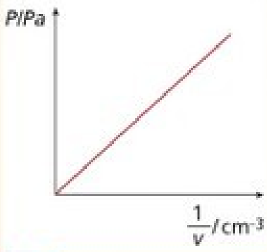
1. Using the apparatus as shown in figure 5.7, seal the cylinder so that no air can enter or leave.
2. Note the volume and pressure of the air inside.
3. Use the piston to reduce the volume by a fixed amount.
4. Again, note the pressure and volume. Because the volume is likely to be small, it might be easier to measure it in cm^3, rather than m^3.
5. Repeat this procedure for several volumes.
6. Record results and plot a graph of P against $\frac{1}{V}$.



5.7 *Apparatus to verify Boyle's law*

Results and Conclusions

A straight-line graph through the origin will verify that, for a fixed mass of gas at constant temperature, the pressure is inversely proportional to the volume, i.e. Boyle's law.



5.8 *P vs.* $\frac{1}{V}$

Accuracy

- It is important to allow the temperature to return to normal after the pressure has been changed.
- Boyle's law tends to be more accurate at relatively low pressures.

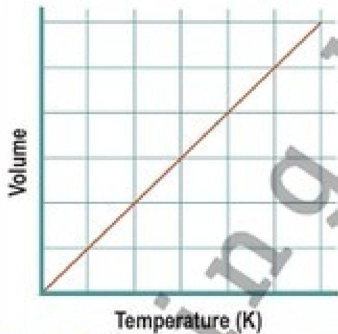
Charles' law

The volume of a gas also changes when the temperature is changed. In 1787, the French physicist Jacques Charles discovered that equal volumes of different gases at constant pressure all expanded by the same amount for a given rise in temperature. Charles' results are summarised in the modern form of Charles' law:

Charles' law – At a constant pressure, the volume of a given mass of any gas is directly proportional to the Kelvin temperature.

5.9 *Jacques Charles*

In mathematical form, V / T = constant, where V = volume of the gas, and T = Kelvin temperature of the gas. For example, if the temperature of 100 cm^3 of nitrogen gas is increased at constant pressure from 300 K to 600 K, the volume increases proportionately, that is, to 200 cm^3.



5.10 *The volume of a gas is directly proportional to the Kelvin temperature at constant pressure Charles*

The Law of Gay-Lussac

Gay-Lussac's law of combining volumes

Very early in the nineteenth century, the French chemist Joseph Gay-Lussac measured the combining volumes of gases in a number of chemical reactions. He found, for example, that two volumes of hydrogen combine with one volume of oxygen to give water. Some other volume ratios for reacting gases are given in Table 5.1.

Table 5.1

Reaction	Gas volume ratio for reactants
Ammonia + hydrogen chloride → ammonium chloride	1:1
Hydrogen + chlorine → hydrogen chloride	1:1
Carbon monoxide + oxygen → carbon dioxide	2:1
Methane + oxygen → carbon dioxide + water	1:2

Notice that all of the ratios were whole number ratios. By 1808, Gay-Lussac was able to state his law of combining volumes:

Gay-Lussac's law of combining volumes – When gases react, the volumes consumed in the reaction bear a simple whole number ratio to each other, and to the volumes of any gaseous product of the reaction, all volumes being measured under the same conditions of temperature and pressure.

There are differences in the naming of these laws in English and Russian scientific literature. The table below gives a brief summary of these differences.

Table 5.2

Russian name	English name	Formula
The Law of Gay-Lussac	Charles' Law	P = const (isobaric process), V/T = const
Charles' Law	The Law of Gay-Lussac or Second Law of Gay-Lussac	V = const (isochoric process), P/T = const

The Combined Gas Law

Boyle's law, Charles' law and Avogadro's law are combined to give the combined gas law:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

where P_1, V_1 and T_1 are the initial pressure, volume and Kelvin temperature respectively, and P_2, V_2 and T_2 are the final pressure, volume and Kelvin temperature respectively.

The most useful application of the combined gas law is to find the volume of a definite mass of gas at s.t.p., when its volume at a different pressure and temperature is known. A knowledge of the volume at s.t.p. of a gas is particularly useful, because the number of moles of the gas in that volume can very easily be calculated.

5.1 Sample Question

If a definite mass of gas occupies 250 cm³ at a pressure of 100,000 Pa and a temperature of 91 °C, what is its volume in cm³ at s.t.p.?

Sample Answer

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

P_1 = 100,000 Pa	P_2 = 101,325 Pa
V_1 = 250 cm³	V_2 = ? cm³
T_1 = 91 °C	T_2 = 273 K
= 91 + 273 K	
= 364 K	

$$\frac{100{,}000 \times 250}{364} = \frac{101{,}325 \times V_2}{273}$$

$$V_2 = \frac{100{,}000 \times 250 \times 273}{364 \times 101{,}325}$$

$$= 185 \text{ cm}^3$$

For you to try

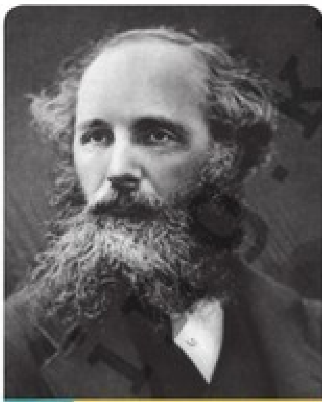
1 What is the volume at s.t.p. of a definite mass of gas that occupies a volume of 530 litres at 20°C and 102,000 Pa?

2 What is the volume at s.t.p. of a definite mass of oxygen gas that occupies a volume of 560 cm³ at 30°C and 100,500 Pa?

3 The volume of hydrogen collected in a reaction between zinc and hydrochloric acid was 240 cm³, measured at the laboratory conditions of 18°C and 101,000 Pa. Calculate the volume of hydrogen at s.t.p.

4 What is the volume at 819 K and 100,000 Pa of a fixed mass of gas that occupies a volume of 5 litres at 91 K and 200,000 Pa?

The Kinetic Theory of Gases



5.11 *James Clerk Maxwell*

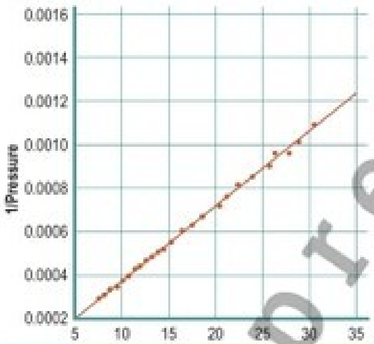
The **kinetic theory of gases** was developed by James Clerk Maxwell and Ludwig Boltzmann towards the end of the nineteenth century. In this theory, it is assumed that:

- Gases are made up of particles whose diameters are negligible compared to the distances between them.
- There are no attractive or repulsive forces between these particles.
- The particles are in constant rapid random motion, colliding with each other and with the walls of the container.
- The average kinetic energy of the particles is proportional to the Kelvin temperature.
- All collisions are perfectly elastic (for example, if a particle travelling at 450 ms^{-1} collides with a wall of its container, it rebounds with the same speed).

5.12 *Ludwig Boltzmann*

This theory can be used to mathematically derive gas laws such as Boyle's law and Charles' law. It can also be used to explain properties of gases such as diffusion. A gas diffuses quickly because its particles are moving constantly and rapidly, frequently colliding with each other and with the walls of its container. This results in the gas quickly spreading out in all directions.

The kinetic theory and gas laws



5.13

Gas laws, such as Boyle's law and Charles' law, are only approximately obeyed by real gases. Figure 5.13 shows a graph of volume versus 1 / pressure using data gathered by Robert Boyle in his original experiments. (The data has been converted to metric units.) The fact that all of the plotted points are not exactly on a straight line indicates that Boyle's law is only approximately obeyed by air, the gas used in those experiments.

Ideal gases and real gases

The kinetic theory is only completely valid for ideal gases. Since the gas laws can be exactly derived mathematically from the kinetic theory, it follows that the gas laws are completely valid only for ideal gases. The behaviour of real gases deviates from that of an ideal gas to a greater or lesser extent, depending on the situation.

> An ideal gas is a gas that perfectly obeys all of the gas laws under all conditions of temperature and pressure.

The behaviour of real gases deviates from that of an ideal gas to the greatest extent at **low temperatures** and at **high pressures**.

- One of the assumptions of the kinetic theory is that the diameters of gas particles are negligible compared to the distances between them. At low temperatures and at high pressures, the diameters are not negligible compared to the distances between them.

- Another assumption of the kinetic theory is that there are no attractive or repulsive forces between these particles. At low temperatures and at high pressures, this assumption is not valid, because the gas particles are in close proximity to each other. Attractive forces between the molecules, such as van der Waals' forces and, if the molecules are polar, dipole-dipole forces or hydrogen bonding, will have noticeable effects when the particles are close to each other. The stronger the intermolecular forces, the more unlike an ideal gas will be a real gas under these conditions.

Real gases behave most like an ideal gas at high temperatures and at low pressures. Under these conditions, the particles of a real gas are relatively far away from each other, and the assumptions of the kinetic theory are reasonably valid. However, the assumption that collisions between molecules are perfectly elastic is never true for a real gas.

For you to try

1. Explain in terms of the kinetic theory of gases why a gas diffuses rapidly.
2. What is an ideal gas?
3. Under what conditions do real gases behave most like an ideal gas?
4. Under what conditions are the assumptions of the kinetic theory of gases least valid?

The Equation of State for an Ideal Gas

Boyle's law, Charles' law and Avogadro's law can be expressed as follows:

Boyle's law:

PV = CONSTANT at constant T and n

Charles' law:

V is proportional to T at constant P and n

Avogadro's law

V is proportional to n at constant T and P

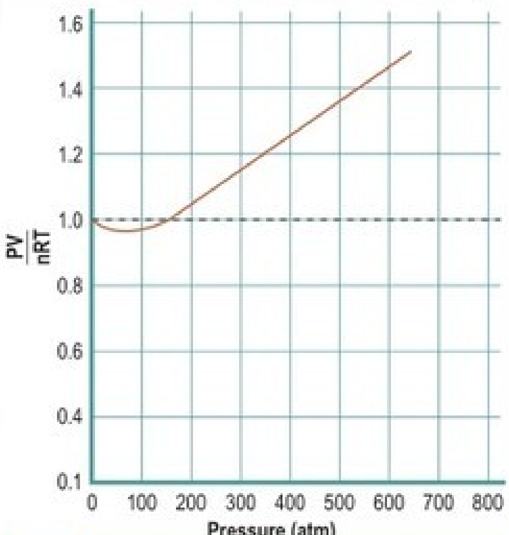
where V = volume, P = pressure, T = Kelvin temperature and n = number of moles.

Combining the three laws, **PV = CONSTANT × T × n**. Written as an equation, this becomes $PV = R \times T \times n$, where R is a constant known as the **universal gas constant**. This relationship is known as the **equation of state for an ideal gas**, and is usually written as follows:

$$PV = nRT$$

The value of R is 8.31 J K^{-1} mol^{-1}.

The relationship $PV = nRT$ is referred to as the equation of state for an **ideal** gas because it is obeyed only approximately by real gases. Figure 5.14 shows how the real gas, nitrogen, deviates significantly from ideal gas behaviour at higher pressures. For an ideal gas, $PV / nRT = 1$, but it is only at lower pressures that nitrogen even approximates to this value. All real gases deviate from ideal gas behaviour; the extent to which a particular gas does so depends on the nature of the gas.



5.14

Calculations involving the ideal gas law

It is important to use units that are consistent with those of the universal gas constant, *R*, when doing calculations involving the ideal gas law. The units of *R* are $J\ K^{-1}\ mol^{-1}$. The units for *V*, *P*, *T* and n that are consistent with this are shown in Table 5.3.

Table 5.3

	Unit
Volume	m^3
Pressure	Pa
Temperature	K
Number of moles	mol

5.2 Sample Question

8.4 g of a gas occupies a volume of 3 l at 77 °C and 100,000 Pa.

(a) How many moles of the gas are present?

(b) What is the relative molecular mass of the gas?

Sample Answer

(a) $PV = nRT$

$P = 100{,}000$ Pa

$V = 3\ l = 3 \times 10^{-3}\ m^3$

$R = 8.31\ J\ K^{-1}\ mol^{-1}$

$T = 77\ °C = 350\ K$

n = amount of gas in moles = $PV / RT = \dfrac{100{,}000 \times 3 \times 10^{-3}}{8.31 \times 350}$

$= 0.103$

(b) 0.103 moles of the gas have a mass of 8.4 g

1 mole has a mass of 8.4 / 0.103 g = 81.55 g

Relative molecular mass = 81.55

For you to try

1 A definite mass of gas has a volume of 2 m^3 at 300 K and 100,000 Pa. How many moles of gas is this?

2 A chemical engineer wants to store a gas produced during a process at a chemical plant at a pressure of 100,000 Pa and a temperature of 20°C. The process produces 2,000 litres of gas per hour, measured at 500,000 Pa and 160°C.

(a) What volume will this amount of gas occupy under the storage conditions?

(b) How many moles of gas are being produced per hour?

3 A mass of 8.4 g of a gas occupies a volume of $5 \times 10^{-3}\ m^3$ at 27°C and 1×10^5 Pa. Calculate the relative molecular mass of the gas.

4 A mass of 5.5 g of a gas occupies a volume of $1 \times 10^{-2}\ m^3$ at 330°C and 1×10^5 Pa. Calculate the relative molecular mass of the gas.

Humidity in air

Air normally contains a certain amount of water vapour in it. Although we cannot see the vapour, human bodies can feel the difference between high and low humidity. Measuring humidity is quite a difficult thing to do. The instruments that measure humidity are called hygrometers. Most hygrometers measure relative humidity rather than absolute humidity.

Absolute humidity is the amount of water vapour contained in a sample of air. It is measured in grams per cubic metre.

Relative humidity is the percentage of water vapour present in a sample of air compared to the maximum amount the sample could hold at that given temperature.

The maximum amount of water vapour that a sample of air can contain depends on the temperature of the air; hotter air can hold more water vapour than cold air. As air cools down the excess water vapour condenses out of the air and drops out as dew. This is why grass normally gets wet at night (when the temperature falls) even if it has not rained.

5.15 *Dew forms on the grass when the temperature of the air falls below the 'dew point'.*

During the day the sun evaporates the dew and plants will transpire additional moisture so that there is more moisture in the air. There is more absolute humidity in the air, but the relative humidity may be lower because warmer air can hold more moisture.

If the air is too dry it can cause skin, eye and lung irritation, and if the humidity is too high it can make it uncomfortable for us as it makes it more difficult to control our body temperature through perspiration.

Humidity also has effects on our buildings and human environment: very low humidity encourages electrostatic charging which in extreme cases can cause injuries and even explosions. High humidity causes mould to grow in buildings and also encourages the growth and reproduction of dust mites!

5.16 *A hygrometer*

An inexpensive hygrometer uses a coiled foil spring coated with hygroscopic salt crystals. It is only accurate to about 5%.

It is particularly important to control humidity in museums where paintings, wooden artefacts and musical instruments are kept. Humidity changes cause dimensional changes, and after many cycles paint can start to crack and instruments may start to warp.

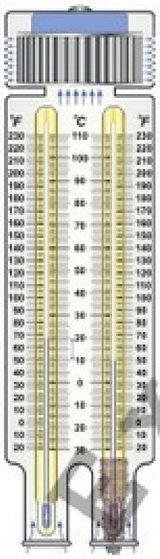
Measurement of relative humidity is normally inferred by measuring some other physical property instead (mass, resistance, capacitance, temperature, length, etc).

The simplest hygrometers use a long human or animal hair. The length of hair increases with humidity. Long hairs increase by only about 2.5% in length as the relative humidity increases from 0% (completely dry) to 100% (saturated air). A long hair is fixed at one end, and the other is wrapped around a small radius pulley on which an indicator needle is fixed.

5.17 *A hair hygrometer uses the extension and contraction of hair as a measure of relative humidity*

An inexpensive way of measuring relative humidity is by means of a metal foil spiral which is coated on one side with salt impregnated paper. When the humidity rises, the salt absorbs some of the moisture and it causes the paper to swell and so the foil straightens out a little. It makes the system act like a bimetallic spring. These methods are only accurate to about 5%.

The most reliable way to measure relative humidity is to find the 'dew point' (the temperature at which water starts to condense out of the air. A mirror is cooled artificially and when the surface starts to go dull, it means that water is starting to form on the surface. A formula is used to calculate the relative humidity from the room dew point temperature.



5.18

One last way of calculating the relative humidity involves using two thermometers, a 'wet' one and a 'dry' one. The combination of the 'wet' and 'dry' thermometers is called a 'Psychrometer'. The wet one has a 'sock' covering its bulb, and the bottom of the sock is in a container with distilled water. The water wicks up (through capillary action) and makes the bottom of the thermometer wet. The sock around the bulb slowly evaporates water into the air, and this evaporation causes the sock to become a little cooler than the dry thermometer. The lower the relative humidity of the air, the faster the sock will evaporate water, and so the 'wet' temperature will be lower that the 'dry' temperature. Conversely, if the air is 100% saturated with humidity already, then no more water will be able to evaporate from the sock, so the 'wet' temperature will be the same as the 'dry' temperature.

To calculate the relative humidity from the 'wet' and 'dry' temperatures a special table called a 'phychrometric chart' is used.

For you to try

1 When you breathe out onto a piece of glass or shiny metal the glass 'fogs up'. Why is that?
2 Why does dew fall during the night, and not during the day?
3 Why is 'dry heat' easier to bear than 'humid heat'?

Surface Tension

There are interesting phenomena that can be noticed at the surface of liquids. Have you noticed how some insects manage to stand on the surface of water without sinking into it? Or that when a water spout falls vertically, after some distance the solid cylindrical flow of water breaks up into separate droplets? Or that soap bubbles tend to be spherical, even if they are blown from a rectangular hoop? These are all called **Surface Tension** effects.

5.19 *Pond skaters' can stand on the surface of water without sinking through it; it is almost like the water has a skin on top of it.*

Liquids behave in such a way as to try to minimise the amount of surface that they are making with the air above or around them. The minimum amount of surface containing a given volume is always a sphere, so water droplets and soap bubbles always try to take on a spherical shape. But why do liquids want to minimise the amount of surface? What physical principle lies behind this?

Atoms in a solid are held in position by rigid bonds to their neighbouring atoms. Molecules in gases do not form bonds to their neighbouring molecules, and so they are not fixed in space; the molecules can fly around in any direction bouncing off the walls of the container. Liquids are somewhere in between. Water molecules deep inside the water are completely surrounded by neighbouring molecules, and they make very weak bonds with them. It requires energy to break these weak bonds. The molecules at the surface of the water do not have neighbouring molecules above them, and so are less strongly bonded and, therefore, are in a slightly higher energy state than the rest of the molecules deep under the surface.

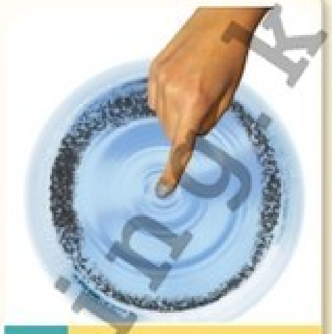
All systems in the universe try to settle in the lowest possible energy state. Things fall downwards, where their gravitational potential energy is lowest. Hot objects normally cool down, until they find the lowest possible temperature that puts them in thermal equilibrium with their surroundings. In the same way, liquids try to arrange themselves in the lowest energy state, and that means having as few molecules on the surface as possible. In this way, liquids try to have the smallest amount of 'skin' possible. This 'skin' seems to be in tension all the time, a little like a stretched rubber membrane.

Experiment 5.2: Surface Tension

A

Method

1 Take a bowl and fill it with water.
2 Sprinkle some pepper or talcum powder lightly on the surface, and notice how it sits on the 'skin' of the surface.
3 Put a small drop of liquid soap on the tip of a finger, and gently touch the surface of the water at the centre of the bowl.



5.20 *Testing for surface tension*

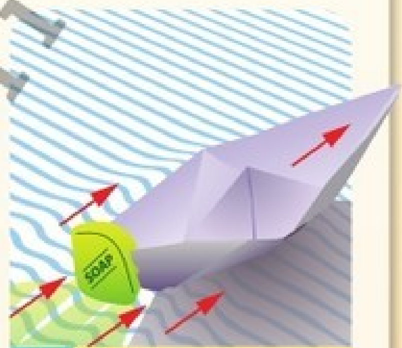
Results and Conclusions

- What do you observe?
- Why do you think this is happening?

B

Method

1 Find a large bowl (at least 30 cm in diameter – the bigger the better).
2 Make a small paper (or aluminium foil) boat which is only around 3 cm long.
3 Float your boat on the surface of the water.
4 Take a small thread of string, and immerse it in liquid soap. Wipe the excess off.
5 Hang the soapy thread off the back end of the boat so that part of it is inside the boat, and the other part is in the water.



5.21

Results and Conclusions

- What do you notice?
- Why do you think this is happening?

Definition of Surface Tension

Surface tension is given the symbol γ, and is defined as the energy (in Joules) per unit area (in square metres).

$$\gamma = \frac{E}{A}$$

where:

E is energy (in Joules) and

A is the area in metres squared

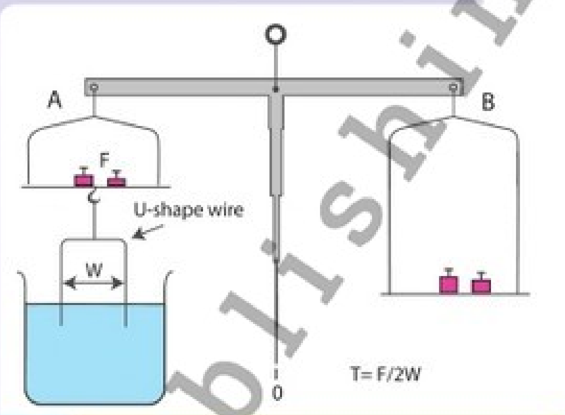
Here is an interesting thing you can do with physics equations, you can break them up and express quantities differently. Energy has identical units to work: so we can replace E with $F \times L$ (Force in Newtons multiplied by Length in metres). We can also replace Area with $L \times L$ (Length in metres multiplied by Length in metres). So the equation now becomes:

$\gamma = \frac{F \times L}{L \times L}$ which simplifies to $\gamma = \frac{F}{L}$, a force per unit length. So an equivalent definition of surface tension is the Force per unit length (the force produced by a unit length of the skin of the liquid).

Experiments can be done to try to measure the surface tension γ of liquids. They need to be sensitive experiments because surface tension is a fairly weak effect. The surface tension of water (to air) is only around 72 mN per metre (a mN is a milli-Newton. 1 mN is approximately the weight of 0.1 g.)

5.3 Sample Question

A student makes a wire U shape as shown in the figure 5.22. He brings the system to balance with the U shape partially immersed in the soapy liquid, but without a film filling the frame. He then immerses the U shape entirely under the surface of the soapy water. He then notices that he needs to add some masses to the right hand side in order to lift the soapy film upwards and lift it to the same balancing position as was achieved before. The width of the U is 10 cm, and the mass he needed to add in order to bring it back into equilibrium was only 0.4 g. What is the surface tension of the soap solution?



5.22

Sample Answer

We start with the equation for surface tension $\gamma = \frac{F}{L}$ but we must notice that the soap film actually has two surfaces facing the air, so the force measured is that due to two surfaces. So the force due to a single surface would be the half of the force he measured.

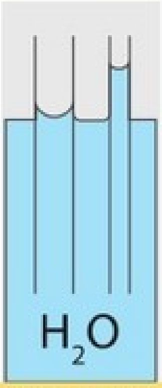
We say $\gamma = \frac{\frac{F}{2}}{L}$, or $\gamma = \frac{\frac{mg}{2}}{L}$. Inserting the measured values we get

$$\gamma = \frac{\frac{0.0004 \times 10}{2}}{0.1}, \ \gamma = 0.02 \text{ N/m} \quad \text{or} \quad \gamma = 20 \text{ mN/m}.$$

Notice that the surface tension of soapy solutions are typically around 1/3 of the surface tension of water. It is this reduction in surface tension that drives the little boat in the experiment above.

Surface Tension and the capillary action of a narrow bore glass tube

A clean narrow glass tube appears to 'suck water up' by a certain amount. The height depends on the internal diameter of the bore.



5.23 *Capillary action of a narrow bore glass tube*

If the adhesion forces between the water molecules and the glass are greater than the cohesion forces between the water molecules themselves, then a concave meniscus, and the tension in the surface pulls the surface upwards until the total upward force provided by the perimeter of the meniscus becomes equal to the weight of the water that is being pulled up. Once this equilibrium is reached, the capillary level stops rising. In equation form we write:

Upward Force = $2 \pi r \gamma$ and downward force = $\pi r^2 h \gamma g$ where r is the internal radius of the capillary bore, h is the height of the water column above the outside level, γ is the density of the water and g is the acceleration due to gravity.

When these are equal $h = \frac{2\gamma}{r p g}$.

5.4 Sample Question

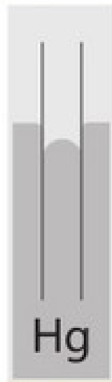
The internal bore of a capillary is 0.2 mm in diameter, calculate the maximum height to which the water may be drawn up if a perfect concave meniscus forms.

Sample Answer

$h = \frac{2\gamma}{r p g}$, $h = \frac{2\,(75 \times 10^{-3})}{0.0001 \times 1000 \times 10}$, $h = 0.15$ m

The water level inside the capillary will rise 15 cm.

In some liquids, the cohesive forces between the molecules are greater than their adhesive forces to the glass. In this case, the reverse thing happens. A convex meniscus forms, and the level of the liquid inside the capillary tube is pushed down. The origin of the force is again surface tension. Mercury is an example of a liquid where the cohesive forces are greater than the adhesive forces towards glass.



5.24 *A convex meniscus*

For you to try

1 When sand on the beach is completely dry, it is easy to push it around, and slides out from under your feet when you walk on it. When it is completely covered by water, it also slides out from under your feet. However, when sand is just wet (but not excessively) it goes very hard. Why do you think this is?

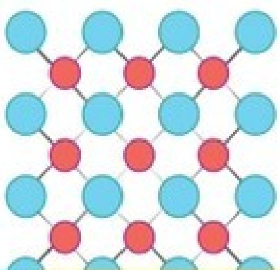
2 Soap bubbles fall slowly through the air. If you blow up from beneath them you can push them up higher again. What do expect the pressure inside the soap bubbles to be: higher, lower or equal to the atmospheric pressure around them?

5.25 *Soap bubbles*

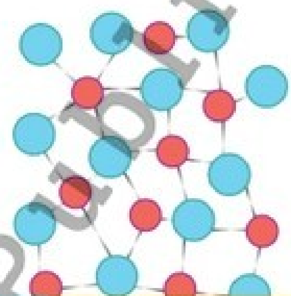
Crystalline and Amorphous solids

All solids have at some point in their existence been liquid. Most of the elements we know are believed to have been formed by fusion in some ancient star that then exploded sending fragments out into space. Even on Earth, some of the rocks have solidified after flowing out from volcanoes as lava.

When we touch a solid and feel it in our hands we cannot know how the atoms are organised inside the solid. All we can normally tell is how dense it is, and how rigid it is. In order to tell something about the arrangement of the atoms inside a solid we need to study them in greater detail. A crude way of testing them is by breaking them and studying the fragments. If the pieces have particular flat faces which are orientated at well defined angles, it is likely that the atoms were organised in a well defined pattern. We say the material is crystalline. However, if the fragments have many random angles and the fracture surfaces are not flat, it is likely that the atoms and molecules in the material were not arranged in a well determined pattern. we say the material is amorphous.



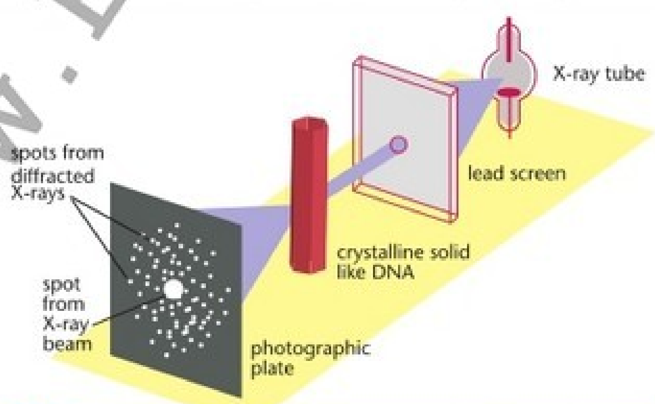
5.26 *Crystalline solid*



5.27 *Amorphous solid*

The process of solidification (whether it is slow or fast, whether in the presence of impurities, etc) determines how the atoms will arrange themselves relative to their neighbours. Generally (though not always), the longer the time the atoms have before they get locked into a solid, the better they arrange themselves in nice orderly patterns (without dislocations or misalignments). When this happens the solid frequently becomes crystalline. By this we mean that atoms are neatly arranged in columns and planes, and a pattern repeats itself again and again. We call this pattern a 'crystal lattice'.

The best way to determine whether a particular material is amorphous or crystalline is to use an 'X ray diffractometer'. X rays have very short wavelengths and can penetrate some way into solids. If the atoms are arranged in a crystal lattice, the positions of the atoms interfere with the X rays passing through the solid and make them come out in very particular directions. If the solid is amorphous, there are no clear patterns in the X rays coming out the other side.



5.28 *X-ray diffraction technique*

An experiment to try out at home

A simple experiment you can try at home involves dissolving some salt in hot water. Mix in as much salt as will dissolve. Then drain the salty water away from any remaining solid salt at the bottom of the container. Pour it into a shallow plate and leave it by a window where the sun can slowly evaporate the water away. After a few days you will start to see salt crystals forming.

Amorphous solids are those where there is no clear arrangement of the atoms inside the solid. Generally, if a solid is flexible (it can be bent), and if it is ductile (it can be extruded or hammered into different shapes) then the solid is amorphous. Rubber, glass and metals are all amorphous solids.

Because of the special geological conditions required to produce crystals, many crystals are quite valuable. They are often cut at very precise angles, and polished to make expensive jewellery.

Crystalline materials often have important commercial applications, and so materials such as quartz and silicon are grown in carefully controlled conditions in order to ensure their crystal lattices are free from dislocations and any contamination.

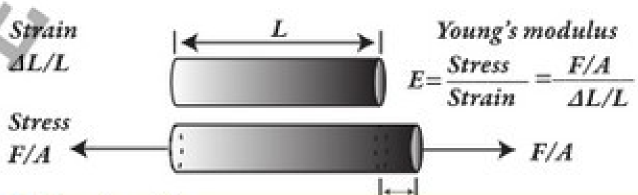
5.29 *Crystals*

For you to try

1 What makes crystalline solids different from amorphous solids?
2 Why is very slow cooling needed in order to encourage solids to become crystalline?

Young's Modulus of Elasticity

No solid is perfectly rigid. That is because individual atoms are bonded to their neighbours with bonds that are both attractive and repulsive. A good mental model of these bonds is to imagine them as springs. If you push two atoms closer together than they want to be, they will try to resist. If you try to pull them further apart than they want to be, they will also resist. The stiffness of these 'springs' determines how rigid the solid will be.



5.30 *Young's modulus*

In Physics and Engineering, the quantity that tells us something about the stiffness of the material is called the Young's Modulus (it is sometimes also referred to as the elasticity modulus). A material with a high Young's Modulus will be very stiff (it will resist deformation), and vice-versa. We can say that the deformation is inversely proportional to the Young's Modulus.

Definition of Young's modulus

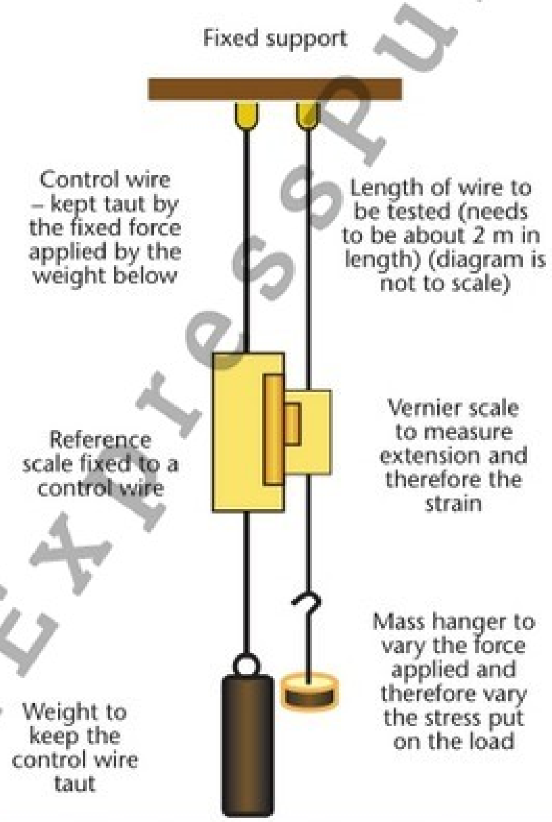
Everyday experience tells us that thicker pieces of material are more difficult to bend than thinner ones. The deformation will be inversely proportional to the cross-sectional area of the sample of material. You may also intuitively feel that it is easier to stretch a long piece of wire by 1 mm than it is to stretch a short wire by 1 mm. For the same force, any extension will be inversely proportional to it length.

Summarising, we say the e**X**tension is proportional to the **L**ength and the applied **F**orce, and inversely proportional to cross-sectional **A**rea and **E**lasticity modulus. In mathematical form we write: $X = \frac{FL}{EA}$ where the meaning of each of the symbols is given in the capital letters in the previous sentence.

We can re-arrange this equation to $E = \frac{\frac{F}{A}}{\frac{X}{L}}$ this may seem like a strange way to re-arrange it, but the reason for doing this is that *F/A* has a special meaning (Pressure), and *X/L* does too (Strain). This leads us to the Definition of the Young's modulus E:

E = Stress / Strain

When using this equation it is very important to convert all the units into the base units of Newtons and metres.



5.31 *Measuring the Young's modulus of a piece of wire using a Vernier scale*

An experiment to measure the Young's modulus of a piece of wire involves very careful loading and measurement of the change of length using either a travelling microscope, or a Vernier scale as shown in the diagram above.

5.5 Sample Question

A copper wire which was 2 m long and had a cross-sectional area of 1 mm^2 was measured to extend by 3 mm when loaded by a weight of 200 N. Calculate the tress and the strain on the wire, and hence the Young Modulus of the copper wire?

Sample Answer

The stress on the wire is the Force divided by the cross sectional area of the wire. Stress = F/A = 200 N / 0.000001 m (notice we have changed the cross sectional area from mm^2 into m^2). Stress = 200 000 000 Pa (1 Newton per m^2 = 1 Pascal). Stress = 2×10^8 Pa.

The strain in the wire is equal to $X / L = 0.003 / 2$ (notice that in this ratio it does not matter if we work in metres or millimetres, as long as we are consistent). Strain = 0.0015 = 1.5×10^{-3}

Now we are ready to calculate Young's modulus for copper.
E = Stress / Strain = $2 \times 10^8 / 1.5\ 10^{-3}$, $E = 1.33 \times 10^{11}$ Pa

This is a very large number! Normally Young's modulus is expressed in GPa (GigaPascals: 1 GPa = 1×10^9 Pa). So we say the approximate modulus of this copper wire is 133 GPa.

Note: The Young Modulus of rubber is quite small by comparison ~ 0.01 GPa, that of glass is 70 GPa, that of steel is 200 GPa, and that of diamond is 1100 GPa. Diamond is the most rigid material that we have discovered in nature!

5.6 Sample Question

The Young's modulus of steel is 200 GPa, how much will a 50 m long steel cable stretch if it is 2 mm diameter and it is loaded with a 400 N weight?

Sample Answer

Let us first calculate the cross-sectional area of a wire that has a 2 mm diameter:
Circular Area = $\pi r^2 = 3.14 \times (0.001)^2 = 3.14 \times 10^{-6}$ m^2

Now, start with the definition of Young's modulus and re-arrange it

$$E = \frac{\frac{F}{A}}{\frac{X}{L}} \quad \text{so} \quad X = \frac{F\,L}{E\,A}$$

$$X = \frac{400 \times 50}{200 \times 10^9 \times 3.14 \times 10^{-6}} \qquad X = 0.032 \text{ m}$$

The cable will stretch by approximately 32 mm.

Sometimes it is possible to solve some problems without actually knowing all of the information needed to calculate the Young's modulus. We do this by using ratios.

5.7 Sample Question

A wire of unknown material and unknown length extended by 15 mm when loaded with the weight of 5 bricks. How much would a similar piece of wire extend by if it was twice as long as the first one and was loaded with 10 bricks?

Sample Answer

Before we start putting any numbers into equations, let us notice that the two wires have the same Young's modulus, and the same cross sectional area. This will help to eliminate some things from the equations.

For the first wire we say $X_1 = \frac{F_1 L_1}{E A}$, and for the second wire we say $X_2 = \frac{F_2 L_2}{E A}$

So taking a ratio of the two equations we say

$$\frac{X_2}{X_1} = \frac{F_2 L_2}{F_1 L_1} \quad \text{so} \quad X_2 = X_1 \frac{F_2 L_2}{F_1 L_1}$$

Notice that it does not matter that we do not know the value of the lengths or the weights so long as we know their ratios. $X_2 = 15 \; \frac{10 \; 2}{5 \; 1} = 60$ mm

For you to try

1 A steel vertical girder which is to be 10 m tall will need to support a mass of two metric tonnes without compressing downwards by any more than 5 cm. What should its minimum cross sectional area be? (Young's modulus of steel = 200 GPa, take gravity as 10 m s $^{-2}$, one metric tonne = 1000 kg)

2 A wire of a given material stretches by 20 mm when loaded by an unknown weight. If we wanted to reduce the stretching to be only 10 mm, what could we do to each of the following:

(A) The length of the initial wire

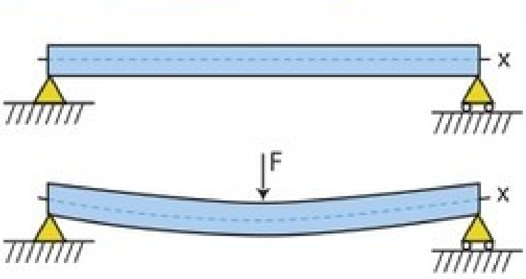
(B) The cross sectional area of the wire

(C) The Young's modulus of the wire

(D) In practice, there is little you can do about the modulus of the wire or its cross-section. If the length of the wire and the load needed to remain the same, what practical solution would you suggest to halve the extension?

Further considerations on material deformation

Engineering components of buildings and bridges are often not only stressed so that they are in compression or tension. Frequently they can be in compression and tension at the same time. This is what happens when a horizontal member is subjected to bending loads. A horizontal beam that is supported at both ends and loaded in the middle will be in tension at the underside and in compression on the topside.

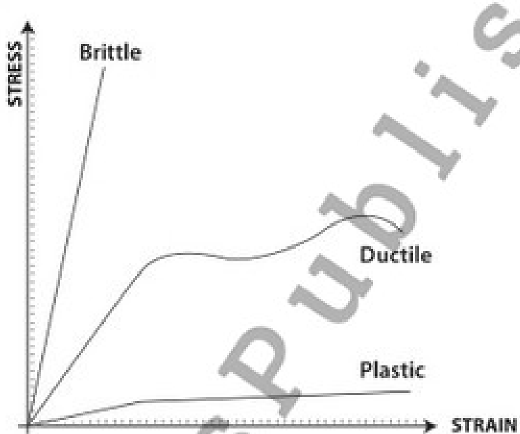


5.32 *Compression and tension on a horizontal beam*

Loaded beams supported at the ends will bend in such a way that the bottom surface is stretched (tension) and the top surface is pushed together (compression).

Materials Scientists (that is the name given to the people who study the performance and characteristics of different materials) often plot Stress against Strain graphs like the one shown below. Different types of materials perform differently. Three major different types are shown:

- **Brittle materials:** these deform elastically (the strain is proportional to the stress) up to a certain point, and beyond that they suddenly break without warning.
- **Ductile materials:** these deform elastically up to a certain point and beyond that they yield, and deform plastically (they remain deformed even after the external load has been removed). After a certain amount of plastic deformation they finally break.
- **Plastic and polymeric materials:** these deform non-linearly; the relationship between stress and strain is not a fixed one.



5.33 *Stress versus Strain graph*

The Stress versus Strain graphs are used by material scientists to characterise the response of different materials to loading.

Fluids and Gases

Fluids and gases have some similarities and some differences. Their main similarity is that both flow easily, and because of this they do not have any shape. They take the shape of their container or the solid boundaries around them.

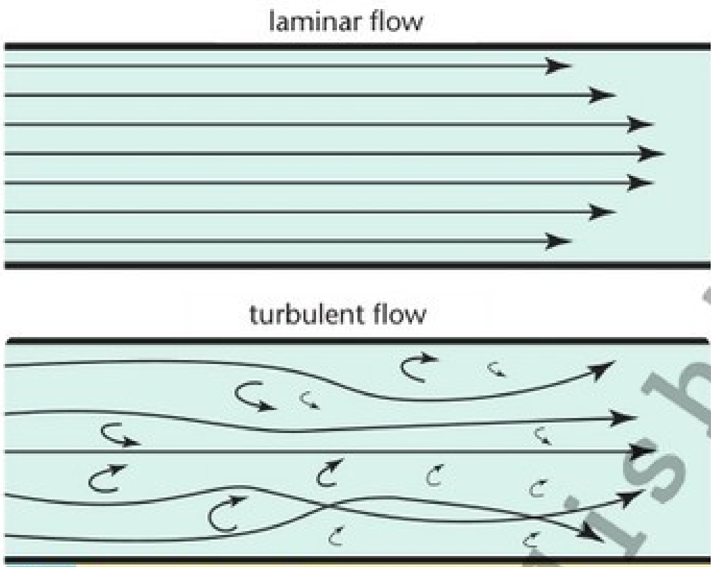
Fluids are on the whole approximately 1000 times denser than gases, for this reason they will stay in a container even if it does not have a lid. Gravity is sufficient to keep liquids inside an open topped container. Gases on the other hand will readily escape through the top of an open top vessel.

Fluids are largely incompressible; their volume is very nearly fixed. Gases on the other hand are very compressible because the molecules in a gas are flying at great distances from each-other.

When gases or fluids are in motion, or when a solid is moving through them, the molecules need to move around them to allow movement to happen.

At slow speeds the movement of the fluid or gas will be laminar. By this we mean that the layers in the fluid flow parallel to each other and there is no mixing. This is sometimes also called streamlined flow.

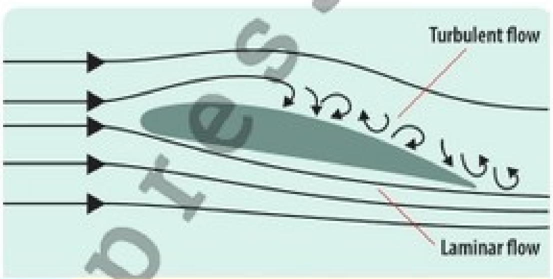
At greater speeds, it is difficult to sustain laminar flow, and turbulence often sets in. In turbulent flow, layers in the fluid no longer move parallel to each other but start to mix together in a rolling type of motion that can be quite chaotic.



5.34 *Laminar and turbulent flow*

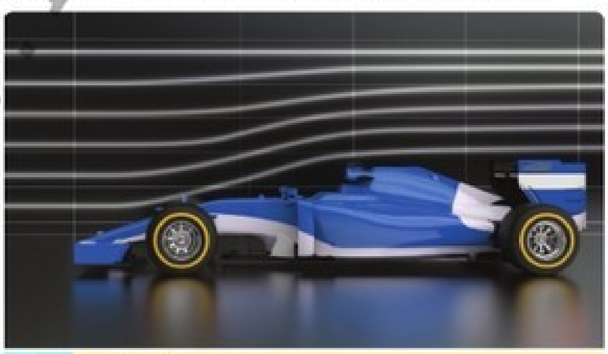
Turbulent flow normally dissipates energy and causes the flow to slow down. It is far more efficient to use a larger bore pipe and allow the liquid to flow slower in a laminar way than to use a narrower bore pipe and force the water through at high pressure.

In the case of an aeroplane or a fast car moving through the air at high speed, turbulent flow increases drag, and makes the aeroplane or the car slow down. To maintain a higher speed it will be necessary to use far more fuel, making the vehicle very inefficient. This is the reason why engineers pay so much attention to the shape of cars and aeroplanes.



5.35 *Laminar and turbulent flow*

At high speeds it is nearly impossible to have perfectly laminar flow. Some turbulence normally sets in. The engineers task is to try to minimise the turbulence. Often a 'Wind Tunnel' is used to study the effects of shape designs before they are put into production.



5.36 *Racing car prototype*

Some wind tunnels have artificial smoke lines injected into them so that the flow can be seen visually. The best designs produce the least disturbance in the smoke lines behind the car.

Effect of depth on the pressure in fluids

Each layer in a fluid needs to support the weight of the layers above it. That means that the pressure at the bottom of a fluid will be higher than the pressure at the top. In gases the increase in pressure with depth is very slight, because gases have low densities. You need to climb a high mountain before you start to notice any changes in pressure. In liquids, however, the pressure changes with depth are very pronounced. If you dive under water, even just 1 metre under the surface you will notice a slight pain in your ears. A depth of only 10 metres in water produces a pressure which is roughly equal to the pressure exerted by the entire atmosphere at sea level!

The pressure P due to a depth of fluid above it is $P = \rho g h$, where ρ is the density, g is the acceleration due to gravity and h is the height of the fluid above. In this equation, the h is sometimes called 'head of pressure'.

5.8 Sample Question

A water tower contains a large tank with its level 20 metres above ground level, calculate the pressure of the water at the foot of the tower. (The density of water is approximately 1000 kg/m^3)

Sample Answer

$P = \rho g h = 1000 \times 10 \times 20 = 200$ kPa

5.9 Sample Question

An engineering press requires a pressure of 350 kPa in order to operate correctly. The engineer wants to design a system whereby this pressure can be delivered by a water tank located uphill from the press. Calculate the minimum height required to deliver this pressure.

Sample Answer

$P = \rho g h$ so $h = P / \rho g$ $h = 350000 / (1000 \times 10) = 35$ metres

For you to try

1 A water pump is capable of pumping water at a maximum pressure of 5 bar. Calculate the maximum height to which water could be pumped. (1 bar = 1 atmosphere = 100 kPa)

2 If this same pump were to be used to pump oil (density of oil = 910 kg/m^3) without doing any calculations, decide whether this pump could pump the oil to

(a) a greater height **(b)** a lesser height **(c)** the same level.

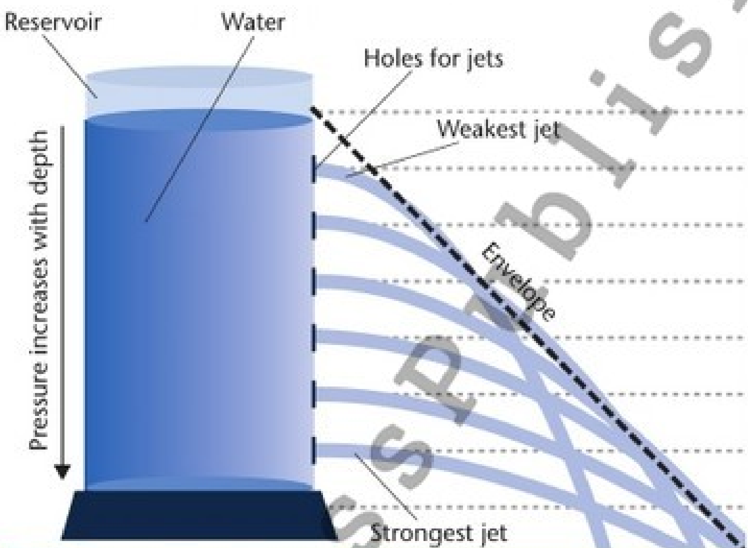
3 What if the water was sea water (ρ = 1029 kg/m^3). How would this affect the level?

Pressure and Velocity in liquids

Evangelista Torricelli (1643) discovered that the speed at which water comes out of a hole on the side of a container increases as the depth of the water inside the container increases. The effect is not linear. Torricelli postulated that the speed at which the water comes out of the container is the same as the water would have achieved if the water had been allowed to fall freely from the same height. We can calculate the speed to depth relationship using the principle of the conservation of energy, equating the gravitational potential energy of the fall to the kinetic energy of the water when it has reached the bottom of the fall.

$$m\,g\,h = \frac{mv^2}{2}$$ Rearranging we obtain $v = \sqrt{2gh}$.

It should be noted that this expression assumes no energy losses due to turbulence or air resistance.

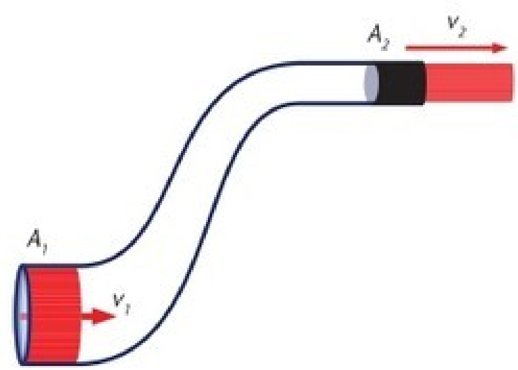


5.37 *Torricelli's law can be demonstrated using a can with multiple holes down the side*

The continuity of flow

When a liquid flows through a long chain of pipes, and these pipes have different diameter in different places, it causes the flow to speed up or slow down. This needs to happen in order to maintain a constant rate of flow throughout the entire length. Where the diameter is smallest, the speed will be highest.

This is a consequence of the conservation of the volume of liquid. The diagram 5.38 gives a clear visual explanation as to why this must be true.



5.38 *Continuity of flow*

The speed of the flow of liquid along the length of a pipe is affected by the cross-sectional area at any given point.

For the flow rate to be constant, the volume per second must be equal everywhere along the length. A volume element is equal to the cross-sectional area A multiplied by a length *L*. If we consider a volume rate, then we must divide this by time. The flow rate at any point will be *A* x *L*/*t*. Notice that *L*/*t* is a velocity, so the flow rate can be expressed as *A* times *v*.

Since the flow rate is constant throughout,

$$A_1v_1 = A_2v_2 = A_3v_3 = \dots$$

This is known as the continuity of flow equation.

5.10 Sample Question

Water is flowing at 20 cm/s along a pipe that has an internal diameter of 3 cm. Further down the line the pipe has an internal diameter of 1 cm. At what speed must the water be flowing at this new point?

Sample Answer

$A_1v_1 = A_2v_2$ so $v_2 = v_1A_1/A_2$.

The ratio of the cross sectional areas is equal to the square of the ratio of the internal diameters, so we can say

$v_2 = v_1(D_1/D_2)^2 = 20\ (3/1)^2 = 180$ cm/s

5.11 Sample Question

The flow of water through a tube is 0.30 m/s where the internal bore is 25 mm in diameter. What diameter bore would be necessary in order to slow down the flow to 1 mm/s?

Sample Answer

The first thing to notice is that there are a number of different units being used in this question, so we must be extra careful.

$A_1v_1 = A_2v_2$ so $A_1 = A_2v_2 / v_1$ so $(D_1)^2 = (D_2)^2\ (v_2 / v_1)$

$(D_1)^2 = (25)^2\ (300 / 1)$ $(D_1)^2 = 187\ 500$ $D_1 = 433$ mm

For you to try

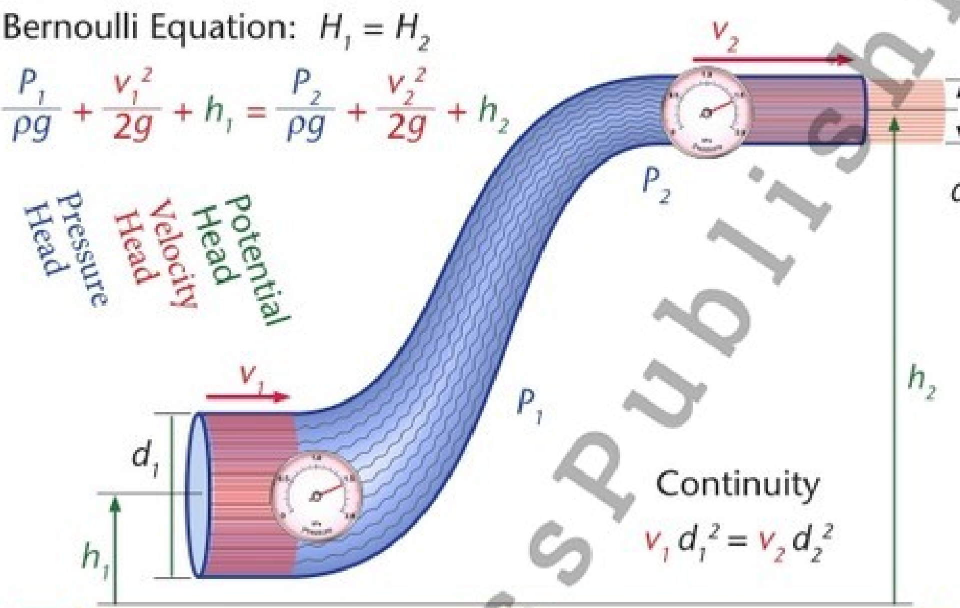
1 If you want to double the speed of flow of water through a section of tube, by what ratio must you reduce its diameter?

2 If you double the diameter of a tube, by what ratio have you slowed down the flow of water through it?

The Bernoulli effect

When water flows through a narrower section of pipe, it is not only the speed of flow that changes; this is what Bernoulli discovered. The increase of speed is fairly intuitive, and it is easy to visualise this in our minds, however, pressure is invisible, and it is not obvious why the pressure should decrease.

So far we have only considered the conservation of water flow. If, additionally, we consider the conservation of energy, it leads us to a fuller description and an equation which includes pressure as well as height (in case the water is flowing upwards or downwards).



5.39 *Bernoulli's equation*

The full Bernoulli equation can also be written as $P + \frac{1}{2}\rho v^2 + \rho gh = constant$.

Often we consider fluids flowing horizontally in which case there is no change in *h*, so the equation reduces to $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$.

Notice some interesting things about this equation. It tell us that if the velocity is zero the $P_1 = P_2$. That is, if the water is stationary (hydrostatic) the pressure is equal. However, if $v_2 > v_1$, then $P_2 < P_1$. Sometimes Bernoulli's equation is summarised as 'High speed, Low pressure'.

5.12 Sample Question

The pressure in a wide bore tube is 50 kPa where water is flowing at 1.3 m/s. What will the pressure be at a constriction where the flow speed is 2.6 m/s? (density of water = 1000 kg/m³)

Sample Answer

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ so $P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = P_2$

50,000 + [1000 (1.3)²]/2 – [1000 (2.6)²]/2 = 47.5 kPa

A difference of approximately 2.5 kPa.

5.13 Sample Question

Water is flowing in a fire hose with a velocity of 0.90 m/s and a pressure of 215 kPa. At the nozzle the pressure decreases to atmospheric pressure. Assuming there is no change in height. Calculate the velocity of the water exiting the nozzle. (The density of water is 1000 kg/m^3 and atmospheric pressure 101 kPa).

Sample Answer

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ (the terms with h cancel as they are equal on both sides)

$\frac{2}{\rho}\left[P_1 + \frac{1}{2}\rho v_1^2 - P_2\right] = v_2^2$

$2/1000\ [215\,000 + (1000 \times 0.90^2)/2 - 101\,000] = v_2^2$

$228.8 = v_2^2$ so $v_2 = 15.1$ m/s

5.14 Sample Question

A water main is pressurised at 200 kPa and water is flowing through it at 5.0 m/s. The pipe needs to pass down through a valley and up the other side. If the valley is 50 m deep, calculate the water pressure in the water main at the bottom of the valley.

Sample Answer

Since the bore and flow rate are the same throughout, we can simplify Bernoulli's equation to

$P_1 + \rho g h_1 = P_2 + \rho g h_2$ so $P_1 + \rho g h_1 - \rho g h_2 = P_2$ or $P_2 = P_1 + \rho g(h_1 - h_2)$

$P_2 = 200\,000 + 1\,000 \times 10\ (50)$

$P_2 = 700$ kPa

For you to try

1 Water is standing stationary in a vertical section of pipe which is 25 m from top to bottom. The pressure inside the pipe at the top is 150 kPa, what will the pressure be at the bottom?

2 A pipeline is transporting oil with a density of 930 kg/m^3. In a section that is 10 cm in internal diameter the pressure is 175 kPa and it is flowing at 4 m/s. Further down the line the internal diameter has been reduced to 7 cm. Calculate the following:

(a) The new speed of the flow at this reduced diameter.

(b) The pressure at the reduced diameter section of pipeline.

Module 6 Thermodynamics and Engines

Learning objectives

- Use internal energy formulae of monoatomic and diatomic ideal gas in problem-solving 10.3.3.1
- Apply the first law of thermodynamics to isothermal and adiabatic processes 10.3.2.2
- Describe the Carnot cycle for an ideal heat engine 10.3.3.3
- Use the formula of energy conversion efficiency of a heat engine in problem-solving 10.3.3.4
- Become familiar with the practical effects of changes on gases 10.3.2.5
- Understand the effects of doing work on a gas (by compression), and of a gas doing work (by expansion)

First Law of thermodynamics

The First Law of thermodynamics is the law of conservation of energy applied to heating, cooling and working. The first law of thermodynamics states:

Heat energy cannot be created or destroyed but it can be transferred from one place to another or converted into another form of energy.

The First Law can be expressed in different ways, according to the sign convention used. In applying the First Law to 'systems' here, it is written:

$$Q = \Delta U + W$$

where:

Q – energy supplied by heat transfer
If energy is **removed** by heat transfer, for example when a gas is cooled, Q is **negative**.

ΔU – change in internal energy.
If ΔU is **positive** there is an **increase** in internal energy, if **negative** there is a **decrease** in internal energy.

Energy supplied or removed by heat transfer = change in internal energy + work done on or by the gas.

Defining a system

In talking about engines and ideas such as their efficiency, we need to define what we mean by the term a 'system'. To solve problems which involve calculating changes: 'system', 'boundary' and 'surroundings' have to be understood.

A system is a region in space that contains a quantity of gas or vapour. In open systems the gas or vapour flows into, out of, or through the region. The gas may pass across the boundary between the system and its surroundings.

Examples of open systems are:

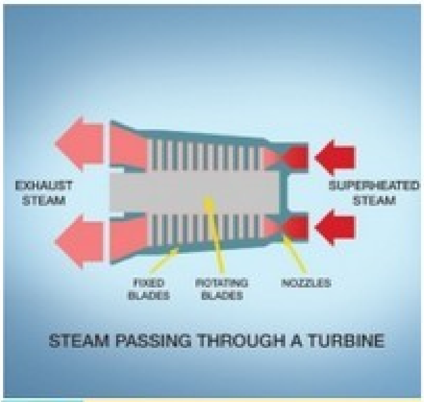
- Steam passing through a turbine.
- Gas expanding through a nozzle from an aerosol can.

In a *closed system* the gas or vapour remains within the region. The boundary between the closed system and its surroundings, however, may not be fixed. It may, for example, expand or contract with changes in the volume of the gas.

Examples of closed systems are:

- Air in a balloon being heated.
- A gas expanding in a cylinder and moving a piston.

In both open and closed systems heat and work can 'cross' the boundary.



6.1 *Steam passing through a turbine*

6.2 *Hot air balloon*

Closed systems

A process is a change from one state to another, where the state of the gas is determined by its pressure (p), volume (V) and temperature (T). In a non-flow process (where the gas does not flow across the boundary) the systems involved are closed systems and there are four basic types of changes or processes to consider:

- isothermal
- adiabatic
- constant pressure
- constant volume

In referring to each of these processes, we will make use of the formula:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

Isothermal change

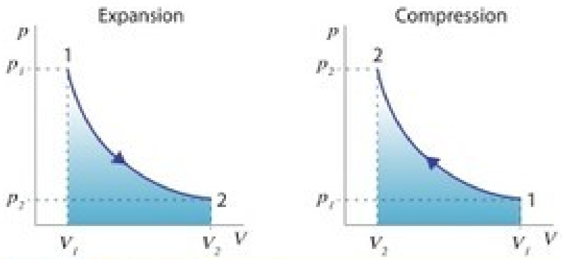
Isothermal change occurs at constant temperature. If the temperature, and hence internal energy, is to remain constant, thermal energy must be supplied to the gas and the gas will expand, doing an amount of work equal to the heat supplied. Since the internal energy does not change, ΔU = zero, so $Q = W$.

Such a process is impossible. It could only happen if the container were a perfect conductor, or the process occurred infinitely slowly allowing time for heat to transfer and the gas to always be in thermal equilibrium with the surroundings. Such a process would be truly isothermal. However, a slow process in a container which is a good conductor will be nearly isothermal.

An isothermal change obeys the law:

$pV = \text{constant}$

$p_1V_1 = p_2V_2$



Constant temperature change from p_1, V_1, to p_2, V_2.
$p_1V_1 = p_2V_2$
Work done= area shaded

6.3 *Isothermal change*

as we have seen in Module 6 this is know as Boyle's law.

6.1 Sample Question

240 cm^3 of air at a pressure of 100 kPa in a bicycle pump is compressed to a volume of 150 cm^3. What is the pressure of the compressed air in the pump?

Sample Answer

$p_1 \times V_1 = p_2 \times V_2$, rearranging to scale up for the new higher pressure

$$p_2 = p_1 \times \frac{V_1}{V_2}$$

$$= 100 \times \frac{240}{150}$$

$$= 160 \text{ kPa}$$

6.2 Sample Question

A 100 cm^3 gas syringe contains 80 cm^3 of gas that was compressed to 60 cm^3. If atmospheric pressure is 101325 Pa, and the temperature remains constant, what is the pressure of the gas in the syringe after compression?

Sample Answer

$p \times V = \text{constant}$

$p_1 \times V_1 = p_2 \times V_2$ e

$$p_2 = p_1 \times \frac{V_1}{V_2}$$

$$= 101\,325 \times \frac{80}{60}$$

$$= 135\,100 \text{ Pa}$$

Adiabatic change

An adiabatic change is one in which no heat passes into or out of the gas. Q = zero, so any work done is at the expense of the internal energy of the gas.

If the gas expands, as in a balloon bursting,

$$Q = \Delta U + W$$

therefore $W = -\Delta U$

the internal energy, hence the temperature, goes down as work is done by the gas in pushing away the surrounding air.

If the gas is compressed, as in the compression stroke of an internal combustion engine,

$$Q = \Delta U + W$$

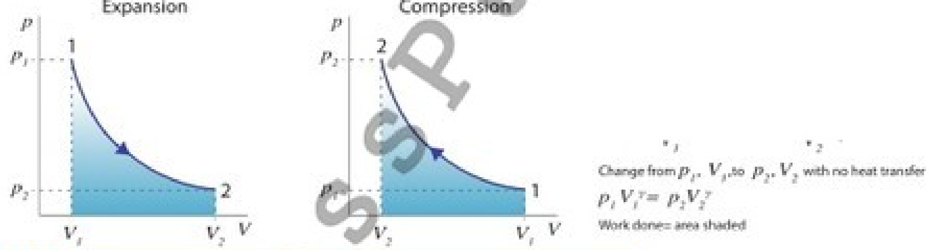
therefore $-W = \Delta U$

the internal energy, and hence the temperature, goes up as work is done on the gas.

The change obeys the law:

$$pV^{\gamma} = constant$$

$$p_1 \times V_1^{\gamma} = p_2 \times V_2^{\gamma}$$



6.4 *Adiabatic change*

Constant pressure

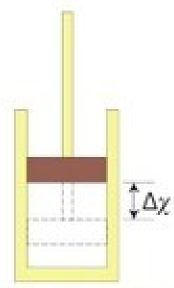
The gas in the cylinder shown in Figure 6.5 is under a constant pressure due to the weight of the piston.

If the gas is heated, the piston moves up a distance Δx. The work done W is $F \Delta x$ where:

F = pressure × area of piston

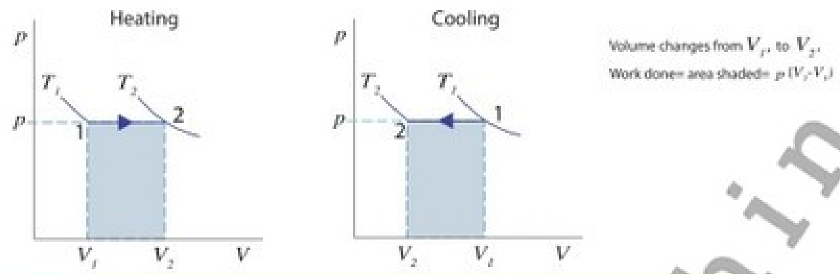
$F = p A$

Therefore $W = p A \Delta x = p \Delta V$



6.5 *Constant pressure in a piston cylinder*

Heating a gas at constant pressure causes an increase in volume (ΔV) and temperature, and external work is done owing to the increase in volume. Cooling the gas will reduce the temperature and volume.



6.6 *Constant pressure change*

Constant volume change

If the gas is heated in a fixed enclosed space, it remains at constant volume and the pressure and temperature, and hence internal energy both increase. Because there is no volume change (the boundary does not move) there can be no work done.

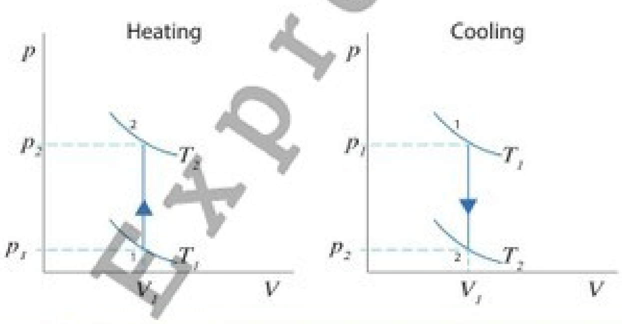
$$Q = \Delta U + W$$

where $W = 0$ (no volume change)

so $Q = \Delta U$

The whole of the energy supplied by heating is stored in the gas in the form of increased internal energy. This process occurs when a camping gas bottle is warmed by the sun throughout the day. There is usually a warning not to store in direct sunlight.

6.7 *Camping gas bottle*



6.8 *Constant volume change*

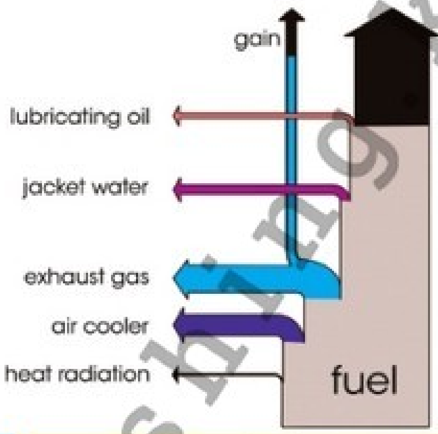
The ratio of C_P to C_V (C_P/C_V) for a gas is known as the **specific heat ratio or adiabatic index** and is commonly denoted by the Greek letter gamma (γ). For diatomic gases which exhibit ideal behaviour at standard conditions, $C_V = 5R/2$ and $C_P = 7R/2$ (where R is universal gas constant). The adiabatic index for such gases is thus given as 7/5 or 1.4.

Engine efficiency

An internal combustion engine is supplied with energy in the form of the chemical energy of the fuel. The chemical energy is transformed into internal energy which the engine in turn converts into mechanical energy. The overall efficiency of the engine is the ratio:

$$\frac{\textbf{Useful energy or work output}}{\textbf{energy input}} \times 100\%$$

We want an engine of a given size to provide as much power as possible for every kg of fuel used, so the more efficient the engine the greater the km per litre of fuel and the less the cost of running the car. Also, if less fuel is used, less CO_2 will be released into the surrounding atmosphere.

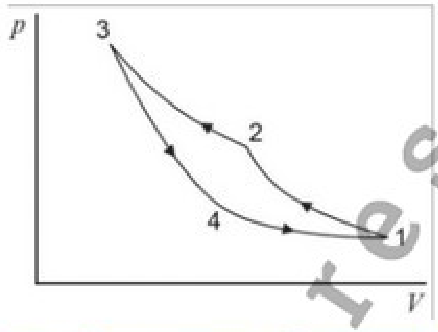


6.9 *Representation of internal combustion engine energy loss*

The Carnot Cycle

Most heat engines contain a working 'fluid' such as gas or steam which is taken through a cycle. During each cycle a net amount of work is done. At the end of each cycle the working fluid is returned to its original pressure temperature and volume, ready to go through the next cycle. There are many different types of heat engine cycle, and only a few give the ideal efficiency.

$$\frac{T_H - T_C}{T_H}$$



6.10 *The Carnot p–V loop*

One cycle which does give this ideal efficiency is the Carnot cycle, (named after Sadi Carnot, 1796 – 1832, who first proposed that the maximum efficiency of a reversible engine depends on the temperatures between which it works). For this reason, the maximum possible efficiency is sometimes referred to as the Carnot efficiency.

In the Carnot cycle the energy input Q_H is by heat transfer during an isothermal expansion, (process $3 \rightarrow 4$) and the energy output Q_C to the sink is by heat transfer during an isothermal compression ($1 \rightarrow 2$).

The rest of the cycle is made up of an adiabatic expansion ($4 \rightarrow 1$) and an adiabatic compression ($2 \rightarrow 3$) where $Q = 0$, so the net work out must equal to $Q_H - Q_C$.

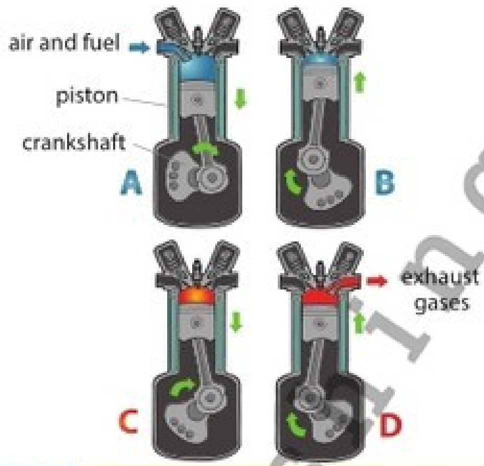
It can be shown that the efficiency of this cycle is given by

$$n_{carnot} = \frac{W_{out}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{T_H - T_C}{T_H}$$

If it were possible to make an engine that operated on the Carnot cycle, it would have a very small power output/engine size ratio because the area of the p–V loop is very small. Compare this to the internal combustion engine loops in Figures 6.16 and 6.17.

Internal combustion engine

The principle of the internal combustion engine is very simple. A mass of air is compressed at a low temperature and expanded at a high temperature. Because the work needed to compress the air at a low temperature is less than the work done by the air when it expands at a high temperature, there is a net output of work. The air in the engine must be heated in order to raise its temperature between the compression and the expansion. It is called an internal combustion engine because this heating is done inside the engine.

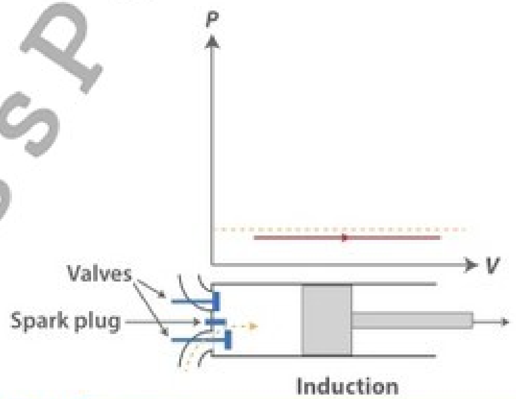


6.11 *The internal combustion engine*

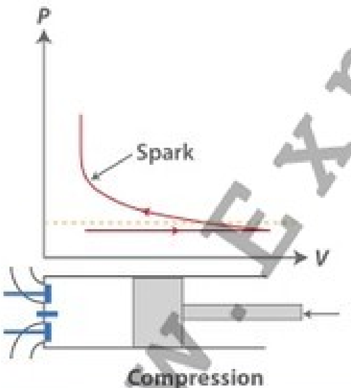
In a petrol or diesel engine a piston moves easily up and down in a cylinder with an almost gas-tight fit between the two. Each movement of the piston up or down is called a stroke. In a four-stroke engine, the fuel is burned once every four strokes. The sequence of operations for one complete four-stroke petrol engine cycle, comprising induction, compression and exhaust strokes is outlined in the four stage diagram 6.11.

Induction

The piston travels down the cylinder. The volume above it increases the mixture of air and petrol vapour is drawn into the cylinder via the outer valve. The pressure in the cylinder remains constant, just below the atmospheric pressure.



6.12 *Induction*



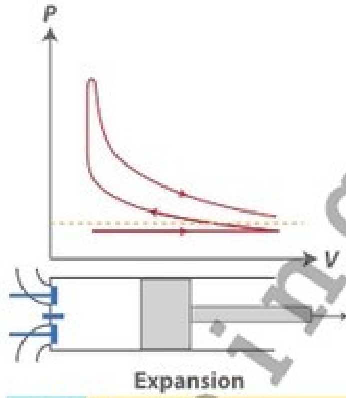
6.13 *Compression*

Compression

Both valves are closed. The piston travels up the cylinder, the volume decreases and the pressure increases. Work is done on the air to compress it. Very near the end of the stroke the air/petrol mixture is ignited by a spark at the spark plug, resulting in a sudden increase in temperature and pressure at almost constant volume.

Expansion

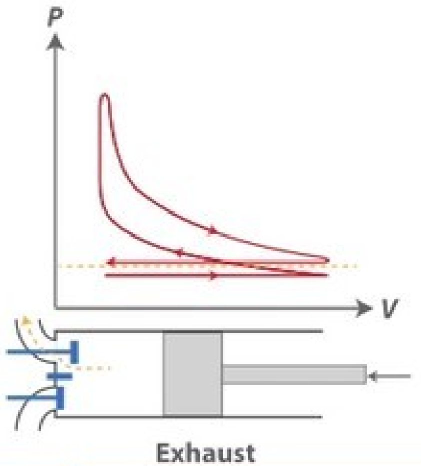
Both valves remain closed, and the high pressure forces the piston down the cylinder. Work is done by the expanding gas. The exhaust valve opens when the piston is very near the bottom of the stroke, and the pressure reduces to nearly atmospheric.



6.14 *Expansion*

Exhaust

The piston moves up the cylinder, expelling the burnt gases through the open exhaust valve. The pressure in the cylinder remains at just above atmospheric pressure.



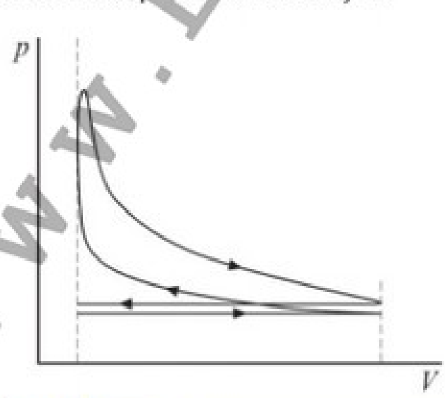
6.15 *Exhaust*

All engines must obey the Second Law of Thermodynamics. This Law tells us that the efficiency of any process for converting heat into work cannot approach 100%. In other words an ideal engine which satisfies both laws of thermodynamics must have a source and a sink. The engine must be subjected to heating from the source and it must reject some energy to the sink. The source must be at a higher temperature than the sink.

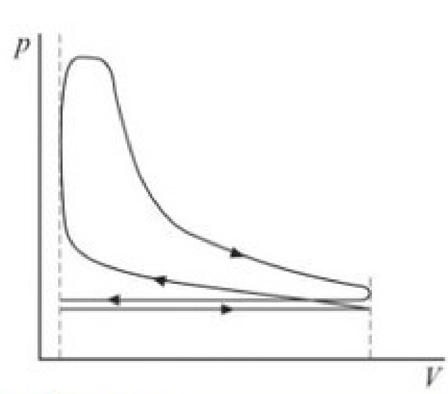
Indicator Graphs

The *p*–*V* graphs below are called indicator diagrams. They are produced using computer software by engineers to evaluate engine efficiency. They vary with parameters such as engine load and speed, and the timing of the spark (petrol), or fuel injection (diesel).

Figures 6.16 and 6.17 show typical indicator diagrams for four-stroke petrol and diesel engines for one complete mechanical cycle.



6.16 *Petrol engine*



6.17 *Diesel engine*

The work done **on** the gas during the compression stroke is given by the area underneath the compression curve, and the work done **by** the gas during the expansion stroke is given by the area underneath the expansion curve. Therefore the **net work done** by the air is given by the **area enclosed by the loop** on the p-V diagram.
If the net work done is divided by the time for one cycle, the indicated power is obtained.
Time for one cycle = 1/cycles per s

Therefore:

indicated power = area enclosed by loop of $p - V$ diagram × number of cycles per second

If there is more than one cylinder in the engine

indicated power = area of $p - V$ diagram × number of cycles per second × number of cylinders

Because there is one power stroke for every two revolutions of the crankshaft this can be changed to:

indicated power = area enclosed by of $p - V$ diagram × $\frac{1}{2}$ (rev min^{-1}/60) × number of cylinders

The area of the small loop formed between induction and exhaust strokes is negative work, and should be subtracted from the main loop area to give the true net work, but in a real indicator diagram the area is so small as to be negligible. In fact, the induction and exhaust strokes usually show up as a single horizontal line.

Some of the power developed by the air in the cylinder is expended in overcoming the frictional forces between the moving parts of the engine and the viscous resistances of the lubricating oil and cooling water. This is called the **friction power**. The output power will, therefore, be less than the indicated power by an amount equal to the friction power.

Calculations of power and efficiency

input power = calorific value of fuel × fuel flow rate

For liquid fuel, the flow rate will be in kg s^{-1} and the calorific value in MJ kg^{-1}.
For a gas, flow rate is usually in m^3 s^{-1} and the calorific value in MJ m^{-3}.

Indicated power = power developed in the cylinders of the engine =
area of p-V diagram × number of cycles per second × number of cylinders

6.3 Sample Question

Test measurements made on a single-cylinder 4-stroke petrol engine produced the following data:

- mean temperature of gases in cylinder during combustion stroke 820 °C
- mean temperature of exhaust gases 77 °C
- area enclosed by indicator diagram loop 380J
- rotational speed of output shaft 1800 rev min^{-1}
- power developed by engine at output shaft 4.7kW
- calorific value of fuel 45 MJ kg^{-1}
- flow rate of fuel 2.1×10^{-2} kg min^{-1}

Calculate:

(a) the rate at which energy is supplied to the engine
(b) the indicated power of the engine
(c) the thermal efficiency of the engine.

Sample Answer

(a) The energy supplied = calorific value x rate of flow
= 45 x 106 J/kg x (2 x 10^{-2} kg/min ÷ 60 s)
= 15 800 J/s

(b) Engine goes through the power cycle once every two revolutions.
There are 15 cycles per second: Indicated power = 380 J x 15 s^{-1} = 5700 J/s

(c) Thermal efficiency = indicated power ÷ input power
= 5700 ÷ 15 000
= 0.38 (38 %)

6.4 Sample Question

The power gained from the fuel of the engine was 15.8 kW. If the power output is 4.7 kW, what is the overall efficiency?.

Sample Answer

Overall efficiency = output power ÷ input power from fuel
Mechanical efficiency = 4700 W ÷ 15800 W
= 0.297 (29.7%)

For you to try

1 A car engine has a power output of 6.2 kW and uses fuel which releases 45 MJ per kg when burned. At a speed of 30 m s^{-1} on a level road, the fuel usage of the vehicle is 18 km per kg. Calculate:
(a) The useful energy supplied by the engine in this time;
(b) The overall efficiency of the engine.

2 The energy content of petrol is about 44.1MJkg^{-1}. Calculate the input power when 5.20 kg of fuel flowed into an engine in one minute.

3 Gas at 120 kPa is rapidly heated from 20°C to 243°C in a sealed container. What is the new pressure of the gas?

4 Diesel fuel has a calorific value of 42.9MJkg^{-1}. Calculate the input power if the flow rate is 6.00×10^{-2} kgs^{-1}.

5 Calculate the thermal efficiency for a car, for which the following data is given:
input power = 462 000 W
indicated power = 122 000 W

6 The area of an indicator diagram gives 960 J. If a four-stroke engine is rotating at 4200 rpm, calculate the indicated power per cylinder.

7 Gasoline vapour is injected into the cylinder of an automobile engine when the piston is in its expanded position. The temperature, pressure, and volume of the resulting gas-air mixture are 20°C, 1.00×10^5 Pa and 240 cm^3, respectively. The mixture is then compressed adiabatically to a volume of 40 cm^3. The adiabatic index of the air-fuel mixture can be taken as $\gamma = \frac{7}{5}$. What is the pressure of the mixture after the compression?

The home refrigerator

This is a beautiful example of the interchange between work and heat. At the heart of refrigerators and air conditioning units is what we call a 'heat pump'. To understand it properly it is best to consider each part of the operation separately:

Step 1

The electrical energy supplied by the power station is used to drive an electrical motor which operates a compressor to pump gas from a low pressure reservoir to a higher pressure reservoir. This work that is being done on the gas molecules causes the gas to heat up (the individual molecules are moving randomly at faster velocities). This may seem like a strange thing to do if you want to try to cool the inside of the refrigerator! The important thing to remember is that compressing a gas is hard work, and it produces heat (you may have noticed the valve of a bicycle tyre gets hot when you actively pump air into it).

Step 2

This heat produced by the compression needs to be dissipated through the use of a 'radiator' that is normally located at the back of the refrigerator. Heat is conducted into a grid of metal wires (sometimes called a 'condenser'). These heated wires then release the heat to the air by convection. By the time the compressed gas comes out the other side of this heat exchanger the gas is nearly back to room temperature. This heat has been 'wasted'! (It could be used to help to dry towels or some other application, but generally no use is made of it).

Step 3

Now the compressed gas is made to pass through a small nozzle (sometimes called an 'expansion valve') so that it can expand into another set of tubes inside the cool box. The action of expanding means that the gas is having to do work in pushing back the frontiers of the space it will occupy. It does this work at the expense of its internal energy, and as a consequence of this the random motion of the molecules slows down and the mean temperature of the gas drops. You may have noticed that when you spray an aerosol can of perfume the nozzle, the can, and even the gas coming out of the can cools down.

Step 4

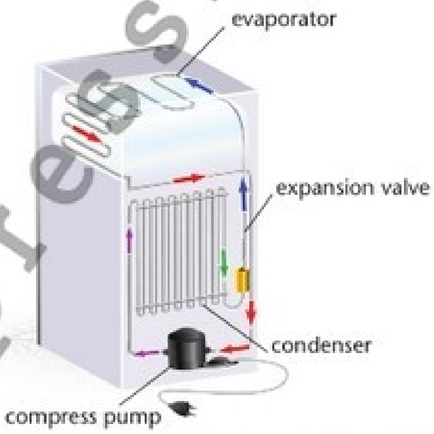
The cooled tubes inside the refrigerator cold box can get so cold that water condenses out of the air on to them, and can even form ice! Now it is time for the food inside the refrigerator to lose its heat to the cold surroundings inside the cold box. This is how the contents of the refrigerator are cooled down. However, the cold gas will gradually warm up as it absorbs the heat extracted from the food inside.

Eventually the low pressure gas will need to be brought out of the cold box and re-compressed (back to step 1), and the cycle repeats again and again. You may have heard the compressor of your home refrigerator switching on and off periodically. Every time that the temperature inside the cold box creeps up above the thermostat set point, the compressor switches on, and it keeps on running until the temperature inside the cold box drops below the thermostat set point.

If you notice that the compressor of your refrigerator seems to be running most of the time, it may mean that:

(a) the door of the refrigerator is not closed properly,
(b) the thermal insulation of the cold box is no longer adequate or
(c) the refrigerant gas inside the system has slowly leaked out.

A refrigerator contains a fixed amount of gas which it compresses (outside) and expands (inside), and in the process 'pumps heat' out of the cold box to the outside world.



6.18 *Refrigerator showing the interchange between work and heat*

For you to try

1. Why does the nozzle of a bicycle wheel get hot when you are pumping up the tyres?
2. Why might a CO_2 fire extinguisher get very cold when it is used? (So cold, that if you hold the venting horn with your bare hands, your fingers will freeze on to it!)
3. Why does a car run less efficiently (getting fewer kilometres for a litre of fuel) if the air-conditioning is switched on?
4. A student notices that if he breathes out against his hand with his mouth wide open, the air coming from his mouth feels warm. However, when he tightens his lips to make a small exit and blows hard on his hand, the air feels cold. Can you explain this difference?

Module 7 Electrostatics

Learning objectives

- Apply the charge conservation law and the Coulomb's law in problem-solving 10.4.1.1
- Apply the principle of superposition for defining the electric field intensity 10.4.1.2
- Apply the Gauss theorem to define electric field intensity of infinite charge planes, full spheres, spheres and endless thread 10.4.1.3
- Calculate potential and work done by electric field of point charges 10.4.1.4
- Use the formula connecting power and energy characteristics of electrostatic field in problem-solving 10.4.1.5
- Compare power and energy characteristics of gravitational and electrostatic fields 10.4.1.6
- Make comparative analysis of electric induction in conductors and polarisation in dielectrics 10.4.1.7
- Research the relationship between condenser capacity and its parameters 10.4.1.8
- Use the formula of series and parallel connection of condensers in problem-solving 10.4.1.9
- Calculate the electric field energy 10.4.1.10

History

7.1 *Lightning is a natural phenomenon caused by electricity*

You do not need any education or specialist knowledge to be aware of electricity. Lightning, and the associated thunder, is a spectacular natural phenomenon that has been known to all cultures throughout history. The symbol of the most powerful Greek god, Zeus, was the thunderbolt, and many other cultures throughout Europe, South America and India have associated lightning with their gods.

In some parts of the world, people are familiar with other natural sources of electricity such as electric eels and some breeds of catfish.

There are also pre-Christian accounts noting that when one rubs fur against the naturally occurring material known as amber, the amber can then be used to lift small and light objects such as pieces of straw.

All of these are phenomena associated with electricity.

Static electricity

If you take a plastic rod and rub it against a different material, such as your hair, you should notice an effect similar to that mentioned above involving amber. You might be able to pick up small pieces of paper using the rod. American scientist and politician Benjamin Franklin (1706–1790) studied this effect using glass rubbed against a piece of silk. He described the charge on the glass as **positive** and that on the silk as **negative**, the first time that these terms had been used.

Experiment 7.1: To investigate electric charge

Method 1

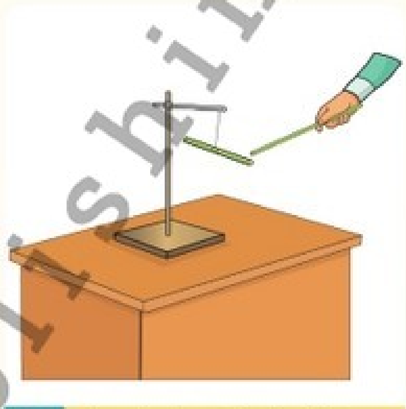
1 Tear up a piece of paper into sections only a few centimetres across.
2 Take a plastic rod and rub it against your hair to charge it.
3 Hold the rod close to the pieces of paper and observe what happens.

Observations

You should find that the pieces of paper are attracted upwards towards the plastic rod.

Method 2

1 Place a plastic rod in a stirrup suspended from a wooden retort stand, as shown in figure 7.2.
2 Take a second plastic rod and rub it against your hair to charge it.
3 Hold the second rod close to the first one and observe what happens.



7.2 *Investigating static electricity*

Observations

You should find that the first rod, suspended in the stirrup, is attracted to the second one, in your hand.

Method 3

1 Take a plastic rod and rub it against your hair to charge it.
2 Place the charged plastic rod in a stirrup suspended from a wooden retort stand, as shown in figure 7.2.
3 Take a second plastic rod and rub it against your hair to charge it.
4 Hold the two charged rods close to each other and observe what happens.

Observations

You should find that the two charged rods repel each other.

Method 4

1 Turn on a tap so that there is a gentle but steady flow of water.
2 Take a plastic rod and rub it against your hair to charge it.
3 Hold the charged rod close to the water stream and observe what happens.

Observations

You should find that the stream of water is deflected by the presence of the electric charge.

7.3 *An effect of static electricity*

Method 5

1 Rub a balloon against your hair to charge it.
2 Hold the charged balloon against a smoothly plastered wall.

Observations

If the charge is sufficient you should find that the balloon sticks (briefly) to the wall.

7.4 *An effect of static electricity*

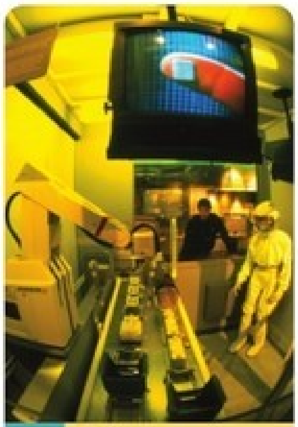
Charges

All of the effects described above come about because of something called **electric charge**, of which there are two types: positive and negative. We often summarise how these charges interact with each other using the key phrase:

Like charges repel, unlike charges attract.

In other words, if we place two positive charges close to each other, they will be repelled from each other. The same happens with two negative charges. By contrast, if we place a positive charge close to a negative charge, the two will be attracted towards each other. Sometimes, if the charges are significant, there can be a spark as the charges jump towards each other.

This can be problematic in many ways. Workers in electronic industries often wear special clothing to prevent damage to circuitry from the small charges that can build up on clothes. Airport workers have to be very conscious of the dangers that come from static charges when they are refuelling aircraft. Workers in flour mills have similar concerns.



7.5 *A clean room in an Intel plant, showing the protective clothing used by technicians in the manufacture of semiconductor chips*

There is an obvious comparison to be made between electricity and magnetism, which also involves like poles repelling and unlike poles attracting. The two phenomena are indeed tightly interconnected, as we will see in more detail when we study the area of electromagnetism. At the same time, it is important to keep the two ideas separate in your thinking.

Electric charge cannot exist by itself. It is a property of tiny particles known as subatomic particles. There is a wide variety of such particles, many of which – such as pions and W bosons – you are unlikely to have heard of, but may learn about when you study modern physics. Generally though, when you encounter electric charge in ordinary situations, that charge is likely to be associated with more familiar particles: protons and electrons.

Protons and electrons are two of the key particles that make up atoms. Recall the basic structure of the atom: the protons are positively charged and are found, alongside a number of non-charged neutrons, in the nucleus; the negatively charged electrons are found circling the nucleus.

It is worth noting that the use of 'positive' and 'negative' as labels to describe the different types of electric charge is a scientific convention. We could just as easily have labelled them 'up' and 'down' charges, or 'left' and 'right'. Positive and negative is just one convenient pair of opposites that is easily understood.

7.6 *The negatively charged electrons are found in shells surrounding the positive nucleus*

Every object contains a very large number of atoms and, therefore, an enormous number of electric charges. A typical plastic rod, like that you used in Experiment 7.1, would contain about 1 000 000 000 000 000 000 000 000 atoms and an even larger number of protons and electrons. However, because it will generally contain the same number of protons and electrons, it has no overall charge. We say that it is **electrically neutral**.

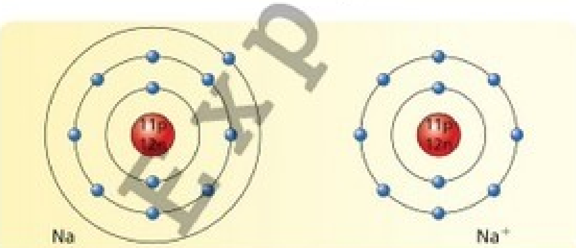
If the rod gains even a few electrons, though, it will have more negative than positive charge and will become negatively charged overall. Similarly, gaining extra protons would make it positively charged.

If you look at the diagram of the atom in figure 7.6, you will see that it is always easier to add or remove the relatively loosely bound electrons on the outside of the atom than to vary the number of protons, which are located deep inside the nucleus. For this reason, it is almost always the addition or removal of electrons that causes an object to become either negatively or positively charged.

If an object is electrically neutral, it has the same number of protons and electrons. **If it gains electrons** it will have more electrons than protons and will be negatively charged. **If it loses electrons** there will then be a majority of protons and the object will be positively charged.

Ions

When atoms either gain or lose electrons and therefore become charged, we refer to them as **ions**. So, for example, if an atom gains electrons it is a **negative ion**. This often happens with atoms such as chlorine. Alternatively, other atoms such as sodium tend to lose electrons and become **positive ions**, as shown in figure 7.7.



7.7 *A sodium atom (on the left) and a positive sodium ion (on the right)*

Charging by contact

As you saw in Experiment 7.1, small electric charges can be built up by rubbing one material against another. When this is done, electrons are physically removed through friction, from one of the materials and added to the other. This is called **charging by contact**.

Charging through contact involves the physical transfer of electrons from one object to another when the objects are in contact.

Because of this effect, mirrors and TV screens should not be cleaned using a dry cloth. The friction between the two builds up a charge, which then causes dust to settle on the mirror or the screen. On a larger scale, a static electric charge built up in this way can create a hazard in flour mills, where a spark from a built-up charge can cause an explosion. Aircraft are always carefully earthed before refuelling to remove any charge that might have built up during a flight.

Charging through induction

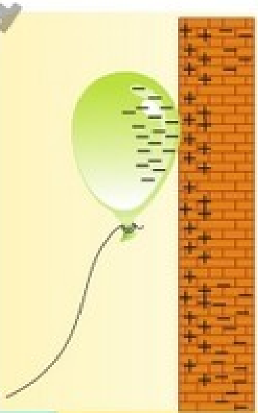
When a balloon is charged by rubbing it against your hair, as in Experiment 7.1, it usually builds up a negative charge by collecting additional electrons in the manner we have already discussed. However, if the balloon is made to stick to a wall as shown in figure 7.9, there must also be a charge of some sort on the wall. It is the attraction of opposite charges, after all, that makes it stick there. Where does the positive charge on the wall come from?

7.8 *Aircraft are earthed before refuelling*

The answer is that the negative charge on the balloon has repelled electrons from the surface of the wall as it approached. This left a small area of positive charge on that portion of the wall, and the force of attraction between the opposite charges was enough to keep the balloon there.

An object that becomes charged in this way – without any contact with another body – has been **charged by induction**.

Something similar happens when a flow of water is displaced by electric charge. A positive rod held near the flow of water will cause negative charges to accumulate in the side of the water that is nearest the rod. The attraction of opposite charges will then cause the flow of water to divert, towards the rod.



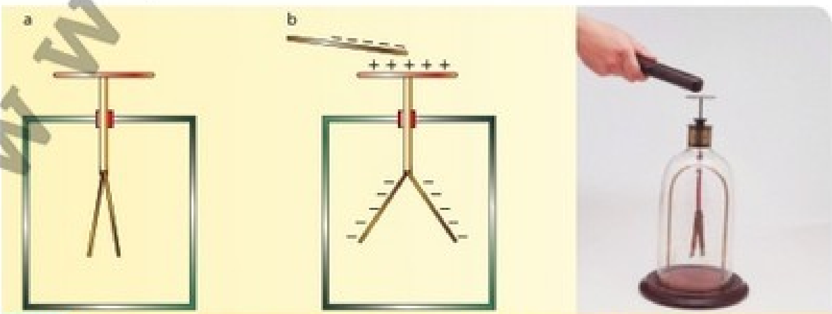
7.9 *The negative charge on the balloon induces a positive charge on the wall*

Induction is the creation, or redistribution, of electric charge on an object through the action of nearby charges.

Gold leaf electroscope

The gold leaf electroscope is a device that allows us to study how an object can be charged through induction. The traditional electroscope consists of a metal cap connected with a metal bar to two gold leaves below, as shown in figure 7.10.

If a negatively charged plastic rod is brought close to the cap, it repels electrons from the cap. They are forced down through the connecting bar to the leaves below. The two leaves then have the same negative charge and are repelled from each other. As they are so light, this repulsion is sufficient to cause them to lift away from each other, as shown in figure 7.10.

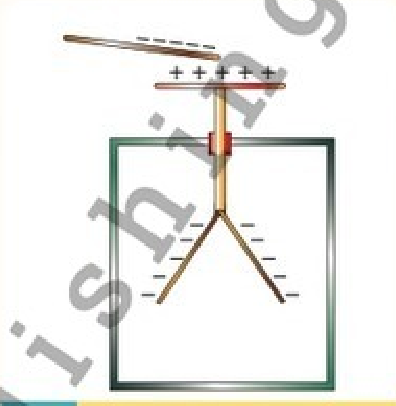


7.10 *The charge on the cap and leaves of a gold leaf electroscope is created without contact and is therefore induced*

Experiment 7.2: To investigate an electroscope, and demonstrate charging by induction

Method

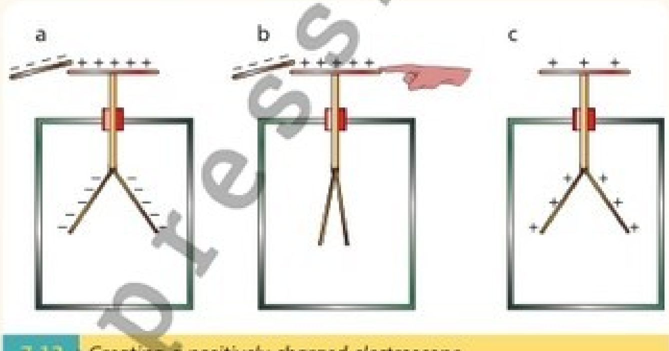
1 Set up a non-charged electroscope.
2 Take a plastic rod and rub it against your hair to charge it.
3 Hold the charged rod close to the cap of the electroscope and observe what happens to the leaves (see figure 7.11).
4 To make the charge 'permanent', bring the charged plastic rod close to the electroscope, and when the leaves are spread out, gently touch the cap of the electroscope with your finger.
5 Remove your finger, and then remove the plastic rod. Observe what happens now.



7.11 *A charged electroscope*

Observations

When you hold the charged rod to the cap of the electroscope, you should see the leaves diverge. When you touch the cap of the electroscope with your finger, you should notice that the leaves collapse. When you remove your finger and the charged rod, you should see the leaves spread out once more. This effect is not truly permanent, as the induced charge will leak into the atmosphere over a number of minutes.



7.12 *Creating a positively charged electroscope*

Earthing

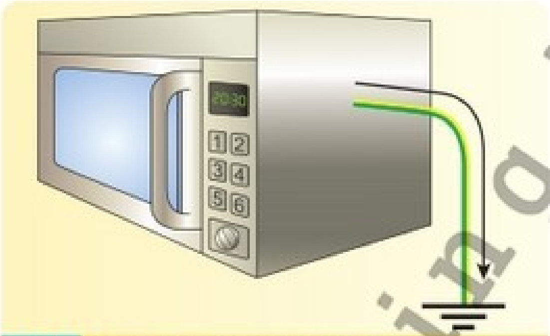
In Experiment 7.2 you found that the leaves on an electroscope will collapse when the cap is touched. This is because we have effectively **earthed** the cap.

An electric charge will always tend to spread itself out as much as possible. For example, if a metal sphere has a negative charge, this means that it has acquired a number of extra electrons. These electrons are all repelled from each other and move as far apart from each other as possible. On a metal sphere this is easy, as both the shape and the material allow the charge to spread itself evenly across the whole surface.

On plastic objects, which do not allow electrons to move easily, the effect can be reduced. The charges are forced to stay in one area.

When a charged object is connected to the earth, the charge spreads out so much it effectively disappears. We refer to this as **earthing** the object. It is built in as a safety feature in electrical wiring, using the earth wire. This is a green and yellow wire that will carry any charge building up on malfunctioning devices to earth before they cause harm to users.

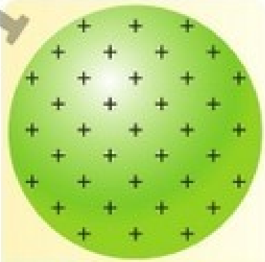
With the electroscope in Experiment 7.2, the charge effectively disappears when we touch it because we are so much larger than it is. We are operating as the 'earth'.



7.13 *In a malfunctioning electrical device, charge is carried to earth to avoid danger*

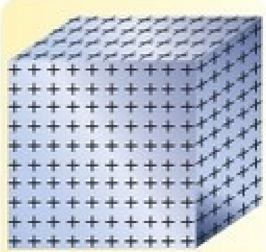
Point effect

We have seen how electric charge will always spread out as much as possible. On a metal sphere this always works well because it is a smooth, symmetrical surface and it easy for the charges to find an arrangement whereby they are all as far away from each other as possible (see figure 7.14).



7.14 *The charge is evenly distributed*

If a metal cube becomes charged, the charges will still spread out to be as far from each other as possible. However, if you look at the corners in figure 7.15, you can see how this becomes problematic: charges are pushed away from the centre of each face of the cube so they can spread out as much as possible, but when they are pushed into a corner, this means that they are being pushed towards the other similar charges on an adjacent face. Nature will try to strike a balance between all the competing forces, but there is no perfect arrangement and there is an inevitable build-up of charge at the corners.



7.15 *There is a build-up of charge at the corners*

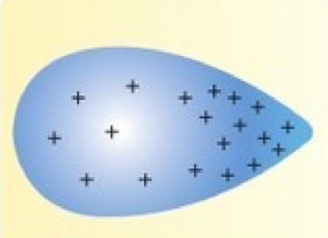
The cube is just one example of a shape in which this problem occurs. In every shape, other than a sphere, something similar happens. This is known as the **point effect**.

The point effect describes how if an object is less curved than a sphere, and particularly if it has sharp points, there will be a build-up of charge.

Point discharge

The point effect can lead to the object becoming discharged.

Remember that air is made up of molecules and that these molecules consists of protons and electrons like all other materials. If there is, say, a very large positive charge at a point on a conductor, this will attract the negatively charged electrons within the molecules in the air towards it and push away the positively charged protons. If the charge is sufficient, the negatively charged electrons can be removed from their



7.16 *Where the curve is reduced and a point develops, there is a build-up of charge* atoms in the air, leaving behind positively charged ions. The electrons are then attracted to the metal surface and neutralise the positive charge that was there, while the positively charged protons are repelled from the surface. The result of this is that the charge is taken from the conductor.

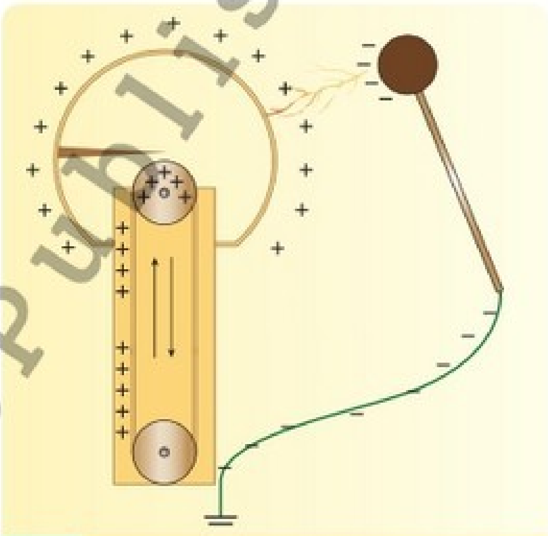
Point discharge is the loss of charge through a point on an object.

The Van de Graaff generator

American physicist Robert J. Van de Graaff (1901–1967) developed his generator in 1929, while trying to develop an early particle accelerator. In the Van de Graaff generator, a rubber belt runs over a pulley at the lower end of a tube and builds up an electric charge through friction. A needle that protrudes from the inside of a metal dome towards the belt makes use of the point effect to draw some of that charge from the belt and transfer it onto the dome. This process repeats continuously.

Although the belt would only ever be able to hold a small charge, the smoothly curved metal dome is perfectly designed to hold a huge charge. Large modern generators of this design can hold enormous electric charges, and rise to a potential of up to 5 million volts.

A second metal sphere is usually used to investigate the charge. A large negative charge builds up on its surface due to induction. The attraction between this negative charge and the positive charge on the dome often leads to large sparks jumping between the two.



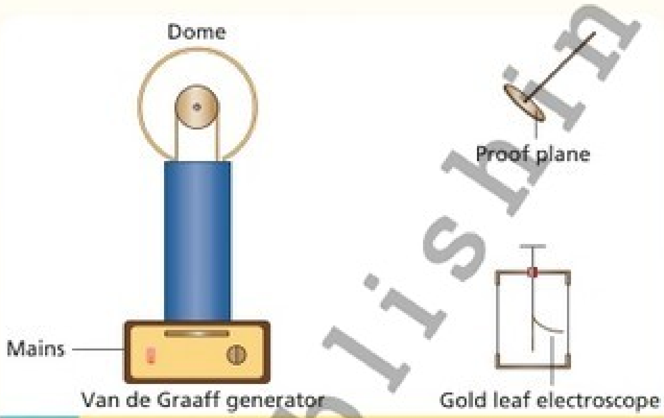
7.17 *A Van de Graaff generator*

7.18 *A girl placing her hand on a Van de Graaff electrostatic generator. Strands of the girl's hair repel each other because they are similarly charged*

Experiment 7.3: To investigate the distribution of charge on a conductor using a Van de Graaff generator

Method 1

1. Set up a Van de Graaff generator and a gold leaf electroscope, as shown in figure 7.19.
2. Bring a proof plane near to the dome of the Van de Graaff generator.
3. Touch the side furthest from the dome to earth it.
4. Remove the connection to earth.
5. Hold the proof plane to the gold leaf electroscope, and check for charge.
6. Repeat at different points on the dome.



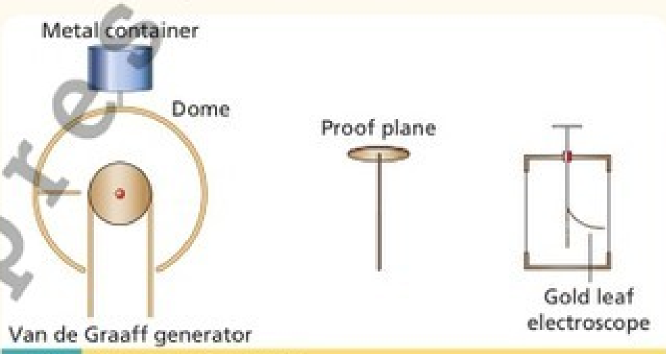
7.19 *Experimental apparatus*

Observations

You should find that the charge is evenly distributed over the dome of the Van de Graaff generator.

Method 2

Repeat Method 1, but this time holding the proof plane inside a hollow metal container on the top of the dome before transferring it to the electroscope (see figure 7.20).



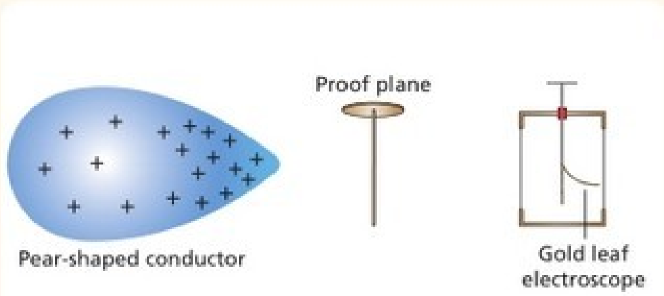
7.20 *Experimental apparatus*

Observations

You should find that there is no charge (and no electric field) inside a metal conductor.

Method 3

Repeat Method 1, but this time using a pear-shaped conductor.



7.21 *Experimental apparatus*

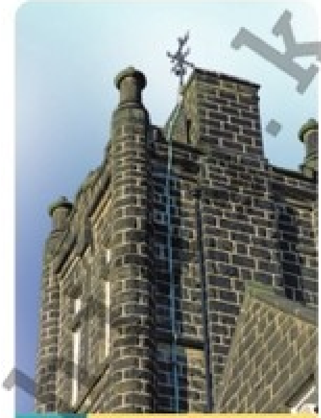
Observations

You should find that there is a larger charge on those parts of the conductor that are more pointed.

Lightning conductors

Most tall buildings have a lightning conductor something like that shown in figure 7.22 attached to the highest point on their roof, and connected to earth by means of a metal strip that runs down through the building.

You are probably already aware of lightning conductors and understand that they are there to protect a building from lightning. It is important to be aware, however, that they do this in two ways. It is true, as is widely understood, that in the event of a lightning strike the lightning conductor provides a relatively safe path to earth for the charge. This can minimise damage, but it does not always eliminate it. The enormous charge running through the metal and the great heat generated as a result can still cause damage in surrounding materials, and there is always a risk that some charge may spark across to the electrical wiring within the building and cause substantial damage there.



7.22 *Lightning conductor on a school tower. The conductor consists of a metal strip, usually copper, of very low resistance connected to the ground below. A good connection to the ground is essential and is made by burying a large metal plate deep in the earth*

It is less widely understood that if lightning strikes the building, the lightning conductor has to an extent already failed. Its primary purpose is to prevent a strike in the first place.

When an electrical storm is developing, a large amount of charge builds up in the atmosphere. This build-up will have the effect of inducing a charge to grow at the point of any lightning conductor in the vicinity: a large positive charge in the atmosphere, for example, will attract a comparable negative charge to the point of the conductor, drawing that charge from the earth.

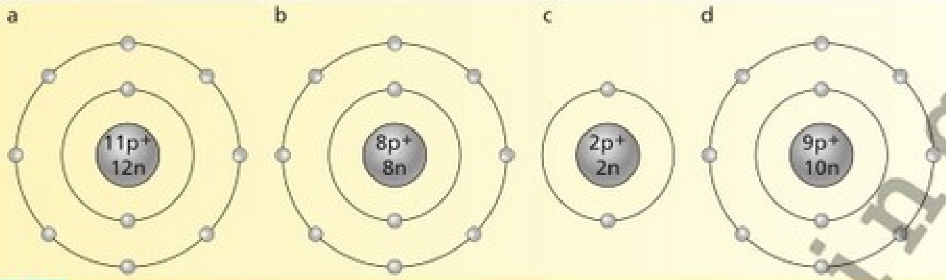
The negative charge on the conductor then draws the positive charge in the atmosphere towards itself. The large electric charges also causes the surrounding air to become charged (or ionised), which makes the air a better conductor than would normally be the case and allows the charge to flow through the air. When the positive charges meet the negative, both are cancelled out. This process will then repeat on an ongoing basis as long as there is charge in the atmosphere, and has the obvious effect of slowly removing most of that charge from the atmosphere, at least in the vicinity of the tall building. In this way a lightning strike is prevented by the lightning conductor.

7.23 *The positive charge in the air attracts a negative charge to the lightning conductor. The two neutralise each other*

For you to try

1. Distinguish between charging by friction and charging by induction.
2. Give examples of how a body can be charged by friction.
3. It is always advisable to clean a mirror with a damp cloth rather than a dry one. Why?
4. What is an ion?

5 What is the overall charge on each of the atoms or ions represented in figure 7.24?

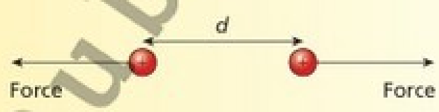


7.24 *Question 5*

6 What do we mean by the term 'earthing'?
7 What is the point effect?
8 Explain how a lightning conductor protects a tall building from damage.

Coulomb's law

We know already that like charges repel each other and that opposite charges attract. Repulsion and attraction are examples of **force**. Forces are vector quantities, which means that they have both a magnitude and a direction.



7.25 *The force between two point charges is related to the product of the charges and the distance between them*

Look at figure 7.25, and ask what force exists on the positive charge. It is immediately obvious what the direction of that force will be: as a positive charge it will be repelled from the other positive charge, a distance d away. However, we also need to be able to determine what the magnitude of that force will be. This is where **Coulomb's law** comes in. Charles-Augustin de Coulomb (1736–1806) was a French physicist who in 1785 laid down the law now named after him:

Coulomb's law states that the force between two point charges is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them:

$$F \propto \frac{Q_1 Q_2}{d^2}$$

where:

F – force
$Q_1 Q_2$ – charges
d – distance between charges

The coulomb is a very large charge. It takes billions upon billions of electrons to build up a total charge of 1 C, for instance. For this reason, you will often find yourself dealing with charges that are a fraction of a coulomb, such as a millicoulomb, or microcoulomb.

$$1\,\text{mC} = 1 \times 10^{-3}\,\text{C}$$
$$1\,\mu\text{C} = 1 \times 10^{-6}\,\text{C}$$

Coulomb's law is an example of an **inverse square law**. This tells us that the force between charges diminishes as charges move apart, and that if we double the distance between them, the force is divided by four (2^2), or if we increase the distance between the charges by a factor of 3, that the force will be divided by nine (3^2), and so on. Newton's law of gravitation is also an inverse square law.

Whenever two measurements are proportional to each other, as here, we can also say that one is a constant times the other. The constant here is represented by the following expression:

$$\frac{1}{4\pi\varepsilon}$$

So the equation becomes:

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{d^2}$$

Permittivity (ε)

Think about two positive electric charges placed close together with air between them. We know that they will create a repulsive force on each other, and we have just seen that Coulomb's law allows us to calculate how large that force will be. Then think about how the two forces might affect each other if they were immersed in water, or a lump of plastic, or metal. It's not difficult to see that the material between the charges will affect the forces created between them.

The **permittivity** (ε) is how we deal with this issue. Each material has a specific permittivity value, and this value is part of the calculation in Coulomb's law. A large value for permittivity indicates a reduced value for the force between charges, whereas a low value for the permittivity indicates a higher value for the force.

You will often come across what we call the **permittivity of free space**, indicated by ε_0. This indicates the permittivity of a vacuum. Air has very little effect on the force between two charges, and so free space, in this instance, is usually taken to represent air as well as a vacuum.

Permittivity of free space, $\varepsilon_0 = 8.854 \times 10^{-12}\ Fm^{-1}$

Relative permittivity

Rather than listing a value for the permittivity of each material, it is often easier to just compare the permittivity with that of free space. Thus, a relative permittivity of 2 indicates a permittivity twice that of free space.

Relative permittivity: $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

7.1 Sample Question

What is the force between two charges of +3 mC and −7 mC, a distance of 2 m apart?

7.26

Sample Answer

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}$$

$$= \frac{(3\times10^{-3})(7\times10^{-3})}{4\pi(8.854\times10^{-12})(2^2)}$$

$$\approx 47186\,N$$

(The charges are attracted towards each other.)

$\varepsilon_0 = 8.854\times10^{-12}\ Fm^{-1}$

Remember, as we saw with Newton's laws, forces always occur in pairs, and the two forces are always equal in magnitude but opposite in direction. In Sample Question 7.1, therefore, the positive charge is attracted towards the negative and the negative towards the positive. And, despite the fact that one charge is more than double the other, the forces are equal in magnitude.

7.2 Sample Question

What is the value of the permittivity of a plastic, with a relative permittivity of 5.9?

Sample Answer

$$\varepsilon = \varepsilon_r\varepsilon_0$$

$$= (5.9)\,(8.854 \times 10^{-12})$$

$$= 5.224 \times 10^{-11}\ Fm^{-1}$$

$\varepsilon_0 = 8.854\times10^{-12}\ Fm^{-1}$

7.3 Sample Question

Two identical charges are placed a distance of 25 cm from each other in a vacuum. The force between them is 1.2 N. What is the size of the charge?

Sample Answer

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}$$

$$1.2 = \frac{1}{4\pi(8.854\times10^{-12})}\frac{Q_2^{\,2}}{0.25^2}$$

$$Q_2^{\,2} = (0.25^2)\,(4\pi)(8.854 \times 10^{-12})\,(1.2)$$

$$Q_2^{\,2} = 8.345 \times 10^{-12}$$

$$Q = 2.89\times10^{-6}\,C$$

7.4 Sample Question

Three positive charges are arranged as shown in figure 7.27. What is the total force on the charge of 2 mC and in what direction?

7.27

Sample Answer

F_4 (force on 2 mC created by 4 mC) $= \dfrac{(4\times10^{-3})(2\times10^{-3})}{4\pi(8.854\times10^{-12})(0.2)^2}$

$= 1.798\times10^{6}$ N, to the right

F_6 (force on 2 mC created by 6 mC) $= \dfrac{(2\times10^{-3})(6\times10^{-3})}{4\pi(8.854\times10^{-12})(0.4)^2}$

$= 6.741\times10^{5}$ N, to the left

$F = 1.798\times10^{6} - 6.741\times10^{5}$

$= 1.124\times10^{6}$ N, to the right

For you to try

1 State Coulomb's law.

2 What is the magnitude and direction of the force on the 5 mC charge shown in figure 7.28?

7.28 *Question 2*

3 A charge of +3 μC is placed at a distance of 30 cm from a charge of +4 μC, in a vacuum. What is the magnitude of the force between them?

4 Two charges are situated a short distance apart in air. The space between them is then filled with oil of relative permittivity 2. Does the force between the charges increase or decrease?

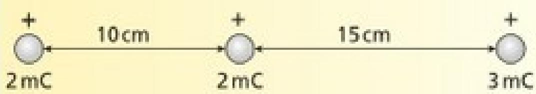
5 A charge of 2 mC is embedded in plastic of relative permittivity 7.1, at a distance of 20 cm from another charge of 4 mC. What is the force between the two charges?

6 A charge of –3 μC is placed at a distance of 1.2 m from a charge of +5 μC, in air. What is the magnitude and direction of the force on the negative charge?

7 In the situation shown in figure 7.29, the 2 mC charges are in air.

(a) Find the magnitude of the force on the central charge.

(b) What is the direction of the force charge?

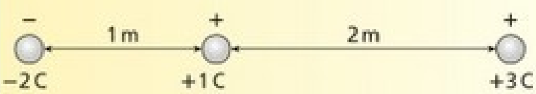


7.29 *Question 7*

8 In the situation shown in figure 7.30, a number of charges are arranged in air.

(a) What is the direction of the force created on the +3 C charge?

(b) What is the magnitude of this force?

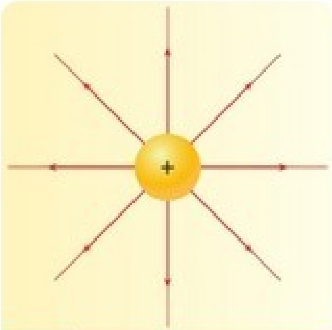


7.30 *Question 8*

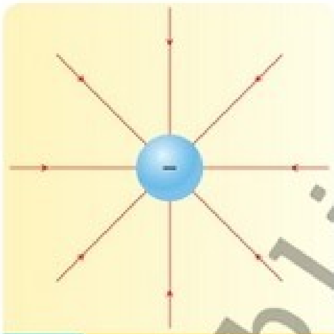
Electric fields

> An electric field is the area around a charge where its effect can be felt.

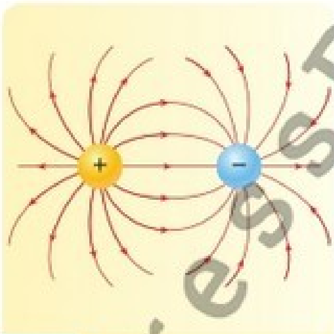
In drawing diagrams to show the shape of the electric field around an electric charge, we always draw the arrows to show the path a positive charge would take. Various examples of such diagrams are shown in figures 7.31–7.34.



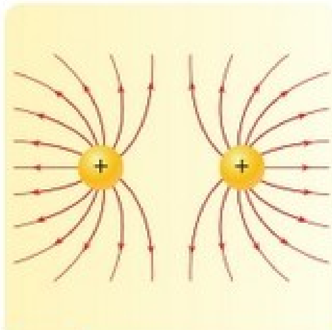
7.31 *The field around a single positive charge*



7.32 *The field around a single negative charge*



7.33 *The field between a positive and negative charge*



7.34 *The field between two positive charges*

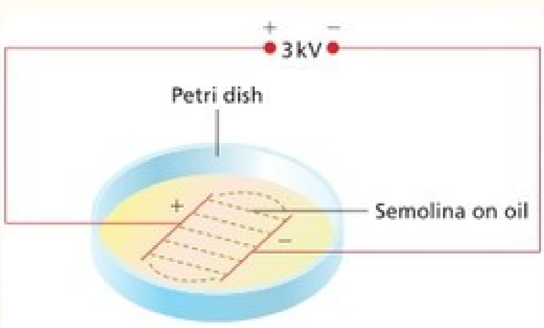
Experiment 7.4: To demonstrate an electric field

Method

1. Add some grains of semolina to a shallow dish of oil and set up the circuit as shown in figure 7.35. Note the very high voltage being used, and be careful!
2. Close the switch and note the movement of the floating particles.

Observations

You should find that the semolina particles become slightly charged at each end, and line up along the lines of force, showing the shape of the field.



7.35 *To demonstrate an electric field*

Electric field strength

As we have seen, when drawing electric fields we use arrows to show the direction a single small positive charge would move. In analysing electric fields mathematically we follow a similar logic. To determine the strength of an electric field at any particular point, for example, we ask what force would exist on a charge of +1 C if it were placed in the electric field at that point.

The purpose of calculating the electric field strength is to have a way in which we can compare the effect of different charges, and different arrangements of charges, with each other.

Electric field strength is the force per unit positive charge at a point in an electric field. It is measured in newtons per coulomb ($N\,C^{-1}$).

$$E = \frac{F}{Q}$$

where:

E – electric field strength

F – force

Q – charge transferred

Electric field strength is a vector quantity.

Another way of looking at this is to use Coulomb's law, taking the size of one of the charges to be 1 C: to find the strength of the electric field around a charge Q, at a distance d from the charge, we simply ask what the force would be on a charge of 1 C if it were placed at that point.

From Coulomb's law we can say:

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{d^2}$$

Remember that this is the force that would exist on a charge of +1 C at that point, if such a charge were placed there. This is, therefore, equal to the value for the electric field strength:

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$$

7.5 Sample Question

A charge of 3 mC is placed in an electric field at a distance from its centre, and experiences a force of 1.2×10^{-3} N. What is the electric field strength at that point?

Sample Answer

$$E = \frac{F}{Q}$$

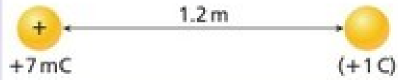
$$= \frac{1.2 \times 10^{-3}}{3 \times 10^{-3}}$$

$$= 0.4\ N\,C^{-1}$$

7.6 Sample Question

(a) What is the strength of the electric field at the point shown in figure 7.36, a distance of 1.2 m from a +7 mC charge?

(b) If we placed a charge of +2 mC at that point, what force would it experience?



7.36

Sample Answer

(a) $E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$

$= \frac{7\times10^{-3}}{4\pi(8.854\times10^{-12})(1.2)^2}$

$= 4.37\times10^{7}\,\text{NC}^{-1}$

(b) $E = \frac{F}{Q}$

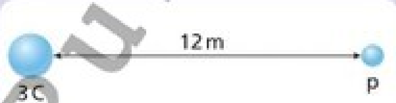
$F = (E)(Q)$

$= (4.37\times10^{7})(2\times10^{-3})$

$= 87\,400\text{ N}$

7.7 Sample Question

What is the electric field strength at the point p shown in figure 7.37?



7.37

Sample Answer

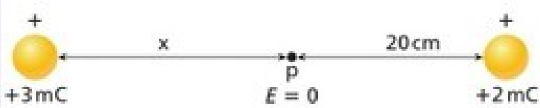
$E = \frac{1}{4\pi\varepsilon}\frac{Q}{d^2}$

$= \frac{3}{4\pi(8.854\times10^{-12})(12)^2}$

$= 1.872\times10^{8}\,\text{NC}^{-1}$

7.8 Sample Question

The electric field is zero at the point p in figure 7.38. What is the distance x?



7.38

Sample Answer

There are two electric fields at p. For the total field to be zero, the fields must be equal in magnitude.

$= \frac{1}{4\pi\varepsilon}\frac{2\times10^{-3}}{0.2^2}$

$= \frac{1}{4\pi\varepsilon}\frac{3\times10^{-3}}{x^2}$

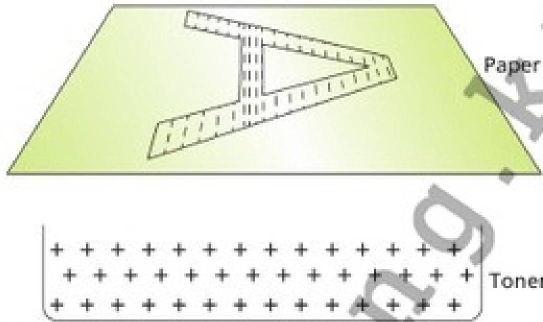
$x^2 = \frac{(0.2)^2(3)}{2}$

$= 0.06$

$= 0.24\text{ m}$

Photocopying

Photocopying makes use of electric fields. A rotating drum inside the photocopying machine is charged and an image of the document to be copied is projected onto the drum. A light then discharges those parts of the document that are white. Charged toner (ink) is then attracted to the remainder, and the copy is printed.



7.39 *The positively charged ink is attracted to the negative charges on the paper and takes on their shape*

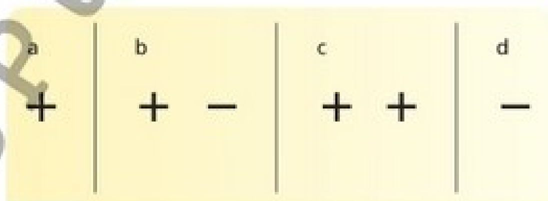
For you to try

1 What is meant by the term 'electric field'?

2 Look at figure 7.40.
  (a) In what direction would a positive charge move if placed in an electric field at the point *p*?
  (b) In what direction would a negative charge move?

7.40 *Question 2*

3 Copy and complete the diagrams in figure 7.41, representing the shape of the electric field in each case.



7.41 *Question 3*

4 A charge of 5×10^{-6} C is placed in an electric field and experiences a force of 3×10^{-3} N.
  (a) What is the electric field strength at that point?
  (b) If a charge of 2 mC was placed at the same point, what force would it experience?

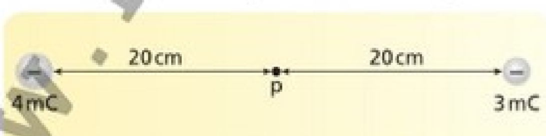
5 What is the electric field strength at the point *p* in figure 7.42?

7.42 *Question 5*

6 A charge of 2 C is placed in an electric field as shown in figure 7.43. What force does it experience?

7.43 *Question 6*

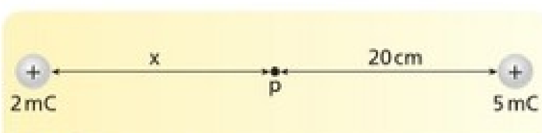
7 Two charges are arranged as shown in figure 7.44.



7.44 *Question 7*

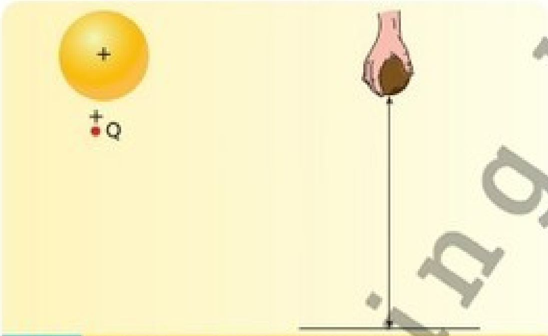
  (a) What is the electric field at the point p?
  (b) What force would an electron experience at that point?

8 The electric field is zero at the point indicated in figure 7.45, a distance x from a charge of 2 mC. What is the distance x?



7.45 *Question 8*

Electric potential



7.46 *The positive charge Q has energy because of the nearby charge. The stone has energy because it has been lifted up*

When we lift up an object such as a stone we give it potential energy, and when we release it, it will fall, converting the potential energy into kinetic energy.

Look at figure 7.46 and you will see that there is an obvious comparison between the stone in that example and the electric charge, *Q*.

The positive charge, *Q*, is close to another positive charge, and if released would move away from it in much the same way that the stone would fall if released. Energy is essentially the ability to make an object move, so this means that the charge, *Q*, has energy due to its position, just as the stone has. There is obviously a distinction between the two situations: the stone has gravitational potential energy, whereas the charge, *Q*, has electric potential energy. However, the comparison is nonetheless perfectly valid.

Potential difference

The comparison can even be extended. We have chosen to measure electrical potential in such a way that we can say a positive charge will move from high potential to low potential, just as a stone will fall from a high point to a lower one. And it is usually the difference in potential between two points that determines whether a charge will move or not, just as it is the difference in height between two points that will determine whether a stone will fall.

However, we should not overstrain the comparison. Gravity and weight are in many ways very straightforward concepts. Essentially, everything falls downwards. With electric charge it can be more complex than this. A positive charge moves away from another positive, as we have seen, but it could also move towards a negative. The presence of many different electric charges creates a complex electric field, and it can be very difficult to predict how exactly a charge will move through that field. The shape of charged objects and the material of which they are made can also have an effect.

A difference in electrical potential can be created through a number of factors, but we can still define the concept in a fairly simple way:

The potential difference (p.d.) between two points is the work done in moving a unit positive charge from one point to the other. The unit of potential difference is the volt.

The potential difference between two points is 1 volt (1 V) if the work done in moving a charge of one coulomb (1 C) from one point to the other is one joule (1 J).

This means that if we let a charge of +1 C move between two points, and the charge gains 15 J of energy in doing so, then the potential difference between the two points must be 15 V.

From this we get the formula:

$$V = \frac{W}{Q}$$

where:

V – voltage
W – work done
Q – charge transferred

Negative charges

You might notice that the above examples are all based on positive charges, and that this was also the case with our definition of electric fields. The nature of the charges involved makes no difference to the size of the measurements that might be taken, such as potential difference and electric field strength. But we must remember that for negative charges, the directions are reversed.

This means that negative charges will move from areas of low potential to areas of high potential. This in turn means that they will move in the opposite direction to that of an electric field.

Zero potential

Although we are normally only interested in measuring a potential difference between two points, rather than the actual level of potential, we still need something to measure this against. The Earth is always taken as being at zero potential (0 V) for this purpose. To return to the comparison with which we started: with falling bodies we are usually only interested in the height through which they can fall, but when we need to we can compare any height to that of sea level.

7.9 Sample Question

The potential difference between two points is 15 000 V. How much work is done in moving a charge of 1.5 μC between the two points?

Sample Answer

$$V = \frac{W}{Q}$$

$$W = QV$$

$$= (1.5 \times 10^{-3})(15\,000)$$

$$= 0.0225 \text{ J}$$

7.10 Sample Question

Two points A and B are separated by a distance of 1.2 m. Point A is at a potential of 25 V, point B is at a potential of 12 V.

(a) What is the potential difference between the two points?

(b) If a charge of 2 C is released directly beside point A, how much work is done is moving it from A to B?

Sample Answer

(a) $25 - 12 = 13 \text{ V}$

(b) $W = QV$

$= (2)(13)$

$= 26 \text{ J}$

For you to try

1 What is the definition of one volt (1 V)?

2 Look at figure 7.47.

(a) In what direction would a positive charge move in this electric field?

(b) In what direction would an electron move?

7.47 *Question 2*

3 In an electric field, a charge of +4 C moves from point A to B, as shown in figure 7.48.

(a) Which of the two points is at a higher potential?

(b) If the charge gains 15 J of kinetic energy as it moves from A to B, what is the potential difference between the points?

7.48 *Question 3*

4 Two points are at potentials of 15 V and 12 V, as shown in figure 7.49.

(a) What is the potential difference between the points?

(b) How much work is done in moving a charge of 2 mC between the two points?

7.49 *Question 4*

Module 8 Capacitance

Learning objectives

- Explain the nature of capacitance
- Outline the design and purpose of a parallel plate capacitor
- Make comparative analysis of electric induction in conductors and polarisation in dielectrics 10.4.1.7
- Research the relationship between capacitor capacity and its parameters 10.4.1.8
- Use the formula of series and parallel connection of capacitors in problem-solving 10.4.1.9

Capacitance

In Module 7 we looked at how to build up a charge on a plastic rod, and we used that rod to examine the effect of static electric charges. We also looked at how to use the Van de Graaff generator to build up a large charge and to examine the associated effects of that charge. Although both objects could build up a charge, it was obvious that the metal dome was capable of holding an enormous charge compared with the plastic rod.

Every object has a limited ability to hold charge. Capacitance is a measure of how much charge it can hold. We can compare this to the concept of capacity. A small tumbler will hold much less water than a pint glass – we say that its capacity is less, just as the capacitance of a plastic rod is less than that of a metal sphere.

The capacitance of a body is defined as the ratio of the charge on the body to its potential:

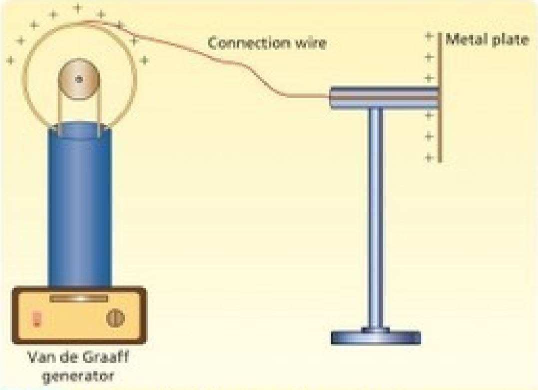
$$C = \frac{Q}{V}$$

The unit of capacitance is the farad:

A body has a capacitance of one farad (1 F) if the addition to the body of one coulomb (1 C) raises the potential of the body by one volt (1 V).

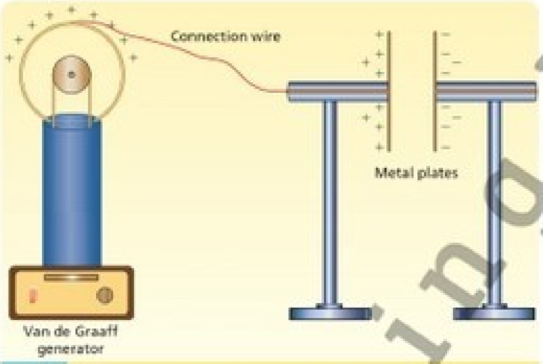
Parallel plate capacitor

If a metal plate is attached to a Van de Graaff generator, as shown in figure 8.1, it will quickly build up a positive charge. The size of this charge is limited by the capacitance of the metal plate, which in turn is controlled by a number of factors such as its shape and the material from which it is made, as well as by its surroundings.



8.1 *A charge builds up on the metal plate*

If we bring another metal plate beside it, however – as shown in figure 8.2 – its capacitance is increased. This happens because the positive charge attracts a negative charge to the surface of the second metal plate. This negative charge then attracts an increased charge to the positive plate, which, in turn, attracts an increased charge to the negative plate.



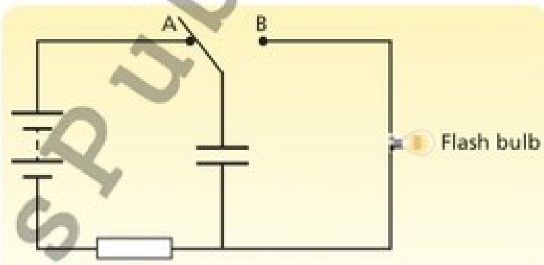
8.2 *A second plate increases the capacitance*

This all happens in a very short period of time, and soon an equilibrium is reached between the two plates. But as the total charge on each plate is higher than it could ever be for a single comparable plate, we can see that this arrangement increases the capacitance of each plate. This is known as a **parallel plate capacitor**, and it is a device that is common in many modern electronic devices.

Uses of a parallel plate capacitor

Electronic camera flash

The bulb used in camera flashes is unusual for a bulb, in that it gives off a very bright light for a short period of time. This can be done using a circuit like that shown in figure 8.3.

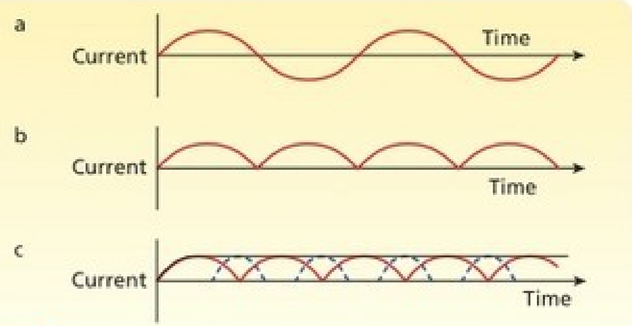


8.3 *Circuit for a camera flash*

When the switch is in position A, a current will flow until the capacitor is fully charged and then stop. When a photograph is taken, the switch is triggered to move across to position B. Now the voltage stored on the capacitor is applied to the light bulb, where it causes a sudden flash of very bright light.

Smoothing currents

Most modern electronic devices require a direct current (d.c.). If they are running off a battery, this is no problem, but if they are connected to the mains, they will be provided with an alternating current (a.c.) instead. There are various ways that this can be changed to d.c., in a process known as rectification. Then the current flows only in one direction and is, therefore, technically d.c., but it is still a very uneven current, as shown in the graphs in figure 8.4.



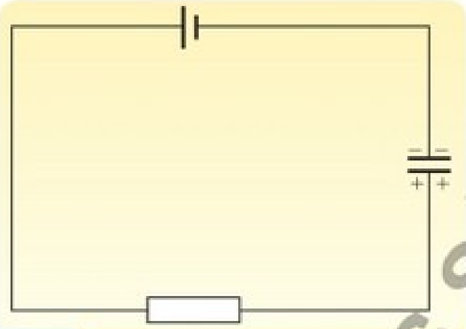
8.4 *Graph a shows a.c.; graph b shows a rectified current; graph c shows a rectified current smoothed out by the use of capacitors*

8.5 *The dial of a radio showing the different stations and frequencies, which are selected using capacitors*

Capacitors in the circuit can be used to smooth off the rectified current. Essentially what happens is that when the value of the rectified current falls (indicated by the red line in figure 8.4, on the previous page), the charge stored on the capacitor flows through the circuit (indicated by the blue line). This creates a smoother flow of total current (indicated by the black line).

Capacitors are also used in the tuning of a radio. A radio aerial picks up a large number of transmitted signals, but when we tune to a station we are choosing a specific value for the capacitance of the circuit, and this has the effect of amplifying only the signal transmitted at a particular frequency.

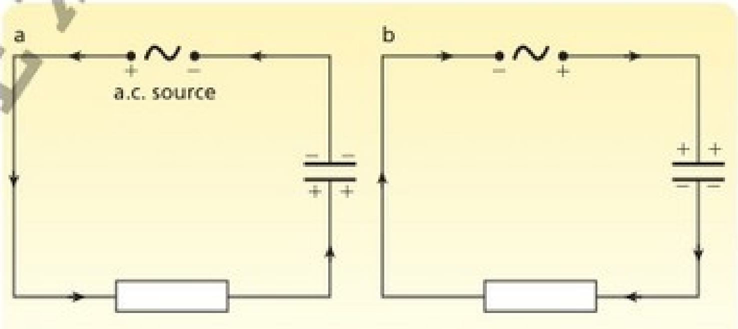
Capacitors conduct a.c., but they block the flow of d.c.


8.6 *In a d.c. circuit, the current will flow only until the capacitor is charged*

If we attach a capacitor to a battery as shown in figure 8.6, a d.c. flows for a short period of time, until the capacitor is fully charged. At that point, because no charge can flow through the capacitor, no current can flow in the circuit. In this way, capacitors always block the flow of d.c.

If the capacitor is attached to an a.c. supply, however, a current flows continuously.

This happens in the following manner. Initially a current flows and builds up a charge on the capacitor, as shown in figure 8.7a. Just as this charge reaches a peak, however, the direction of flow of the current reverses. This causes the capacitor to lose its charge and then build up a charge in the opposite direction, as shown in figure 8.7b. Again, just as this charge reaches a peak, the process is reversed. This process continues, repeatedly charging the capacitor, discharging it and recharging with the opposite polarity. This means that, although no current flows across the capacitor, a current is always flowing in the circuit.



8.7 *In an a.c. circuit containing a capacitor, current flows constantly*

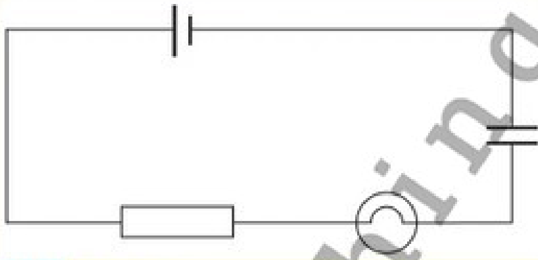
Experiment 8.1: To demonstrate that capacitors allow a.c. to flow, but block d.c.

Method 1

1 Set up a d.c. circuit as shown in figure 8.8.
2 Observe what happens to the bulb.

Observations

You should find that the bulb lights for only a short time. This is because once the capacitor is fully charged it blocks the flow of direct current.



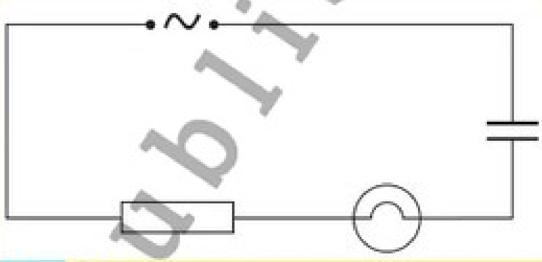
8.8 *A d.c. circuit*

Method 2

1 Set up an a.c. circuit as shown in figure 8.9.
2 Observe what happens to the bulb.

Observations

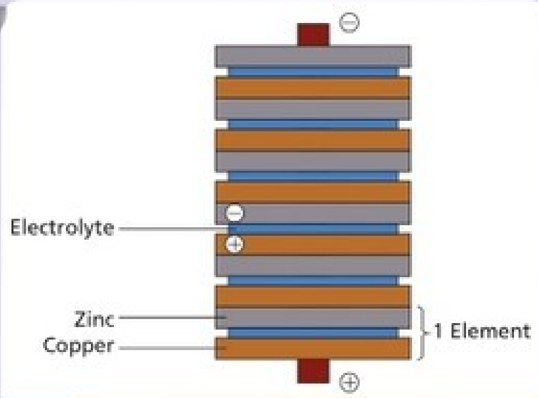
You should find that the bulb remains lit while the circuit is complete. This is because the capacitor allows an alternating current to flow.



8.9 *An a.c. circuit*

Alessandro Volta

Alessandro Volta (1745–1827) – after whom the volt is named – lived in Como in Northern Italy, where he was a teacher of physics in the local school. He read a piece of work by Benjamin Franklin that described something he called 'flammable air', and he was fascinated. He searched for such a material and finally succeeded in isolating the gas involved. It is now known to us as methane.



8.10 *A battery, based on work done by Volta*

Another Italian, Luigi Galvani, had discovered a phenomenon whereby a frog's legs could be made to twitch by joining them together with different metals. He thought the effect was caused by electricity inside the animal, but Volta realised that the key to the effect was in fact the use of two different metals. He went on to show that an electric current can be created using two different metals separated by a chemical known called an electrolyte. In doing so, he created the first electric cell. A series of these cells connected in series then gave rise to what we now call a battery.

8.1 Sample Question

If a charge of 12 C increases the voltage of an object by 3 V, what is the capacitance of the object?

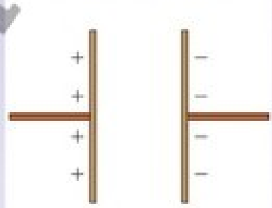
Sample Answer

$$C = \frac{Q}{V}$$
$$= \frac{12}{3} = 4\ \text{F}$$

8.2 Sample Question

(a) In the capacitor shown in figure 8.11, which plate is at a higher potential?

(b) If the capacitance of the capacitor is 6 μF, and each plate holds a charge of 3 μC, what is the potential difference between the plates?



8.11 *Question 8*

Sample Answer

(a) The positive plate is at a higher potential.

(b) $C = \frac{Q}{V}$

$$V = \frac{Q}{C}$$
$$= \frac{3 \times 10^{-6}}{3 \times 10^{-6}}$$
$$= 0.5\ V$$

For you to try

1 What is meant by the term 'capacitance'?

2 In what unit is capacitance measured? What is the definition of this unit?

3 Outline three uses of capacitors.

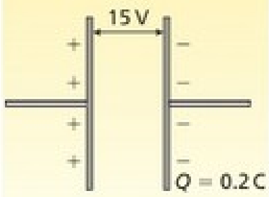
4 If a charge of 15 C increases the voltage of an object by 2V, what is the capacitance of the object?

5 If a charge of 1.5 μC increases the voltage of an object by 2V, what is the capacitance of the object?

6 If a charge of 1.2 mC increases the capacitance of an object by 2 μF, what is the voltage of the object?

7 What capacitance would you expect to find on an object that is at a voltage of 20 V and holds a charge of 2 μC?

8 In figure 8.12, where there is a potential difference between the two plates of 15 V, and the charge on each plate is 0.2 C, what is the capacitance of the capacitor?



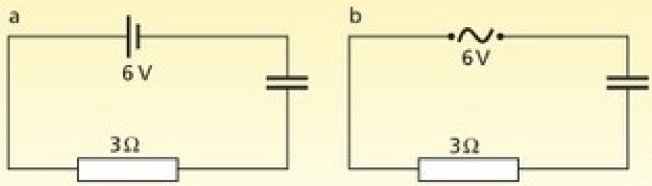
8.12 *Question 8*

9 **(a)** In the capacitor shown in figure 8.13, which plate is at a higher potential?

(b) If the capacitance of the capacitor is 3 μF, and each plate holds a charge of 2 μC, what is the potential difference between the plates?

10 Through which of the circuits in figure 8.14 will a current continuously flow? What is the maximum voltage across each capacitor?

8.13 *Question 9*



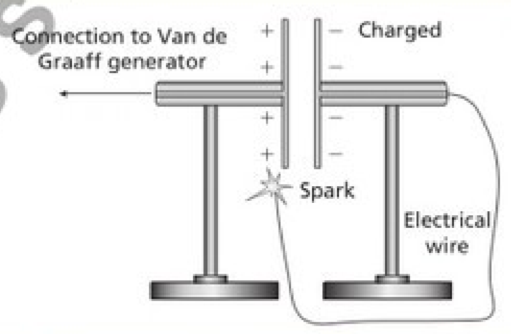
8.14 *Question 10*

Experiment 8.2: To demonstrate that a capacitor stores energy

Method

1 Set up the apparatus as shown in figure 8.15.

2 Allow the capacitor to build up a charge. Bring a conducting wire from one plate towards the other and observe what happens.



8.15 *To demonstrate that a capacitor stores energy*

Observations

As the wire approaches the plate you should see and hear a spark.
The presence of light and sound energy indicates that energy had been stored on the capacitor.

The energy (*E*) stored is given by the formula:

$$E = \frac{1}{2}CV^2$$

Factors determining the capacitance of a capacitor

The capacitance of a capacitor is controlled by three factors: the common area of the plates (or A, the area of overlap), the distance between the plates (d), and the permittivity (ε) of the material between the plates. This can be demonstrated using an electroscope (see Experiment 8.3, overleaf).

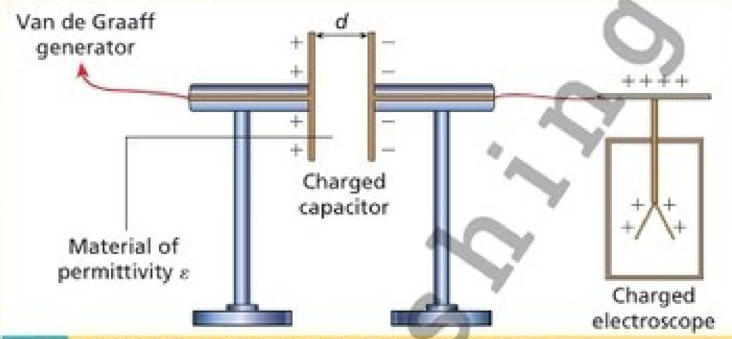
Experiment 8.3: To demonstrate the factors that control capacitance

Method

1 Set up the electrical arrangement shown in figure 8.16.

2 Separately carry out these three investigations and observe what happens:

- Slowly vary the distance between the plates
- Vary the area of overlap of the plates
- Add and/or remove various materials *(e.g. glass, plastic)* between the plates.



8.16 *Investigating the factors that control capacitance*

Observations

You should find that when the plates are drawn apart, the leaves collapse, indicating that the stored charge, and therefore the capacitance, is inversely proportional to d.

Similarly, you should find that reducing A and varying the material between the plates affects the capacitance according to the formula:

$$C = \frac{\varepsilon A}{d}$$

8.3 Sample Question

A parallel plate capacitor consists of two plates a distance (*d*) of 1.5 mm apart, with a common area (*A*) of 15×10^{-3} m². The material between the plates has a relative permittivity of 2.8. What is the capacitance of the capacitor?

Sample Answer

The positive plate is at a higher potential.

$$\varepsilon = \varepsilon_r \varepsilon_0$$
$$= (2.8)(8.854 \times 10^{-12})$$
$$= 2.479 \times 10^{-11} \text{ F m}^{-1}$$
$$C = \frac{\varepsilon A}{d}$$
$$= \frac{(2.479 \times 10^{-11})(15 \times 10^{-3})}{1.5 \times 10^{-3}}$$
$$= 2.479 \times 10^{-10} \text{ F}$$

8.4 Sample Question

What is the energy stored by a 330 mF capacitor where there is a potential difference of 8.0 V between the plates?

Sample Answer

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2}(330 \times 10^{-3})(8)^2$$

$$= 10.56\text{J}$$

For you to try

1. What are the factors that determine the capacitance of a capacitor?
2. Briefly outline an experiment to show that energy is stored on a charged capacitor.
3. **(a)** Which of the arrangements in figure 8.17 would you expect to have the greater capacitance? The only difference in each case is the distance between the plates.
   (b) In which arrangement would the most energy be stored?
4. A parallel plate capacitor consists of two plates a distance of 1×10^{-3} m apart, with a common area of 2×10^{-3} m^2. The material between the plates has a relative permittivity of 2.3. What is the capacitance of the capacitor?
5. A parallel plate capacitor consists of two plates a distance of 0.01 mm apart, with a common area of 2×10^{-3} m^2. The material between the plates has a relative permittivity of 3.1. What is the capacitance of the capacitor?
6. A capacitor has a capacitance of 2 μF, and consists of two plates a distance of 15 μm apart. The material between the plates has a relative permittivity of 2.5. What is the common area of the plates in the capacitor?
7. A capacitor of capacitance 120 μF consists of two plates with a potential difference between them of 1.2 mV. How much energy is stored on the capacitor?
8. What is the energy stored by a 33 000 mF capacitor charged to 10.0 V?
9. **(a)** What is the charge held by a 500 mF capacitor charged to a p.d. of 3.5 V?
   (b) How much energy is stored on the capacitor?

Capacitance and Physical dimensions of parallel plates

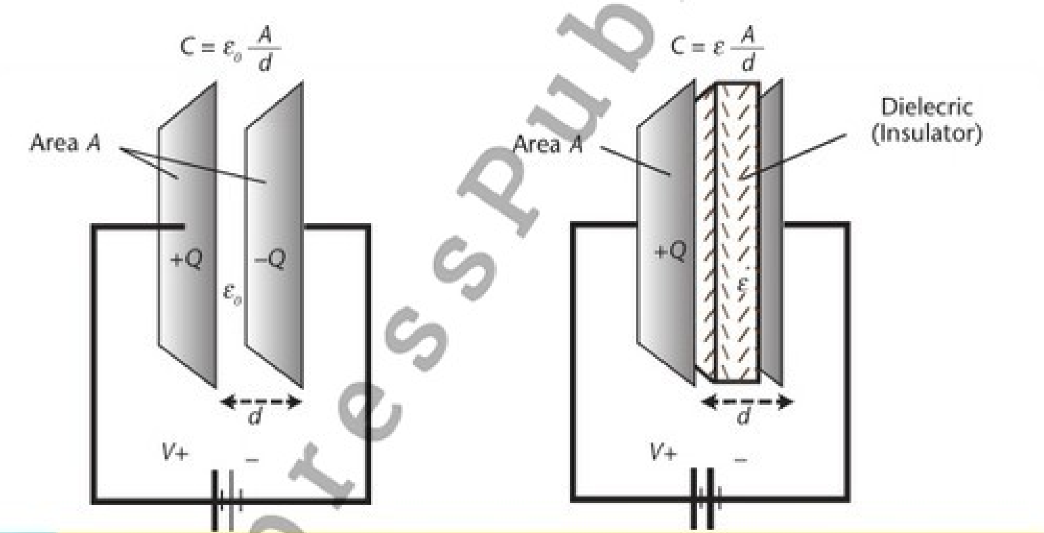
When any object is given charge it will change its potential (or voltage). Smaller objects will increase in voltage rapidly when given a some charge, but larger objects will increase their potential (or voltage) more slowly. The capacitance of parallel plates held in close proximity is proportional to the area A of the plates, and inversely proportional to the separation d between the plates:

$C = \xi_0 A/d$ where ξ_0 is the permittivity of the air between the plates.

The permittivity of free space (a vacuum) is a physical constant equal to approximately 8.85×10^{-12} farad per meter (F/m).

Note: The action of adding a dielectric in the space between two parallel plates and so increasing the capacitance has similarities with the action of inserting ferroelectric material (like iron) into the core of a coil to increase the magnetic field strength produced.

The capacitance of capacitors can be increased further by filling the space between the parallel plates with a dielectric which can become polarised when in an electric field. In this case the permittivity increases by a facto caller the relative permittivity ξ_r.



8.17

8.5 Sample Question

Two metal parallel plates with dimensions 10 x 15 cm are separated by a gap of 2 mm. Calculate the capacitance if there is only air in the gap between them.

Sample Answer

$C = \xi_0 A/d$, (watch out with the units – convert them to metres)

$C = 8.85 \times 10^{-12} \times 0.1 \times 0.15 / 0.002 = 6.6 \times 10^{-11}$ Farads.

Notice this is a very small number. Capacitances of components normally are very small. Often quoted in pico-Farads, or nano-Farads.

8.6 Sample Question

With the same plates as above, what distance between the plates would be necessary in order to make a capacitor with a capacitance of 300 pF (pico Farads)?

Sample Answer

$C = \xi_0 A/d$, so $d = \xi_0 A/C$. $d = 8.85 \times 10^{-12} \times 0.1 \times 0.15 / 300 \times 10^{-12}$

$d = 0.00044$ m $= 0.44$ mm (notice this is a very small gap).

8.18 *Commercial capacitors normally look like cylinders with two legs*

Because the gaps are so small, there is often the danger of the plates touching each other and shorting out. Partly for that reason it is safer to place a thin insulating sheet between them to prevent the shorting. Once this thin sheet is in place, it is possible to make the plates very thin too (as thin as aluminium foil), and the whole arrangement can be rolled up into a cylinder and placed inside a little can. It has the added advantage that the thin insulating material also has a relative permittivity which increases the capacitance.

8.7 Sample Question

Two strips of aluminium foil which are 2 cm wide and 3 m long are separated by a dielectric film that is 0.01 mm thick. An additional thin dielectric strip is placed over the outside of one of the aluminium foils so that it can be rolled up without shorting the two foils together. If the relative permittivity of the dielectric sheet is 4, calculate the capacitance of the system.

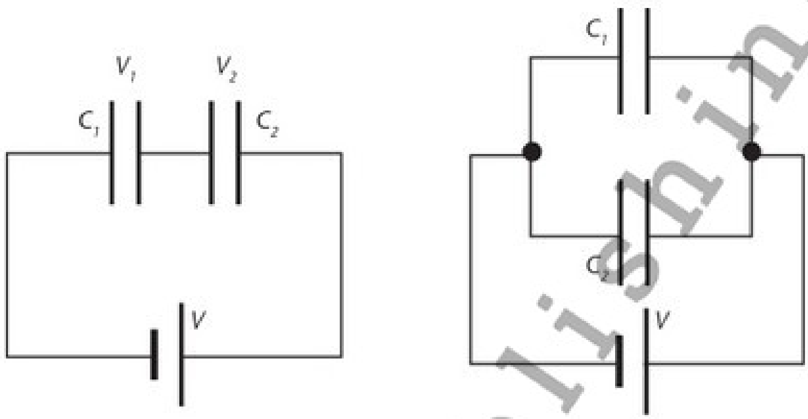
It must first be noticed that because of the rolling up of the sheets, each 'plate' now has a facing plate in front of it and also behind it. This effectively doubles the capacitance.

Sample Answer

$C = \xi_0 \xi_r A/d$, so $C = 2 \times 8.85 \times 10^{-12} \times 4 \times 0.02 \times 3 / 0.00001 = 1.42 \times 10^{-7}$ F. (normally labelled as 142 nF)

Connections of multiple capacitors

Just as resistors can be connected in series and in parallel, capacitors can also be connected in series and parallel. However, the effect they have is the opposite you might expect.



8.19 *Connections of multiple capacitors*

Resistors: key points

When resistors R_1, R_2, and R_n are connected in ***series***, their total resistance is $R_{total} = R_1 + R_2 + ... + R_n$

When resistors R_1, R_2, and R_n are connected in ***parallel*** their total resistance is $1/R_{total} = 1/R_1 + 1/R_2 + ... + 1/R_n$

However, when capacitors C_1, C_2 and C_n are connected in ***series***, their total capacitance is $1/C_{total} = 1/C_1 + 1/C_2 + ... + 1/C_n$

When capacitors C_1, C_2 and C_n are connected in ***parallel***, their total capacitance is $C_{total} = C_1 + C_2 + ... + C_n$

Notice that the equation for capacitors in ***series*** is similar to that for resistors in ***parallel*** and the equation for capacitors in ***parallel*** is similar to that for resistors in ***series***!

8.9 Sample Question

Two capacitors of 20 nF are connected in parallel. What will the total capacitance of the combined system be?

Sample Answer

$C_{total} = C_1 + C_2 = 20 + 20\ nF = 40\ nF$. That was too easy; let's connect them in series instead.

8.10 Sample Question

What would the capacitance be if they had been connected in series instead?

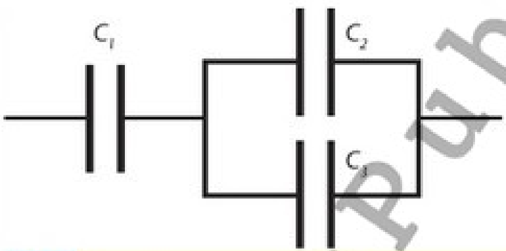
Sample Answer

$1/C_{total} = 1/C_1 + 1/C_2 = 1/20 + 1/20 = 2/20 = 1/10$ so $C_{total} = 10$ *nF*.

Notice that the total capacitance is less that the capacitance of one of them.

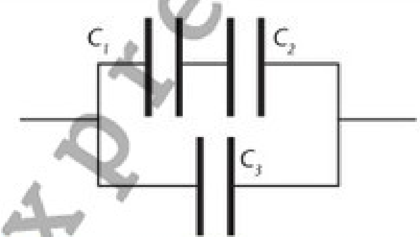
For you to try

1 In the figure below, $C_1 = 20$ nF, C_2 and $C_3 = 40$ nF each. Calculate the total capacitance of the whole circuit.



8.20

2 In the figure below, $C_1 = 20$ nF, $C_2 = 30$ nF and $C_3 = 40$ nF. Calculate the total capacitance of the whole circuit.



8.21

3 This one is a little more tricky, and requires some more thinking: How could you connect 20 nF capacitors (as many as you need) to obtain a capacitance of 15 nF?

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Module 9 Conduction

Learning objectives

- Use the Ohm's law for a circuit unit with mixed connection of conductors 10.4.2.1
- Research mixed connection of conductors 10.4.2.2
- Research connection between electromotive force and source voltage in different regimes of its work (working regime, idle speed, short circuit) 10.4.2.3
- Apply the Ohm's law in complete circuits 10.4.2.4
- Define electromotive force and internal resistance of current source 10.4.2.5
- Use the formulae of work, power and efficiency of current source in problem-solving 10.4.2.7
- Describe electric current in metals and resistance-temperature relationship 10.4.3.1
- Discuss perspectives of developing high-temperature superconducting materials 10.4.3.2
- Describe an electric current in semiconductors and to explain the application of semiconductor devices 10.4.3.3
- Research volt-ampere characteristics of a filament lamp, resistor and semiconductor diode 10.4.3.4
- Use the laws of electrolysis in problem-solving 10.4.3.5
- Define an electronic charge in the electrolysis process by experiments 10.4.3.6
- Describe an electric current in gases and vacuum 10.4.3.7
- Explain the operating principle and application of cathode ray tubes 10.4.3.8

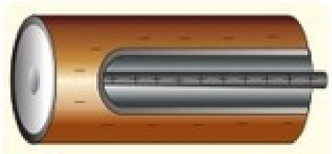
Electric current

In the 1700s Italian physician and physicist Luigi Galvani (1737–1798) noticed that he could cause a frog's legs to twitch by creating a circuit that connected the legs together using two different metals. He thought his experiment showed that animals generate their own electricity. Although it is true that muscles are controlled by tiny electric currents flowing through the body's nervous system, this was not in fact what Galvani had discovered.

Later experiments by another Italian physicist, Alessandro Volta (1745–1827), showed that the arrangement Galvani had used, making use of two different metals, was actually generating an electric current itself – through a process now known as **electrolysis**. Essentially, a very slow chemical reaction was taking place between the materials involved by forcing electrons to move through the connecting wires.

Volta replicated the arrangement in a device now known as an electric cell to create an electric current. Several cells joined together form a **battery**.

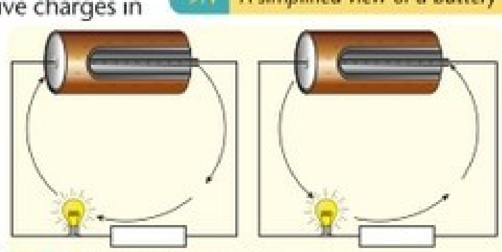
The battery



9.1 *A simplified view of a battery*

A simplified – but functional – view of a battery is to think of it as containing two chambers, one full of positive charges and the other full of negative charges, as shown in figure 9.1.

There is an attraction between the positive and negative charges in the battery, but they are separated by a barrier that prevents them moving to connect to each other. When we join the two ends, or **poles**, of the battery together using electrical wire, however, we provide a path through which the charges can combine. With this simplified arrangement we cannot say whether it would be the positives that move to meet and neutralise the negatives, or vice versa.



9.2 *Simple electric circuits*

We now know that it is in fact the negative charges – the electrons – that move. However, the study of electricity was well advanced – indeed many cities had access to electric light – before the electron was discovered. And in that era the assumption had become common that it was the positive charges that move.

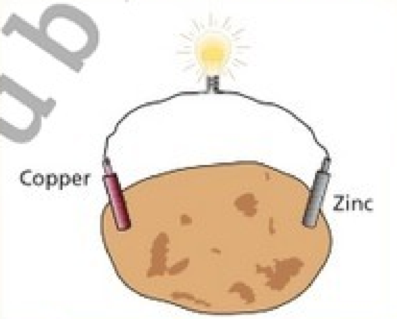
The reality is that it makes very little difference to the study of electric circuits which particle actually moves. The key thing is that when electric charge moves through the circuit, it must also move through whatever device we place in the circuit, whether that be a simple device such as a light or a heater, or a much more complex one such as a computer or a TV set.

Because it makes very little practical difference to how we imagine a circuit working and which charge we picture as moving, the tradition from the nineteenth century is often maintained today and current is represented as going from plus to minus rather than the opposite. When we do this it is called **conventional current**, and we will generally show current travelling in this manner in this book.

Experiment 9.1: To construct a potato-fuelled circuit

Method 1

1 Take a potato, two different strips of metal (e.g. one strip of copper and one strip of zinc), two wire clips and a small light-emitting diode (LED). Assemble as shown in figure 9.3
2 Observe what happens to the LED.



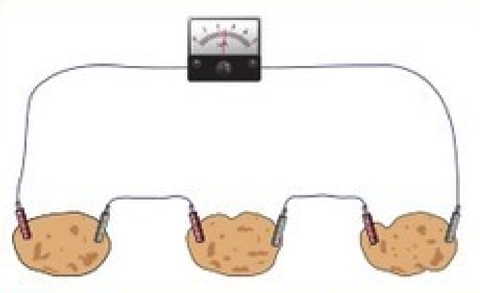
9.3 *A potato acting as a cell*

Observations

You should find that the light turns on. This is due to a chemical reaction slowly taking place between the zinc, the copper and the potato.

Variation

1 Set up the apparatus shown in 9.3, but replace the LED with a micro-ammeter.
2 Measure the current.
3 The voltage and current can be increased by using two or more potatoes connected 'in series', as shown in figure 9.4.
4 You can also replace the potato with a weak acid, such as vinegar. (It is the very small quantity of acid in a potato that allows the chemical reaction to take place.)



9.4 *Several cells creating a potato battery*

Electric circuits

Figure 9.5 represents a basic circuit and shows some of the symbols most commonly used in electric circuits.

An electric current is a flow of charge. A current of one amp (or ampere) is a movement of one coulomb per second:

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$$I = \frac{Q}{t}$$

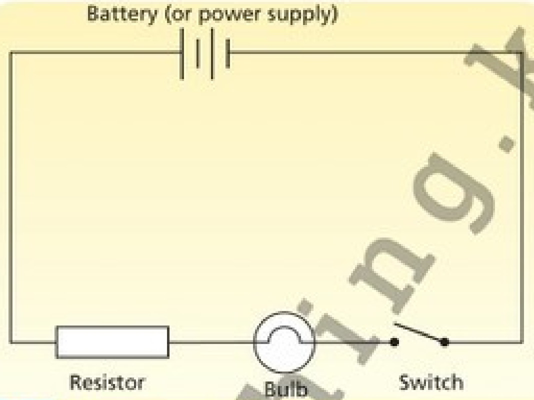
where:

I – current

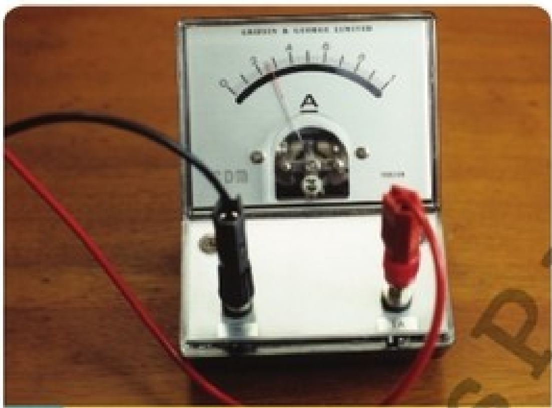
Q – charge transferred

t – time

If a current of one amp is flowing in a conductor, one coulomb is the quantity of charge passing a point on that conductor in one second.



9.5 *A simple electric circuit*



9.6 *An ammeter showing a current of about 2.6 A*

This gives us a definition of the **coulomb**:

The coulomb (C) is a very large charge, and so we often encounter smaller units such as millicoulombs (mC) or microcoulombs (μC). The same follows for current, and most modern electronic devices run off a current of a fraction of one amp (1 A).

Conductors and Insulators

A conductor is a material that allows electrons to move through it freely. The type of bonding common in most metals allows electrons to move easily from one atom, or group of atoms, to another. This is why metals are generally described as good conductors. In most other materials, the electrons are tightly connected to a particular atom or group of atoms and cannot move easily. This means that they are generally not good conductors, and they are described as **insulators**.

Although it is true that, generally speaking, metals are good conductors and other materials are not, you should not think of this as a rule that is always followed. There are many exceptions. Carbon, for example – particularly in the form of graphite – is a non-metal that is a good conductor. Ionic compounds such as sodium chloride (table salt) are not good conductors in the solid state but allow a large current to flow when dissolved in water.

We all learn to be careful about electricity and water because of the danger of electrocution. However, pure water is in fact not a conductor. It is the dissolved solids within it that allow water to conduct electricity. Water is such a good solvent, though, that it almost always contains dissolved solids, especially ionic compounds, and as such water is usually thought of as a conductor.

Voltage

We have already learnt about potential difference (p.d.) and voltage. Remember that voltage measures the energy lost or gained by a positive charge of 1 C as it moves from one point to another.

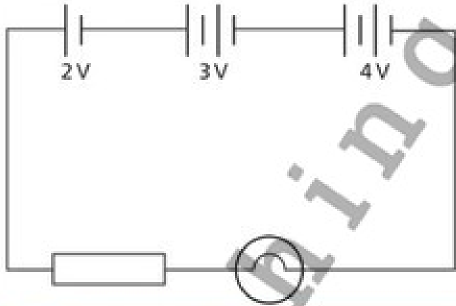
A voltage (V), when applied to a circuit, is known as the electromotive force (emf).

A cell is a commonly used source of emf. Several cells joined together form a battery and give a higher voltage: other sources are the electrical mains and a thermocouple.

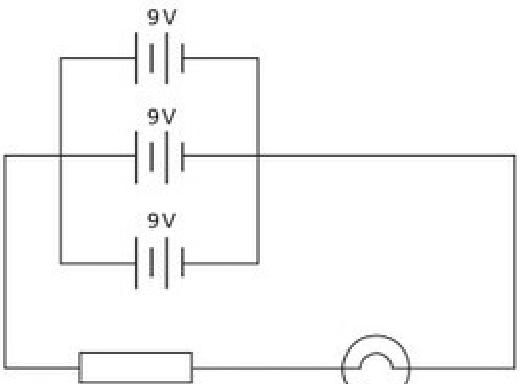
Batteries in series and parallel

A number of batteries connected in series creates a larger voltage, as shown in figure 9.7, where the total voltage in the circuit would be (2+3+4) = 9V.

When batteries are connected in parallel their lifetime is extended, but the voltage is not increased. In the circuit shown in figure 9.8, the total voltage is 9 V.



9.7 *Batteries in series*



9.8 *Batteries in parallel*

9.9 *Typical 1.5 V batteries*

Ammeters and Voltmeters

Ammeters and voltmeters are commonly used in studying electric circuits. The most basic design of each usually includes a circular dial and a needle, like those shown in figure 9.10. The current is read by noting where on the scale the needle is pointing.

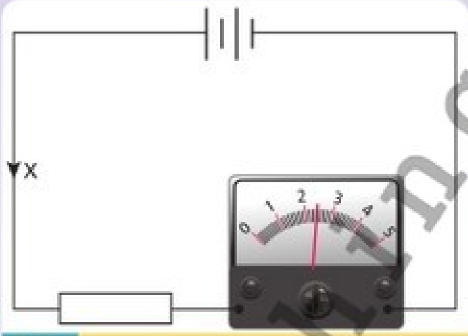
9.10 *An ammeter and voltmeter*

Although useful when you are learning about circuits, meters such as ammeters and voltmeters are rarely used now by electricians and engineers. Instead they use a modern device known as a multimeter. This has a central dial that can be turned to different positions depending on whether you want to measure voltage, current or resistance. To allow greater accuracy, there are different settings for measuring high currents as opposed to low currents, etc.

9.11 *A modern multimeter which can act as a voltmeter, ammeter or ohmmeter*

9.1 Sample Question

Identify the current X in figure 9.12.



9.12

Sample Answer

X = 2.4 V

9.2 Sample Question

If a current of 2 mA flows for 3 s, how much charge passes a particular point in the circuit?

Sample Answer

$I = \frac{Q}{t}$

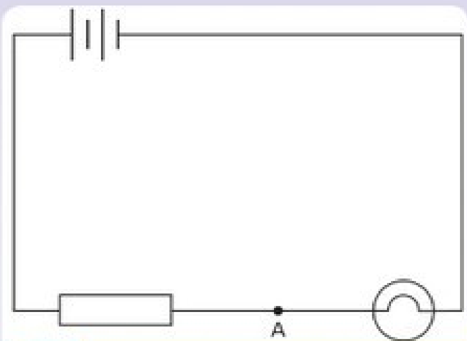
$Q = It$

$= (2 \times 10^{-3})(3)$

$= 6 \times 10^{-3}$ C

9.3 Sample Question

If a current of 3 A flows in the circuit shown in figure 9.13, how many electrons pass point A every second?



9.13

Sample Answer

$Q = It$

$= 3 \times 1 = 3$ C

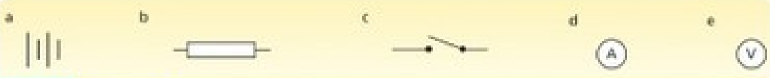
N (number of electrons) $= \frac{3}{1.6 \times 10^{-19}}$

$= 1.875 \times 10^{19}$ electrons

Take the charge on the electron to be 1.6×10^{-19} C.

For you to try

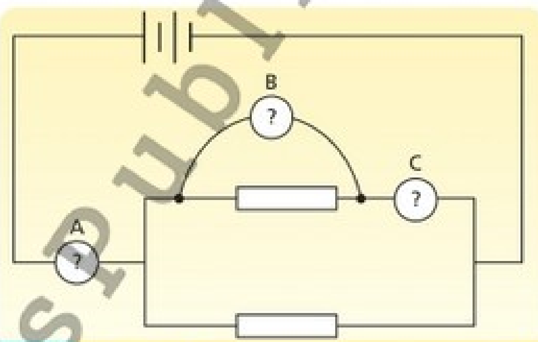
1 When an electric current flows through a metal wire, which charges are moving?
2 What do we mean by the term 'conventional current'?
3 Draw a diagram to show how a potato can be used to drive a current through an LED.
4 What is meant by the term 'emf'?
5 Identify the electrical components represented by the diagrams in figure 9.14.



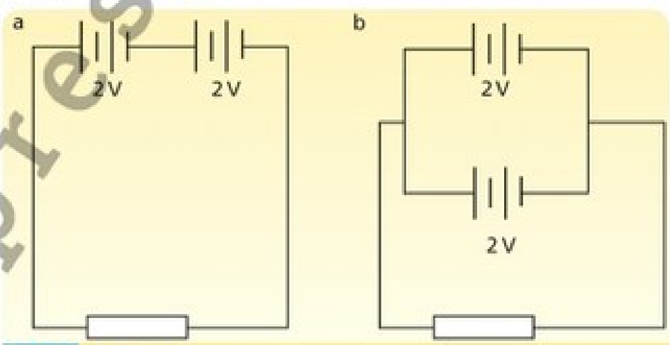
9.14 *Question 5*

9.15 *Question 6*

6 If the current in figure 9.15 flows for 5 s, how much charge has passed a particular point in the circuit?
7 If a current of 2 mA flows for 3 min, how much charge has passed a particular point in the circuit?
8 If a current of 2 μA flows for 3 h, how much charge has passed a point in the circuit?
9 A charge of 5 C passes a point within a circuit over a period of 2 s. What is the current?
10 If 4.8×10^{20} electrons flows pass a particular point in an electric circuit in 1 min, what is the current?
11 In figure 9.16, identify which meters are ammeters and which are voltmeters.
12 What is the total voltage in each of the circuits shown in figure 9.17?

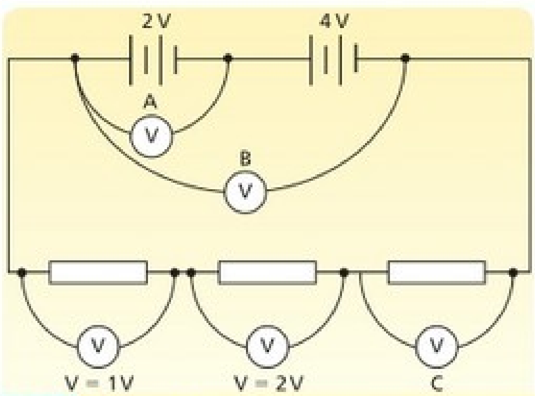


9.16 *Question 11*



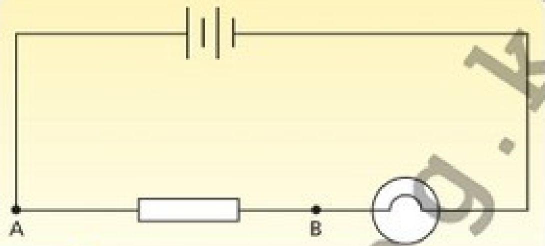
9.17 *Question 12*

13 What voltage would you expect to see on each of the voltmeters shown in figure 9.18?



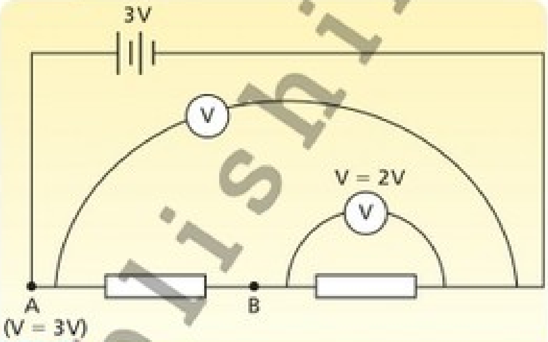
9.18 *Question 13*

14 Look at figure 9.19. If the potential at point A is 9 V, and the potential drop across the resistor is 3 V, what is the potential at point B?



9.19 *Question 14*

15 If the potential at point A is 3 V, and the reading on the voltmeter is as shown in figure 9.20, what is the potential at point B? And what would you expect the reading on the second voltmeter to be?



9.20 *Question 15*

Resistance

The flow of an electric current through metal wires is often compared to the flow of water through pipes. Just as higher water pressure creates an increased current in the flow of water, a higher voltage tends to increase the current in an electric circuit. And just as the flow of water can be reduced by constrictions in a pipe – such as the kinks that can develop in a hose – the flow of an electric current can be reduced by something called **resistance**.

Sometimes we deliberately introduce resistance into part of a circuit, so that the current is not too big. But whether we want a high resistance or a low one, there is always some resistance on a circuit. Even metals, such as copper, that we use to make conducting wires are not perfect conductors. Electrons still have to do some work to travel through the wires, and this tells us that they carry a resistance.

In fact, many resistors are manufactured from metals such as nickel–chrome (nichrome) that would often be thought of as conductors, but if manufactured to the right specifications offer enough resistance to allow us to control a current.

A good insulator would block a current entirely and would therefore not be a useful resistor.

Ohm's law

The connection between voltage, current and resistance was discovered by German physicist and teacher Georg Ohm (1789–1854) in 1827.

Ohm's law states that, for a metallic conductor at constant temperature, the current will be proportional to the voltage: $V \propto I$.

When two items are proportional, we can say that the ratio of one to the other is a constant:

$$\frac{V}{I} = \text{a constant.}$$

Ohm realised that the value of this constant in an electric circuit is essentially the same thing as the resistance:

$$\frac{V}{I} = R$$

where R – resistance.

This gives us a key formula for the study of electric circuits as well as a useful way in which to define exactly what we mean by the term 'resistance':

$$V = IR$$

> The resistance of an object in an electrical circuit is defined as the ratio of the voltage across it to the current flowing through it:
>
> $$R = \frac{V}{I}$$

The **unit of resistance** is the **ohm**, and its value is set using the above formula:

> A conductor has a resistance of one ohm (1 Ω), if a current of one amp (1 A) flows when a voltage of one volt (1 V) is applied.

Ohm's law should not be thought of as a fundamental law of nature. Unlike Newton's laws, for example, there are many situations in which Ohm's law is not valid. We will study these in more detail, but the current flowing through a gas, an ionic solution and a vacuum tube are just some of the situations in which the law is not followed. However, in studying standard electric circuits, Ohm's law is a very useful guide to the relationship between current, voltage and resistance.

9.4 Sample Question

If a voltage of 9 V is applied across a resistor and a current of 3 A flows through it, what is the resistance of the resistor?

Sample Answer

$$R = \frac{V}{I}$$

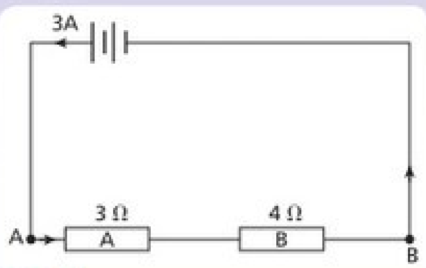
$$= \frac{9}{3} = 3\,\Omega$$

9.5 Sample Question

A current of 3 A flows through two resistors of 3 Ω and 4 Ω, as shown in figure 9.21.

(a) What is the potential drop across each of the resistors?

(b) What is the voltage between points A and B?



9.21

Sample Answer

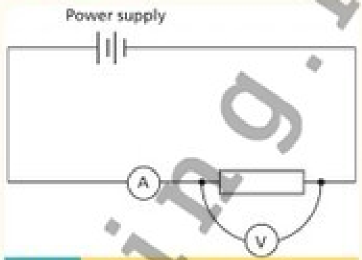
(a) $V_A = (3)(3) = 9\,V$; $V_B = (3)(4) = 12\,V$

(b) $V_{AB} = 9 + 12 = 21\,V$

Experiment 9.2: To investigate Ohm's law

Method

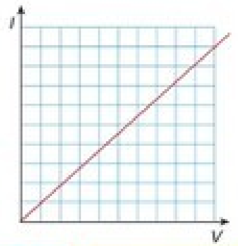
1 Set up a circuit as shown in figure 9.22, allowing you to measure both the current through, and the voltage across, the resistor.
2 Set the power supply to a low voltage. Note the readings on the ammeter and voltmeter.
3 Increase the voltage using the power supply and again note the readings on the two meters.
4 Repeat for several settings of applied voltage and record your results.
5 Draw a graph of voltage against current using your results.



9.22 *Experimental circuit*

V/V						
I/A						

9.23 *Use a table like this one to record your results*



9.24 *A straight line through the origin*

Results and conclusions

A straight line through the origin (as on the graph in figure 9.24) will confirm that voltage is proportional to current (Ohm's law).

Resistance and Temperature

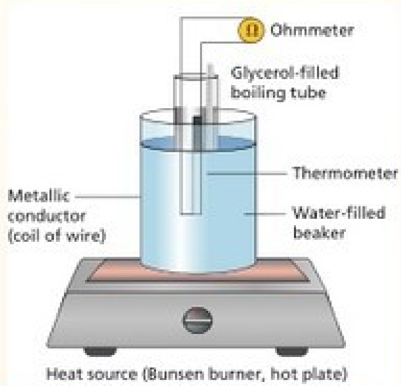
The resistance of an object depends (among other things) on its temperature. The variation of resistance with temperature, however, is not the same for all materials.

The connection between resistance and temperature can be investigated experimentally.

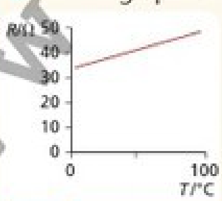
Experiment 9.3: Investigation of the variation of resistance with temperature for a metal

Method

1 Set up the apparatus as shown in figure 9.25.
2 Use the thermometer to note the temperature of the glycerol, which is also the temperature of the coil of wire.
3 Record the resistance of the coil of wire using the ohmmeter.
4 Slowly heat the beaker.
5 For each 10°C rise in temperature record the resistance and temperature using the ohmmeter and the thermometer.
6 Plot a graph of resistance against temperature.



9.25 *Experimental apparatus*



9.26 *Resistance vs. temperature for a metal*

Results

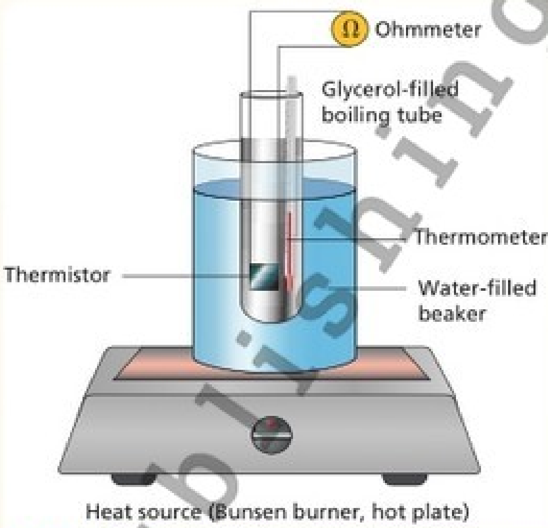
You should find that your graph is similar to that shown in figure 9.26.

It is important to heat the water slowly, to avoid a situation in which the liquid will be much hotter than the wire.

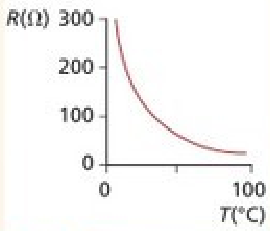
Experiment 9.4: Investigation of the variation of resistance with temperature for a thermistor

Method

1. Set up the apparatus as shown in figure 9.27.
2. Use the thermometer to note the temperature of the glycerol, and therefore of the thermistor.
3. Record the resistance of the thermistor using the ohmmeter.
4. Heat the beaker.
5. For each 10°C rise in temperature, record the resistance and the temperature using the ohmmeter and the thermometer.
6. Plot a graph of resistance against temperature.



9.27 *Experimental apparatus*



9.28 *Resistance vs. temperature for a thermistor*

Results

You should find that your graph is similar to that shown in figure 9.28.

Accuracy

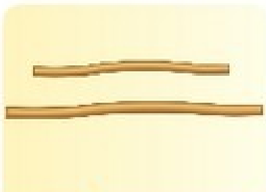
The glycerol is used to ensure better thermal contact between the thermistor and its surroundings.

You will note from experiments 9.3 and 9.4 that the resistance of metals tends to increase with temperature, whereas the resistance of semiconductors decreases as the temperature increases.

Resistivity

Look at the two wires represented in figure 9.29. Which one do you think has the greater resistance?

As long as everything else is the same, the longer wire will have the greater resistance. Electrons have to work to move through even the best conductors – therefore the longer the wire the more work they have to do, and the greater the resistance.



9.29 *Resistance is proportional to length*

Now look at the two wires in figure 9.30. Again, which would you expect to have the greater resistance?

9.30 *Resistance is proportional to 1/area*

This time the narrower wire will tend to have the higher resistance. Remember that an electric current is the movement of electric charge. The wider wire – with the greater diameter – provides a wider path for the electric charge to move. As more charge can move through the wire, the resistance is reduced.

Now look at the two wires in figure 9.31. One is made of copper and the other nichrome. You cannot tell by looking, but even if the length and diameter of the two wires were identical, the nichrome would have the greater resistance. This is because of a property called **resistivity**.

9.31 *Resistance is proportional to resistivity*

We cannot say that metals have low resistance. It is often true, but not necessarily true. A narrow wire thousands of kilometres long, for instance, does not have a low resistance. We can, however, say that metals **tend to** have a low resistivity.

To summarise:

As well as temperature, the resistance of an object also varies with:

- The length, l, of the object (the longer the object, the greater the resistance)
- The cross-sectional area, A, of the object (the wider the object, the lower the resistance)
- The material from which the object is made (the property of a material that controls the resistance is the resistivity, ρ, of the material).

This leads to a key formula:

$$R = \frac{\rho l}{A}$$

The resistivity of a material is the resistance of a unit cube of that material:

$$\rho = \frac{RA}{l}$$

The unit of resistivity is the ohm-metre (Ωm).

Experiment 9.5: Measurement of the resistivity of a wire

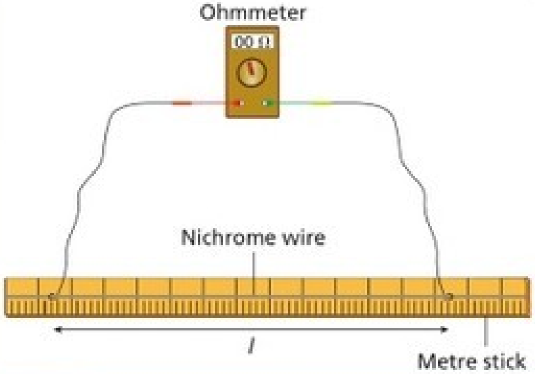
Method

1 Set up the apparatus as shown in figure 9.32. Ensure that the resistance of the leads is zero when the crocodile clips are connected together.

2 Stretch a length of nichrome along a metre stick as shown in figure 9.32. Ensure there are no kinks or 'slack' in the wire.

3 Note the resistance, R, for a particular length, l, of wire.

4 Increase the distance between the crocodile clips and note the new values of R and l.



9.32 *Experimental apparatus*

5 Use the micrometer to find the diameter of the wire at different points, taking the zero error into account. Find the average value of the diameter, *d*.
6 Repeat this procedure for a number of different lengths of wire.

Results

For each set of results, calculate the resistivity using the formula:

$$\rho = \left(\frac{R}{l}\right)A$$

where:

$$A = \pi r^2$$

The average value is the resistivity.

Accuracy

- Kinks in the wire will affect the measurement of both length and cross-sectional area.
- The use of greater lengths will reduce the percentage error.

9.6 Sample Question

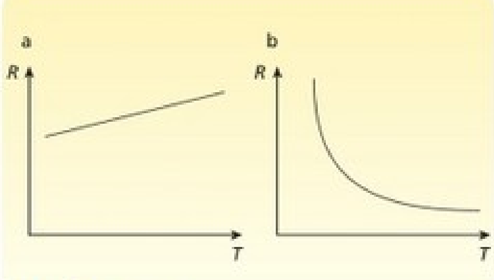
A piece of metal has a length of 15 cm, a resistivity of 2.3 Ωm and a circular cross section of diameter 1 mm. What is its resistance?

Sample Answer

$$A = \pi r^2$$
$$= \pi(0.5 \times 10^{-3})^2$$
$$= 7.85 \times 10^{-7}\ \text{m}^2$$
$$\rho = \frac{RA}{l}$$
$$R = \frac{\rho l}{A}$$
$$= \frac{(2.3)(0.15)}{7.85 \times 10^{-7}}$$
$$= 439490.4\ \Omega$$

For you to try

1 What is resistance?
2 What is the definition of 1 Ω?
3 The graphs in figure 9.33 represent the variation of resistance with temperature. Which one would you expect to represent a metal and which one would you expect to represent a thermistor?
4 What voltage will produce a current of 3 A in a resistor of resistance 15 Ω?



9.33 *Question 4*

5 Arctic explorers sometimes use socks lined with electrical wire for heating. They run off a 9 V battery and draw a current of 0.1 A. What resistance does the electrical wire have?

6 If a current of 3 A flows through a resistor when a voltage of 6 V is applied to it, what is the resistance of the resistor?

7 A toaster is plugged into a mains source providing 230 V. The resistance is 30 Ω. What current flows in the toaster?

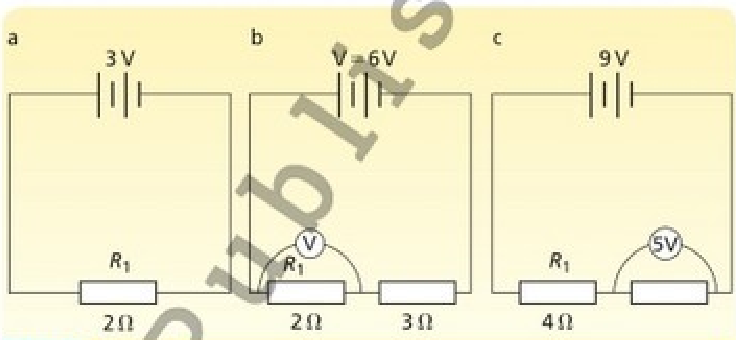
8 A piece of metal has a length of 30 cm, a resistivity of 4.5 Ωm and a circular cross section of diameter 1 mm. What is its resistance?

9 If a transatlantic cable is 5000 km long and consists of several strands of copper (resistivity $1.7 \times 10^{-8}\,\Omega$m) 1 mm in diameter, what is the total resistance of one such strand?

10 Does the resistance of a copper wire increase or decrease when both length and cross-sectional area are doubled? Explain your answer.

11 Copper and nichrome have very different resistivities. If we take two pieces of wire of the same length, one made from nichrome and one from copper, is it possible that they would have the same resistance?

12 In each of the circuits shown in figure 9.34, what current would you expect to flow in the resistor R_1?



9.34 *Question 12*

Resistors in series

When an electric circuit is arranged so that the same electric current flows through several resistors, we say that those resistors are **in series** with each other.

Derivation

For this setup, the total voltage drop from A to B is given by:

$V_{AB} = V_1 + V_2 + V_3$

where V_1 is the voltage drop across R_1, and so on.

Following Ohm's law, we can say:

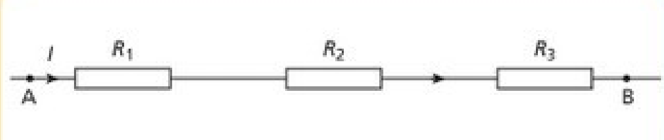
$R = \frac{V}{I}$

which gives:

$IR_T = IR_1 + IR_2 + IR_3$

Dividing by I:

$R_T = R_1 + R_2 + R_3$

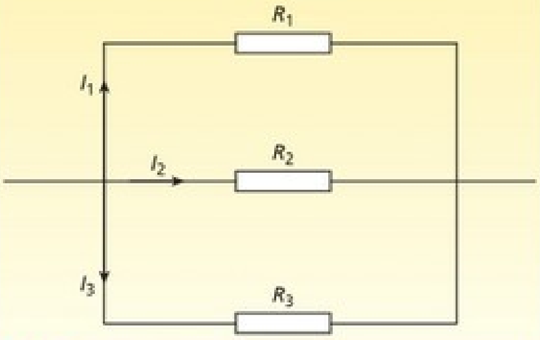


9.35 *Resistors in series*

This tells us that, to find the total resistance when a number of resistors are joined together in series, we simply add together all the individual resistances.

Resistors in parallel

When resistors are used in a circuit in such a way that the current flowing must at some point be divided between alternative paths, we say that the circuit is connected **in parallel**. The example in figure 9.36 shows three resistors in parallel.



9.36 *Resistors in parallel*

We do not mean geometrically parallel: the angle between the resistors in the diagram is of no significance.

Derivation

In the circuit shown in figure 9.36, the total current is given by:

$I_T = I_1 + I_2 + I_3$

The total voltage drop across the group of resistors is the same, regardless of the path followed, so, as $I = \frac{V}{R}$:

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

And dividing across by V gives:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

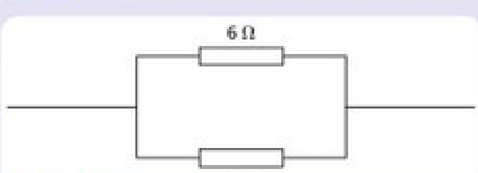
This is the formula that we must follow if we are to find the total resistance of a circuit in which resistors are wired in parallel.

9.7 Sample Question

What is the total resistance of the circuits in figures 9.37 and 9.38?

Sample Answer

9.37 *(a)*



9.38 *(b)*

(a) $R^T = 20 + 30 + 5 = 55\,\Omega$

(b) $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{1}{6} + \frac{1}{9} = \frac{5}{18}$$

$$R_T = \frac{18}{5} = 3.6\,\Omega$$

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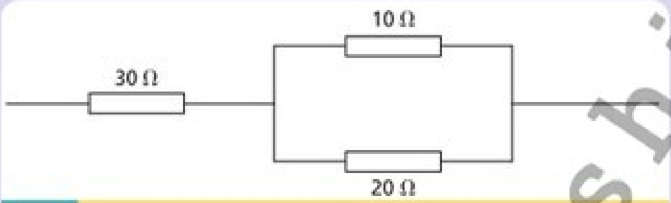
Sample question 9.7 deals with relatively straightforward situations in which resistors are either in series or parallel. However, it is possible to have a combination of both occurring in one circuit. This is dealt with in sample questions 9.8 to 9.11.

9.8 Sample Question

What is the total resistance of the circuit in figure 9.39?



9.39

Sample Answer

$R_{||}$ = total resistance of the resistors in parallel

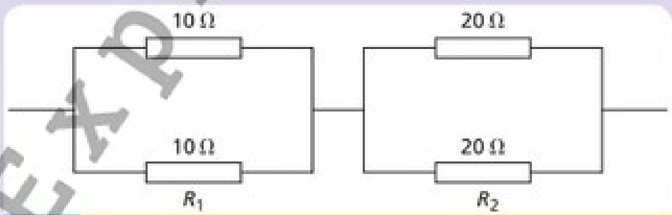
$$\frac{1}{R_{||}} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$$

$$R_{||} = \frac{20}{3} = 6.67\Omega$$

$$R_T = 30 + 6.67 = 36.67\Omega$$

9.9 Sample Question

What is the total resistance of the circuit in figure 9.40?



9.40

Sample Answer

Divide into R_1 and R_2.

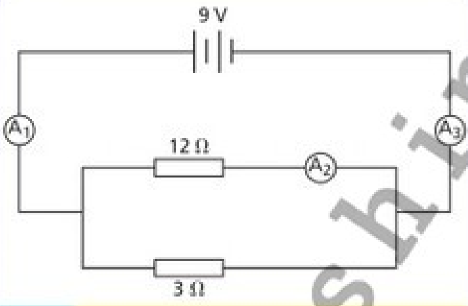
$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10},\ R_1 = \frac{10}{2} = 5\Omega$$

$$\frac{1}{R_2} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20},\ R_2 = \frac{20}{2} = 10\Omega$$

$$R_T = R_1 + R_2 = 5 + 10 = 15\Omega$$

9.10 Sample Question

(a) What is the total resistance of the circuit in figure 9.41?
(b) What current would you expect to see in each ammeter?



9.41

Sample Answer

(a) $\frac{1}{R_T} = \frac{1}{12} + \frac{1}{3} = \frac{5}{12}$

$R_T = 2.4\,\Omega$

(b) $V = IR$

$I_T = \frac{V}{R} = \frac{9}{2.4} = 3.75\,\text{A}$

So $= A_1 = A_3 = 3.75\text{A}$

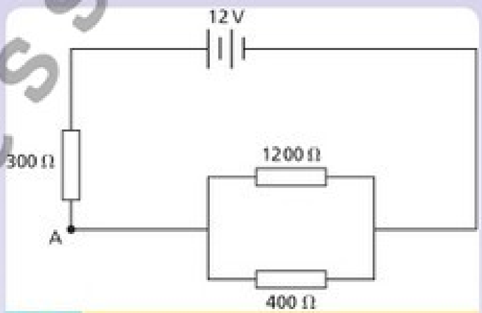
The total current splits between the two paths. The current will split in the opposite proportion to the resistances; the resistors are in the ratio 12:3 or 4:1, so the current will split in the ratio 1:4. The smaller current will flow in the 12 Ω resistor:

$A_2 = \frac{1}{5} \times 3.75 = 0.75\,\text{A}.$

9.11 Sample Question

Look at the circuit shown in figure 9.42.

(a) What is the total resistance?
(b) What is the total current?
(c) What current flows through the 400 Ω resistor?
(d) What is the potential at point A?



9.42

Sample Answer

(a) $R_{||}$ = resistance of the 1200 Ω and 400 Ω system

$\frac{1}{R_{||}} = \frac{1}{1200} + \frac{1}{400} = \frac{1}{300}$

$R_{||} = 300\,\Omega$

R_T = total resistance = 300 + 300 = 600 Ω

(b) $I = \frac{V}{R_{||}} = \frac{12}{600} = 0.02\,\text{A}$

(c) The resistors are in the ratio 12:4 or 3:1, so the current is in the ratio 1:3. The current through the 400 Ω resistor will be the larger:

$I400 = \frac{3}{4} \times 0.02 = 0.015\,\text{A}$

(d) V = the potential drop across the 300 Ω resistor.

$V = IR$

$= (0.02)(300) = 6\text{V}$

Potential at A = 12 − 6 = 6V

For you to try

1 What is the total resistance in each of the arrangements shown in figure 9.43?

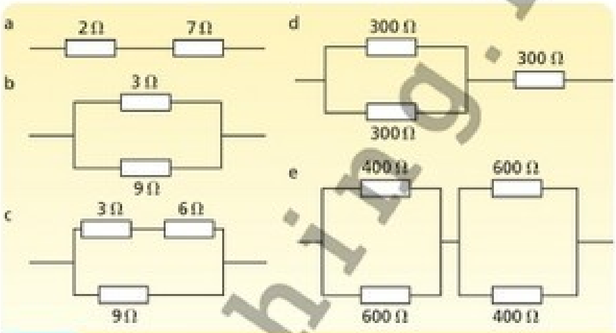
2 How many 2 Ω resistors must be connected in parallel to create a total resistance of 0.25 Ω?

3 Look at the circuit in figure 9.44.
   (a) What is the total resistance?
   (b) What is the current flowing in each of the ammeters?

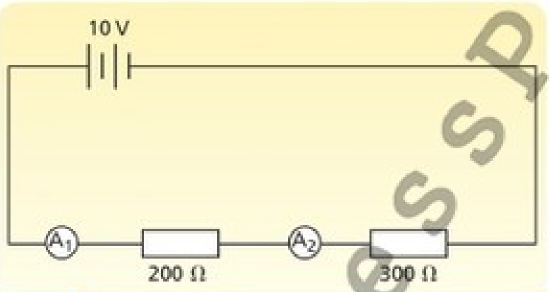
4 What is the total resistance of the circuit in figure 9.45? What is the total current flowing in each ammeter?

5 Look at the circuit in figure 9.46.
   (a) What is the potential difference across each of the resistors?
   (b) What is the emf of the circuit?

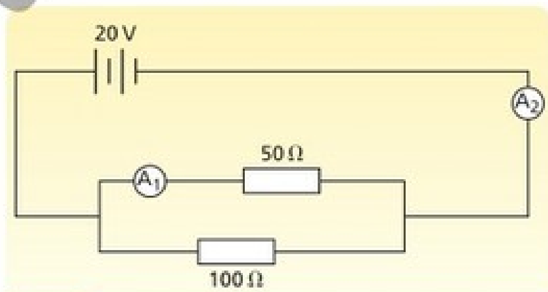
6 The total resistance of the circuit in figure 9.47 is 25 Ω.
   (a) What is the resistance of the resistor marked X?
   (b) What current flows through that resistor?
   (c) What is the potential at points A, B and C?



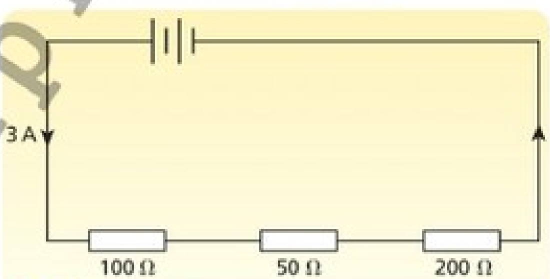
9.43 *Question 1*



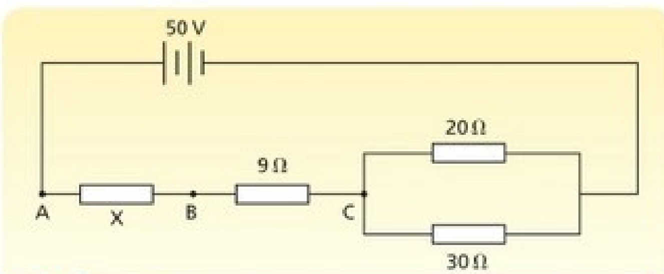
9.44 *Question 3*



9.45 *Question 4*



9.46 *Question 5*



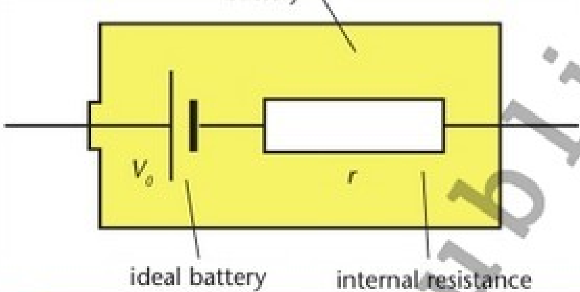
9.47 *Question 6*

Electromotive force

At this point it is helpful to learn the meaning of a new term. Electromotive force (EMF for short) is a term used to describe the voltage across a battery, a generator or some other source of electricity. The reason why it is helpful to use EMF is that it helps us to remember that this is a source of energy, whereas other voltage differences can be drains of energy.

Batteries and Internal resistance

Normally we only think as batteries as sources of constant voltage, however, real batteries do not behave that way. The more current is drawn from them, the more their apparent EMF falls. It is almost as if a battery consisted of an ideal battery and a resistor connected in series. Sometimes the EMF is referred to as V_0 to denote the fact that it is the voltage of the battery when zero current is flowing.



9.48 *A real battery behaves like an ideal battery and a resistor in series.*

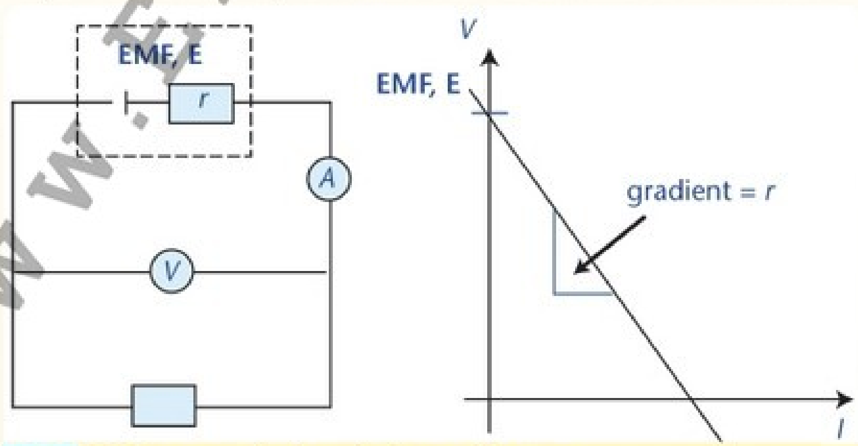
$$\varepsilon = I(R + r)$$

where:

- ε – electromotive force in volts, V
- I – current in amperes, A
- R – resistance of the load in the circuit in ohms, Ω
- r – internal resistance of the cell in ohms, Ω

Experiment 9.6: To measure the EMF and internal resistance of a battery

In order to measure the internal resistance of a battery we need an ammeter a voltmeter and a few resistors (or a variable resistor). The circuit should be connected as shown below.



9.49 *Circuit to measure the internal resistance of a battery*

The procedure consists of measuring the voltage across the battery, and the current flowing through the resistor at the same time. This procedure should be repeated for several different values of resistance. It is good if the range of the values of the resistors is as wide as possible, and the resistances start from values as low as 5 Ohms.

The measurement data can then be plotted on a graph as shown in the figure above, with the currents along the X axis and the Voltages along the Y axis. A line of best fit is then drawn through the data so that it intercepts the Y axis. Now we are ready to analyse the graph.

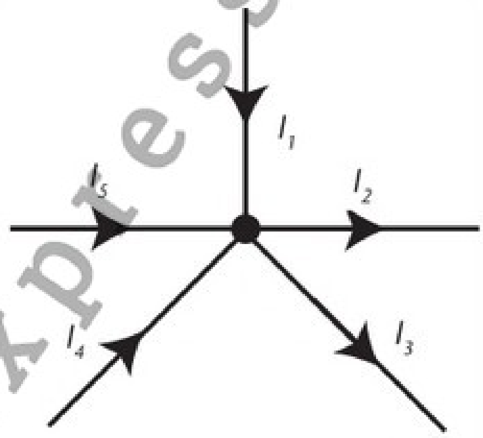
The intercept with the Y axis represents the Voltage when no current is flowing. This is Vo or the EMF of the battery. The slope of the line is the internal resistance of the battery. The better the quality of the battery, the smaller the slope will be. In real batteries, often the internal resistance of the battery changes as they age. The older they get, the higher the internal resistance. Finally they reach the point that when a small current flows, the little energy that is delivered is dissipated inside the battery; they become useless!

Kirchoff's Laws

When circuits are simple it is quite easy to visualise how the current is flowing and to calculate any variables we need to find. However, real circuits can be quite complex, and it is helpful to have some laws to guide our understanding.

Kirchoff's Current Law

At any node (or junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.



$$I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

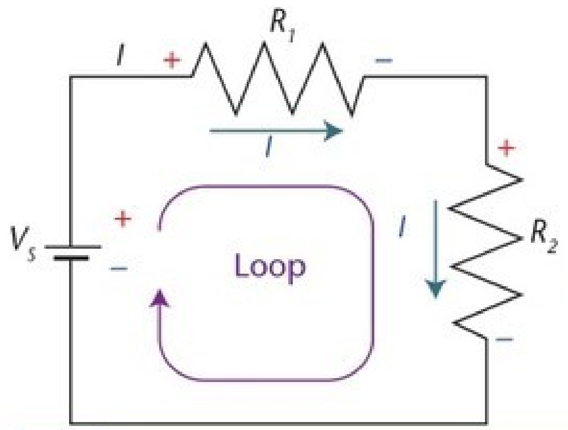
9.50

Notice that this is based on the principle of conservation of charge. Any flow of charge coming into a junction must be equal to the charge leaving the junction. Otherwise charge would be either disappearing or being created at that junction.

Notice also that this is true even for batteries: any current leaving the battery must be equal to the current returning to the battery.

Kirchoff's Voltage Law

The sum of the EMFs in any closed loop is equivalent to the sum of the potential drops in that loop.



$$V_S + (-IR_1) + (-IR_2) =$$
$$\therefore V_S = IR_1 + IR_2$$

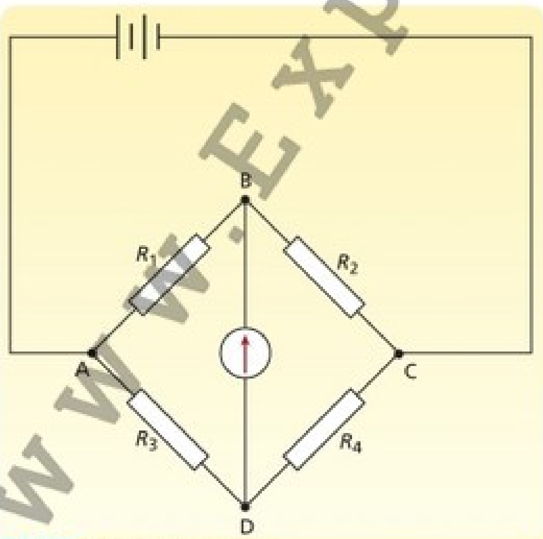
9.51 *Circuit to measure the internal resistance of a battery*

You may wonder why the law speaks about "the sum of the EMFs", this is because it is possible to have more than one battery in a circuit, and the batteries do not all have to be together (although we normally do place them next to each-other).

Notice that this is based on the principle of conservation of energy. Remember that a volt is a joule per coulomb, so what this law is effectively saying is that the sum of the joules of energy given by the batteries to the coulombs in a circuit (the EMFs) is equal to the sum of the joules spent by the coulombs in the resistors, lamps, or other drains in the circuit. Energy is neither created nor destroyed, it is simply redistributed.

The Wheatstone bridge

Charles Wheatstone (1802–1875) was an English scientist and inventor from the Victorian era who invented a number of diverse things, ranging from a new type of code to a concertina. However, he will be mainly remembered for a device he developed that allowed us to measure electrical resistance: the **Wheatstone bridge**.



9.52 *A Wheatstone bridge*

The Wheatstone bridge is essentially a circuit as shown in figure 9.52, with a galvanometer connecting the two points B and D.

The values of the resistances R_1, R_2, R_3 and R_4 can all be varied. Generally, if resistors are chosen at random, you would expect current to flow through the galvanometer. However, when an arrangement of resistors is found for which no current flows through the galvanometer, we can say that the bridge is balanced, and the following formula applies:

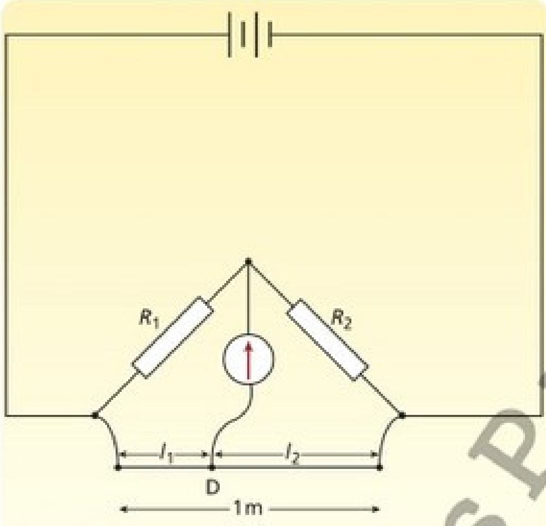
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

If one, and only one, of the resistors is of unknown resistance, this formula allows us to calculate its value: i.e. if R_1 is of unknown value, that value can be found using:

$$R_1 = \frac{R_2\ R_3}{R_4}$$

The circuit was originally used mainly as a way to calculate resistance. Although it is still a highly accurate way of doing this, a modern ohmmeter is generally a more convenient device. The same circuit is still of great importance, though, because it can be adapted for use as a thermostat.

In an electric oven when you set the temperature, for instance, to 180° C, you are in fact picking a value for a variable resistance in a Wheatstone bridge. As the oven heats up, the resistance changes. When the desired temperature is reached, the bridge 'balances', and this causes the heating element to be switched off. Once the oven begins to cool, the resistance again changes, the bridge becomes unbalanced and the heater switches on again. In this way the temperature is kept as close as possible to the set value.



9.53 *A metre bridge*

Similar devices can be used to switch a central heating system, or a water heating system, on and off. They can also be used as a 'fail-safe' to prevent any device overheating.

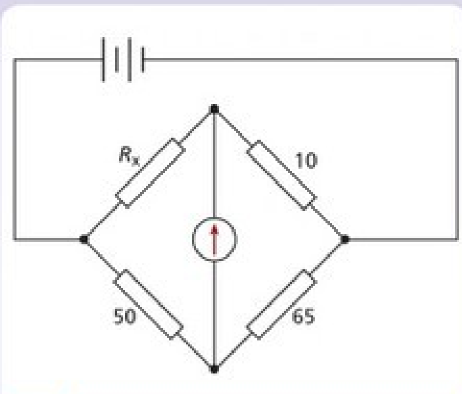
As a way of measuring resistance, the main difficulty with the Wheatstone bridge is in finding the values of the resistances that will allow the bridge to balance. This can be made easier using a **metre bridge**. Instead of replacing resistors, the sliding connection at D just has to be moved along the metre-length of conducting wire, until the position where no current flows through the galvanometer is found (see figure 9.53).

As resistance is proportional to length, the formula is effectively replaced with:

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

9.12 Sample Question

The Wheatstone bridge in figure 9.54 is balanced. What is the value of the unknown resistance Rx?



9.54

Sample Answer

$$\frac{R_1}{R_2} = \frac{R_2}{R_3}$$

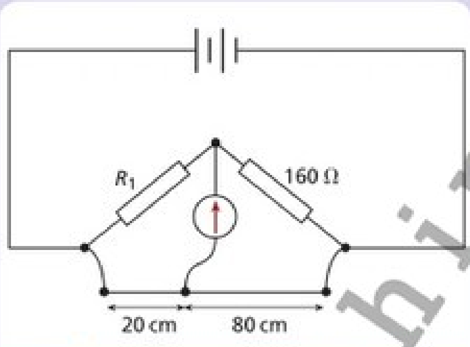
$$\frac{R_x}{10} = \frac{50}{65}$$

$$R_x = \frac{10 \times 50}{65}$$

$$= 7.69\,\Omega$$

9.13 Sample Question

The metre bridge shown here is balanced. What is the value of the unknown resistor R_1?



9.55

Sample Answer

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$

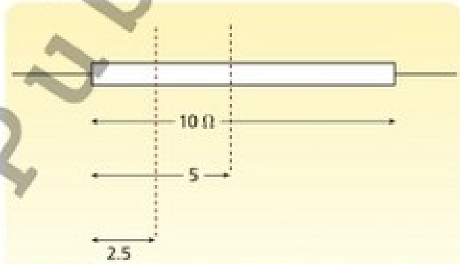
$$\frac{R_1}{160} = \frac{20}{80}$$

$$R_1 = 160 \times \frac{20}{80}$$

$$= 40\,\Omega$$

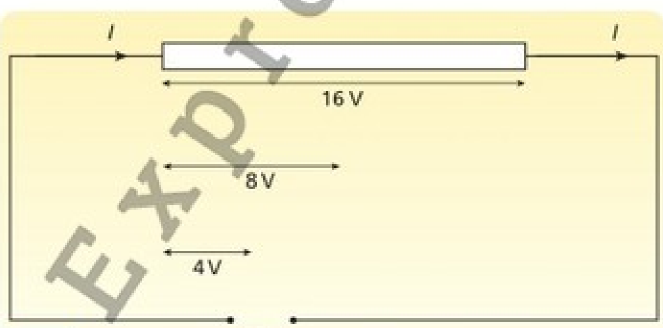
Potential divider

We have seen already that the longer a resistor is, the larger its resistance becomes. If a current flows only across a section of a resistor, the resistance is proportionately reduced, as shown in figure 9.56.



9.56 *Resistance is proportional to length*

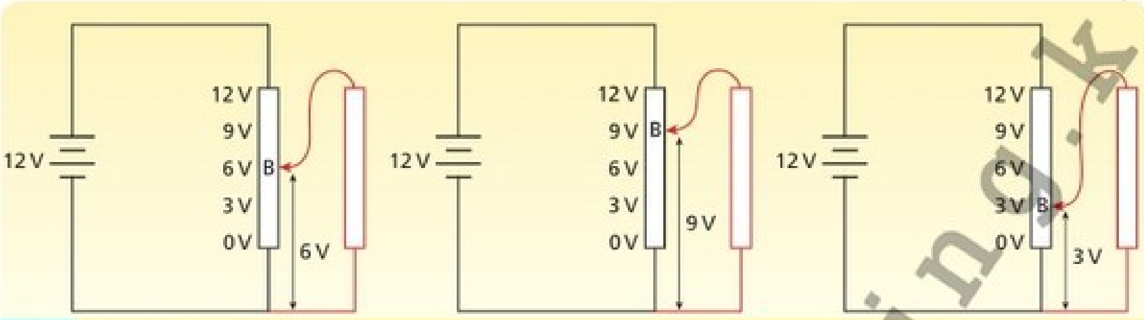
Following on from the formula $V = IR$, we can see that the potential drop across a resistor is also in proportion to the fraction of a resistor across which a current flows (see figure 9.57).



9.57 *The drop in voltage is also proportional to length*

A potential divider works in this way, making two connections to a resistor across which a current flows. In this way, smaller voltages can be supplied.

Potential dividers are often used in circuits like that shown in figure 9.58. The voltage driving a current through the part of circuit to the right (coloured red) is controlled by the position of the connection labelled B.

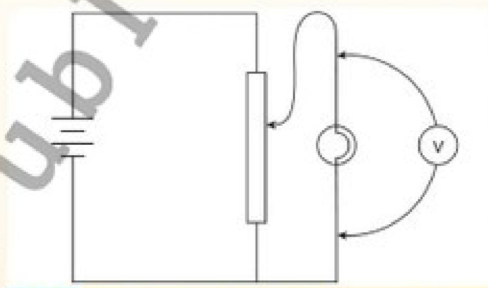


9.58 *A potential divider*

Experiment 9.7: To demonstrate the use of a potential divider

Method

1. Set up a circuit as shown in figure 9.59.
2. Move the 'slider' along the variable resistor and note how the reading on the voltmeter to the right increases or decreases.



9.59 *To demonstrate the use of a potential divider*

Observations and conclusions

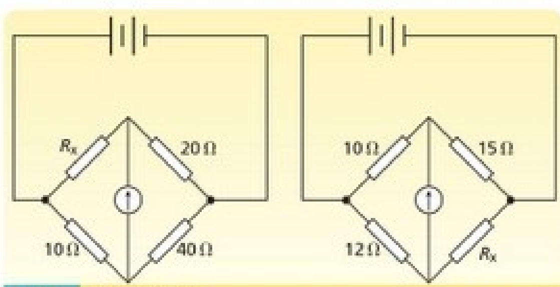
The voltmeter to the right measures the output of the circuit. This is the voltage that drives a current through the bulb – higher voltages drive higher currents and make the light brighter.

Potential dividers (sometimes called potentiometers) are often used in laboratory investigations as they allow us to supply a carefully controlled voltage to a circuit. They are also often used as the volume control in a radio. They can also be used to control a dimmer switch.

9.60 *Volume control*

For you to try

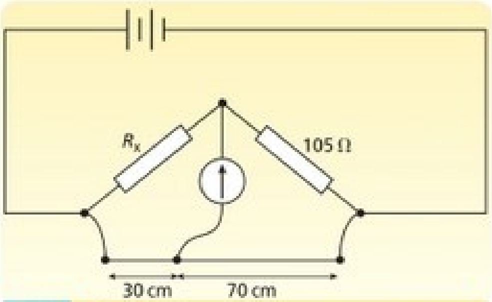
1. What was the original purpose of a Wheatstone bridge?
2. What other uses does the Wheatstone bridge circuit have?
3. Draw a circuit for the Wheatstone bridge.
4. The Wheatstone bridges shown in figure 9.61 are all in balance. What is the value of the unknown resistance Rx in each case?



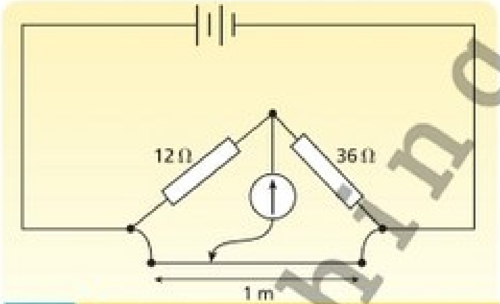
9.61 *Question 4*

5 The metre bridge in figure 9.62 is balanced. What is the value of the unknown resistance?

6 Look at figure 9.63. At what position on the metre-length of wire should you place the sliding connector so that the bridge would balance?

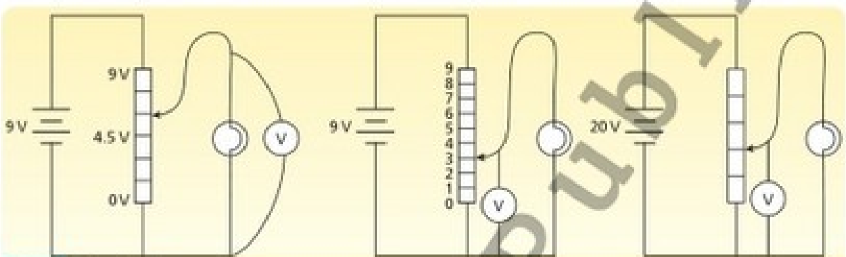


9.62 *Question 5*



9.63 *Question 6*

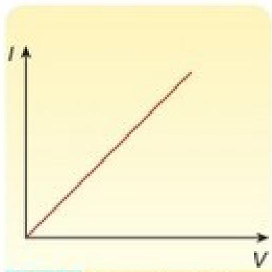
7 Figure 9.64 shows a potential divider. What reading would you expect to see in each of these situations?



9.64 *Question 7*

Voltage–current graphs

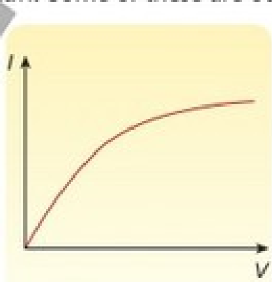
We looked at Ohm's law and learnt that for metals and some other conductors, the current flowing through a resistor will be proportional to the voltage across it. This means that the relationship between voltage and current follows the graph shown in figure 9.65 – a straight line through the origin.



9.65 *Current against voltage*

However, it should be remembered that Ohm's law applies only in limited circumstances. In particular, it applies mainly to metallic conductors and only if those conductors are maintained at constant temperature. There are many electric circuits that use components that do not follow Ohm's law. Some of these are outlined below.

9.66 *A filament bulb*



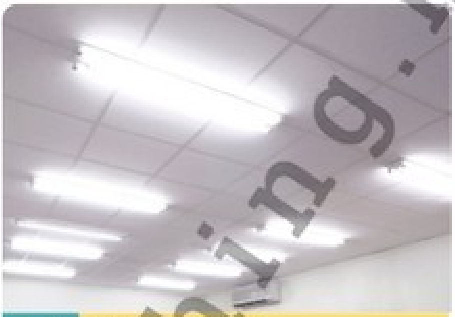
9.67 *I against V for a filament bulb*

Filament bulb

The traditional light bulb used in most homes consists of a very narrow piece of tungsten metal, known as the filament. Although a conductor, the very narrow cross section of the wire creates a high resistance in the tungsten, and when current flows through it a great deal of heat is produced. This causes the bulb to give out light. However, because the temperature of the wire varies, it is obvious that Ohm's law cannot apply. The relationship between current and voltage in this example is shown in the graph in figure 9.67.

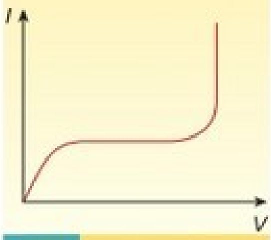
Gas

Fluorescent lights are often used in schools and factories because they require only a very low current and are, therefore, relatively cheap to run. Within a tube there is a mix of stable gases between the two electrodes at either end. The attraction of negative to positive causes some electrons to move from the negative towards the positive connection. As these electrons move, they collide with the gas particles that fill the tube. This collision can have sufficient energy to 'knock' electrons off the gas atoms, which leaves both an extra free electron and a positively charged ion (a charged atom). This happens repeatedly throughout the gas, so that a great deal of ions are created. These positive ions then move through the tube towards the negative connection.



9.68 *Fluorescent lighting*

9.69 *Conduction in a gas*



9.70 *I against V for a gas*

In this way electric charge moves through the gas and an electric current flows. However, there is a limit to how many ions can be created. This means that although an increase in the voltage will increase the current, the connection is not linear and instead follows the curve shown in the graph in figure 9.70.

Vacuum

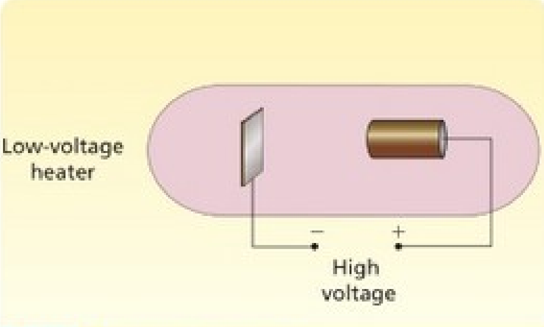
In a cathode ray tube (CRT), an electric circuit is created with a gap between two electrodes (the negative **cathode** and positive **anode**). You have learnt already that current generally cannot flow once there is a gap in the circuit, but there are two important aspects of this circuit that allows a current to flow.

Firstly, a heater is placed behind the negative connection – the cathode – and this provides additional energy to the system. The electrons on the cathode use this energy to escape from their atoms and from the surface of the metal. This process is known as **thermionic emission**.

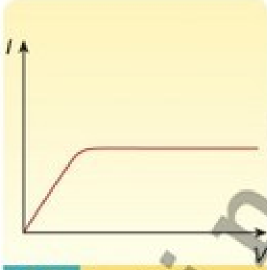
Secondly, a vacuum has been created in the space between the two electrodes. The absence of air molecules means that there is no impediment to the movement of the electrons from the cathode to the anode. This allows the electrons to cross the gap and to continue to flow through the circuit.

There is a limit, however, to the total number of electrons that can be released at the cathode via thermionic emission. This means that above a specific level of applied voltage, the current no longer increases significantly. This accounts for the levelling off of the graph.

9.71 *A cathode ray tube (CRT)*

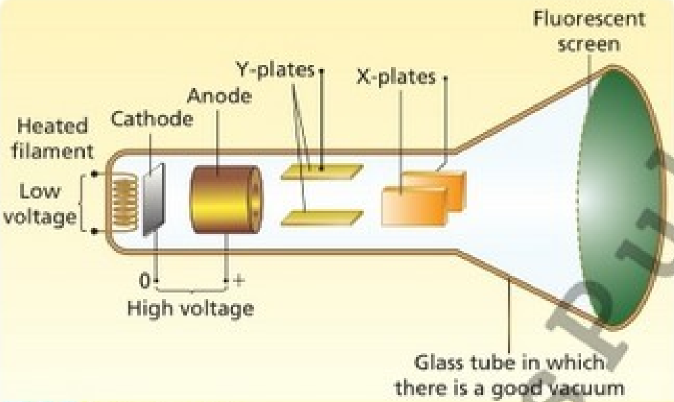


9.72 *Conduction in a vacuum*



9.73 *I against V for a vacuum*

Cathode ray tubes

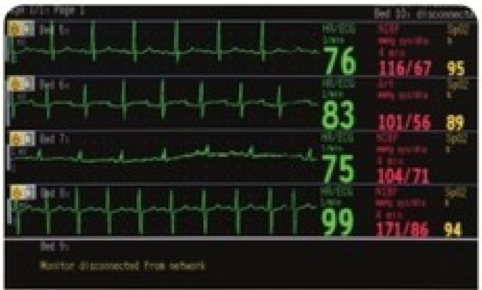


9.74 *Cathode ray tubes*

Key points in the operation of a CRT

- Thermionic emission occurs at the cathode.
- This releases electrons, which move towards the anode at great speed (above $10^7\,\mathrm{m\,s^{-1}}$).
- Due to their high speed, most electrons pass straight through the anode and continue in a straight line.
- They strike the screen, producing a spot of light at its centre.
- The potential and shape of the anode allows us to control the focusing of the spot.
- The beam can be deflected by the electric field across the X- or Y-plates (electromagnets can also be used).
- The brightness is controlled by a grid between the cathode and the anode. It is negative with respect to the cathode. The more negative it is, the more electrons are repelled back towards the cathode, and the fewer electrons there are to strike the screen.

In a device such as an oscilloscope, the X- and Y-plates are connected to some external source. If they are connected to a microphone, for example, and held close to a source of a musical note, the charges on the plates vary in a way that follows the variations in the sound waves. This allows the sound wave to be 'seen' on the screen. Alternatively, they can be connected to small sensors attached to a person's chest (see figure 9.75). The beating of the heart creates small variations in the current passing through these



9.75 *An oscilloscope (CRT) display*

sensors, and these variations are seen on the screen. This allows medical staff to study the rhythm of a beating heat. There are many other similar uses.

In the traditional design of a TV set, the beam of electrons and, therefore, the dot of light would travel in lines across the screen several hundred times every second, and there were three separate beams controlling each of the three primary colours. Variations in the strength of each beam allowed the colours to vary and for the pictures with which we are all so familiar to be formed.

9.76 *Components of a CRT display for a TV. These are the components that are behind a TV screen*

In recent years, all of this technology has been replaced with flat screens. These are not just a modernisation of the traditional technologies. They operate on very different principles, and in fact a number of competing technologies all vary significantly from each other.

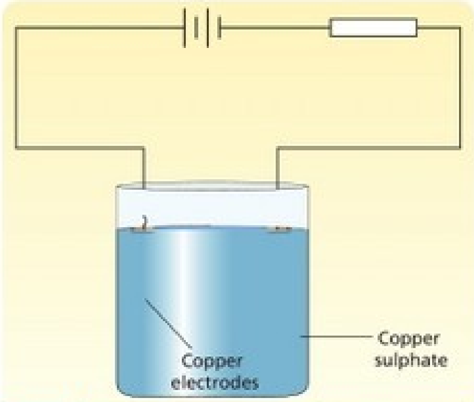
Long after the old-style TV screens are forgotten though, or consigned to museums, the CRT will still be covered in the study of physics because of its historic significance in discovering the electron.

Electrolysis

Electric current is generally carried by electrons, as we have seen. However, we have also seen that sometimes it is carried by ions (charged atoms). This happens when current passes through a gas, for example, and also when it passes through an ionic solution.

In copper sulfate, for example, the individual molecules contain two charged particles: the positively charged copper ions (Cu^{++}) and the negatively charged sulfate group (SO_4^{--}). The attraction between positive and negative is generally what holds this molecule together.

However, if the copper sulfate is dissolved in water and two electrodes are placed in the solution as shown in figure 9.77, the molecules begin to break up. This is because the positive copper ions, while still attracted to the negative sulfate ions, are also attracted to the negative electrode; some of the copper ions will leave their molecules and move towards that electrode. When they get there, they collect electrons from the electrode and become copper atoms, which usually then form a coating on the electrode. A similar process can be used to create a thin layer of gold or silver on the surface of otherwise cheap jewellery.



9.77 *Conduction in an ionic solution*

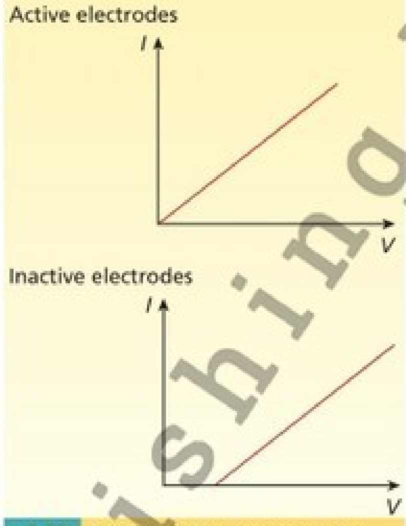
Meanwhile, the sulfate ions move over to the positive electrode and give up their electrons to the circuit.

As this process carries on over and over again there is a constant flow of electrons to the liquid and out of the liquid. Though the current crosses the liquid itself by the movement of the ions within, it does not mean that a current is not flowing: as long as there is constant movement of charge, the current is clearly flowing.

This process can take place with what we call **inactive electrodes** – electrodes that do not take place in a reaction themselves but instead serve as the surface at which electron transfer can take place. An example of an inactive electrode is a carbon rod.

We can also have **active electrodes** – electrodes that are themselves part of the electrolysis process. For example, during electroplating, electrolysis deposits a thin layer of one metal on another metal in order to improve beauty or resistance to corrosion. Using copper electrodes in a copper sulfate solution is an example of the use of active electrodes.

With active electrodes, this creates a circuit that obeys Ohm's law. With inactive electrodes, where a few volts have to be applied before a current begins to flow, the graph is a straight line, but it does not go through the origin and, therefore, it is not obeying Ohm's law.

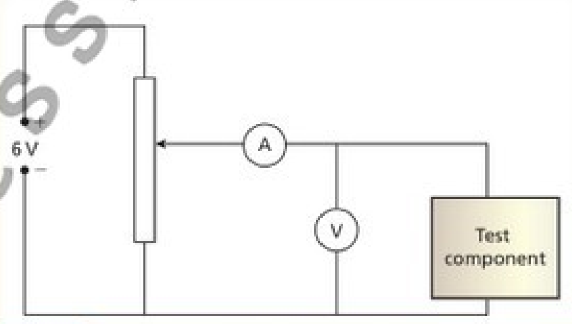


9.78 *I against V for an ionic solution*

Experiment 9.8: Investigation of the variation of current with p.d. for various electrical components

Method

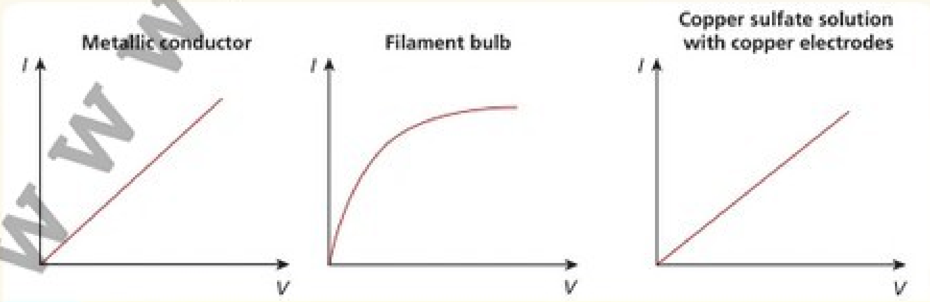
1 Set up the circuit as shown in figure 9.79, using a wire as the test component, and set the voltage supply (e.g. at 6 V d.c.).
2 Move the slider along the resistor to obtain different values for the voltage *V* and hence for the current *I*.
3 Obtain a number of values for *V* and *I* and plot a graph of *I* against *V*.
4 Repeat, replacing the wire with other devices: a filament bulb, copper sulfate solution.



9.79 *Experimental circuit diagram*

Results

You should find that your graph is similar to those shown in figure 9.80.



9.80 *I against V curves*

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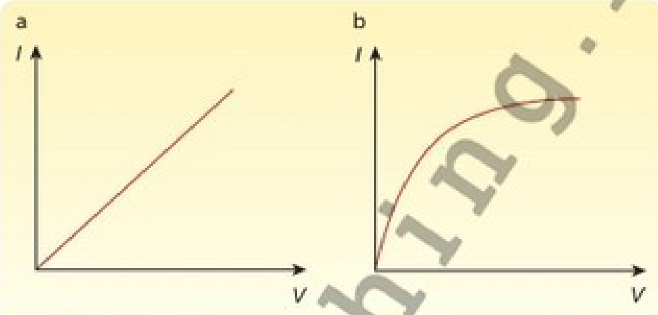
For you to try

1 One of the graphs in figure 9.81 represents the voltage – current relationship for a metal and the other for a filament bulb. Which is which?
2 Which of the two graphs in figure 9.81 follows Ohm's law?
3 Explain why the graph for the filament bulb in figure 9.81 has the shape that it does.



9.81 *Questions 1–3*

4 Which other graph would you expect to follow Ohm's law?
5 Draw the *I–V* graph for a gas and briefly explain why it has the shape that it does.
6 Draw the *I–V* graph for a vacuum and briefly explain why it has the shape that it does.
7 What are ions?
8 Give two circuits in which the principal charge carriers are electrons and two in which the principal charge carriers are ions.
9 Tables 9.1 and 9.2 represent the results of an investigation into the *I–V* graphs for two components, A and B. Sketch the graph for each one and suggest which component each may be.

Table 9.1 Component A

***V*/V**	0	2	4	6	8	10	12
***I*/A**	0	0.6	1.1	1.9	2.4	3.0	3.5

Table 9.2 Component B

***V*/V**	0	2	4	6	8	10	12
***I*/A**	0	0.9	2.0	2.5	3.1	3.9	4.1

Power in electrical circuits

Power is defined in mechanics as 'work done divided by the time taken to do the work'. In electrical circuits we can still calculate powers, but we must note a few changes. Work is measured in joules, the unit of energy. In a circuit, charges carry energy from the battery and dissipate it somewhere else in the circuit where the current experiences some resistance.

The voltage tells us how much energy is given to each coulomb, and the current tells us how many coulombs per second are going past any point in the circuit. So the amount of energy per second can be written as joules/coulombs x coulombs/second. The coulombs cancel out, leaving joules/second which is the definition of power.

In this way we have shown that Power (watts) = Voltage (volts) x Current (amperes). It may seem obvious that Voltage is measured in volts, but remember that we could also have written this expression as Power (watts) = EMF (volts) x Current (amperes). The first expression would tell us how much power is being dissipated in any given component in the circuit, this later expression (involving EMF) tells us how much power the source is delivering to the circuit.

Normally in circuit calculations, we assume that the wires have no resistance at all. This means that no energy is ever wasted in transmission: it is all delivered from the battery to the components. In real life, even the best conductors have some resistance, and this causes some power to be wasted along the way.

The heat generated in a circuit is given by Joule's Law:

$$Q = I^2 R t$$

where:

I – current
R – resistance
t – time

Some thoughts about 'superconductivity'

Scientists have discovered that at very low temperatures (around –200°C) some materials become 'superconductors'. By this we mean that their resistance is exactly zero. This means that once a current starts to flow through them it can flow indefinitely without causing any heating. Unfortunately it is very expensive to achieve the low temperatures needed to reach superconductivity; it requires liquid nitrogen. Although nitrogen is abundant in the atmosphere, it is expensive to cool it down to the point where it condenses out of the atmosphere.

We do not know if in the future it might be possible to discover materials that are superconductive at room temperatures, however, if this were to become possible, it would revolutionise the way in which we transport electricity. Some of the consequences of superconductivity at room temperature would be the following:

- No need for dangerously high voltages
- Much greater efficiency in all electrical appliances
- Fantastically efficient electric cars
- Mobile phones that only need to be charged once per month

Can you think of any other benefits from superconductivity?

9.14 Sample Question

What is the power of an electric heater that operates at a voltage of 220 V and draws a current of 10 amperes?

Sample Answer

Power = Voltage x Current, so Power = 220 x 10 = 2200 watts

9.15 Sample Question

An electric hair dryer operates at a voltage of 220 V, the manufacturer states that its power is 750 W. What is the current flowing through it?

Sample Answer

$P = V I$ so $I = P/V$ $I = 750/220 = 3.41$ amperes

9.16 Sample Question

A water pump needs to lift water from a well that is 10 metres deep. We require the pump to be capable of delivering 20 litres per minute. The pump will be operated from a 220 V supply. Assuming the efficiency of the electric pump to be around 25%, calculate how much current the pump will draw.

Sample Answer

First of all, let us calculate what power is needed:
The amount of gravitational potential energy being delivered to the water is = *mgh*.
The density of water is 1 kg/litre, so the mass is 20 kg.
Energy supplied = 20 kg x 10 ms^{-2} x 10 m = 2000 Joules.
We require this much energy in 1 minute (= 60 seconds).
Power needed = Energy/time = 2000/60 = 33.3 W.
The question asks us to assume that the conversion will only be 25% efficient, that means that we will need a pump that is 4 times more powerful.
That is 33.3 x 4 = 133.3 watts.
Power = Voltage x Current, so Current = Power/Voltage = 133.3 / 220 = 0.6 amps.

For you to try

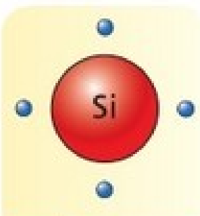
For all the questions that follow, assume a mains voltage of 220 V.

1 A mains bulb has a resistance of 800 Ohms. What will its power output be when connected to a mains voltage of 220 V?
2 An electric kettle is rated at 2000 W. If it is switched on for 3 minutes, how much energy has been converted?
3 An electric heater needs to have a power output of 3500 W, what should its resistance be?
4 Two electrical heaters are connected to the mains in series with each-other. The first has a resistance of 50 Ohms, and the second a resistance of 35 Ohms. What will their power outputs be?

Semiconduction

A semiconductor is a material with a resistivity that lies between that of a conductor and that of an insulator.

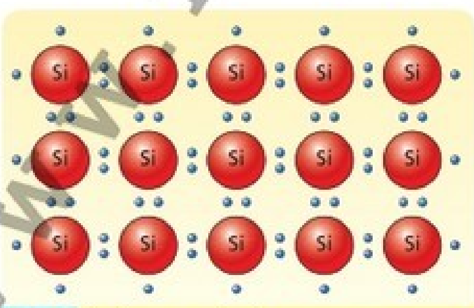
There are many examples of semiconductors. Germanium, gallium arsenide and zinc sulfide are just a few of those used. However, the best-known semiconductor and the example that we will study in detail is silicon.



9.82 *Silicon atom*

Silicon has a valency of four. This means that it has four electrons in its outermost shell but, like other atoms, would 'prefer' to have eight.

In a silicon crystal, in order to achieve a situation in which each atom is surrounded by eight electrons, the atoms arrange themselves in such a way that each atom shares an electron with each of four neighbours (see figure 9.82).



9.83 *A silicon crystal at low temperature*

At a temperature of 0 K, this a perfect insulator. No electrons are free to move and they are therefore not free to carry a current.

Above 0 K, the added heat energy is distributed throughout the crystal as kinetic energy. From time to time, an electron will gain sufficient energy to escape from its place and move through the crystal. It leaves a 'hole' behind it.

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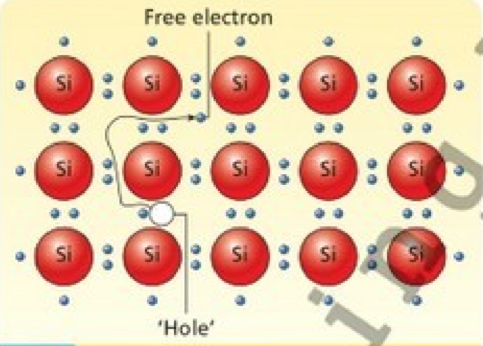
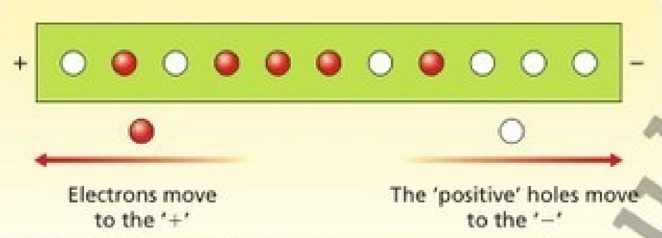
Intrinsic conduction

Once electrons gain enough energy to escape from their places, they are available to carry a current if the crystal is placed in a circuit. In such a case, the free electrons will move towards a positive connection, and the holes will end up effectively moving towards the negative connection. They have no charge, of course, but as they will always move towards a negative connection, it is useful to think of them as being positive; and they are often referred to as **positive holes**.



9.84 *A silicon crystal at high temperature*

We can simplify a crystal as shown in figure 9.85.



9.85 *Intrinsic conduction in silicon*

Extrinsic conduction

Conduction can be improved by either adding extra free electrons **(n-type doping)** or extra holes **(p-type doping)**.

n-type doping

When constructing a silicon crystal, we can introduce a small number of phosphorous atoms. These will try to fit into the overall crystal structure, sharing one electron with each of four neighbours. But they have five electrons in their outer shell and will, therefore, have one 'extra electron'. This is likely to break free from the atom and to become available for conduction. As there are extra free electrons, conductivity is improved. Addition of a small quantity of an impurity in this way is known as doping.

p-type doping

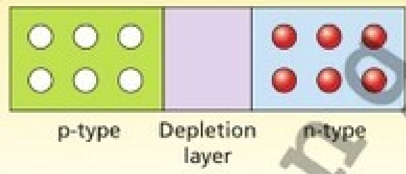
If we add a small quantity of boron to a growing crystal of silicon, the boron atoms will try to fit into the structure of the crystal. Each one has only three electrons in its outer shell, however, so an extra 'hole' will be created. As we have seen, holes act like positive charges and are sometimes referred to as positive holes – they are often thought of as actually carrying the current. In p-type material, we even say that holes are the **majority charge carriers**. In reality, they act as stepping stones for the moving electrons, which improves the conductivity.

Doping is the addition of small quantities of an impurity to a semiconductor in order to improve conductivity.

p-n junction

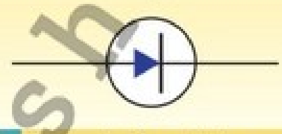
n-typing and p-typing are really only of any use to us when we place the two together at a p-n junction. The simplest example of this is in a **diode**.

At the junction of the two materials, the extra free electrons tend to occupy the extra free holes. This is known as the **depletion layer**. It acts as a block of insulating material in the middle of the diode (see figure 9.86).



9.86 *A p–n junction*

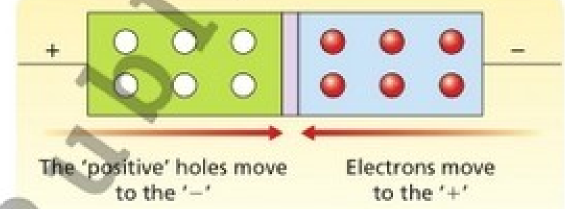
The symbol for a diode is shown in figure 9.87. The significance of the diode is that it allows current to flow in one direction and not the other.



9.87 *The symbol for a diode*

Forward bias

If the diode is placed in a circuit so that the p-type material is attached to a positive connection, current will flow. This is because the electrons are drawn towards the positive, and the holes to the negative: the depletion layer shrinks and allows current to pass through it (see figure 9.88).



9.88 *In forward bias, the depletion layer shrinks and a current flows*

Reverse bias

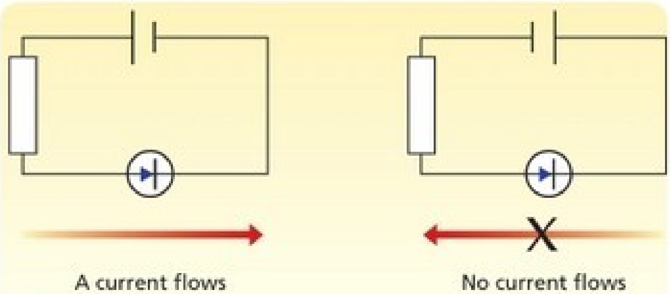
For reverse bias, the p-type material is attached to the negative terminal, and the n-type to the positive. The electrons are drawn backwards from the centre and the depletion layer grows. A current will not be able to move through the device (see figure 9.89).

Using electrical symbolism, these circuits could be shown as in figure 9.90.

9.89 *In reverse bias, the depletion layer grows and no current can flow*

One use of a diode is in **rectification** – the conversion of a.c. to d.c. – which we came across in Module 8 (see page 144). The diode blocks the current flow in one direction, allowing it through only when it is in forward bias. The current is not constant and it flows only in one direction and is therefore d.c.

Another example of a diode is the **light-emitting diode** (LED). LEDs are diodes in which a current crossing the p-n junction causes a small amount of light energy to be released.



9.90 *Circuit diagrams to represent the situations in forward bias (when a current flows) and in reverse bias (when no current flows)*

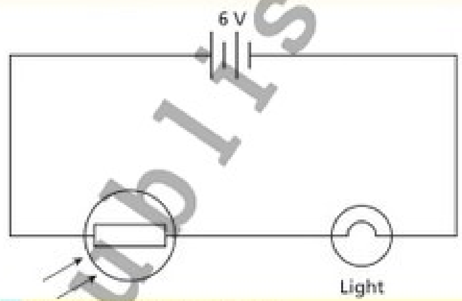
For you to try

1 What is a semiconductor? Give three examples.
2 Briefly explain each of the terms 'intrinsic conduction', 'extrinsic conduction', 'p-type doping' and 'n-type doping'.
3 What is a p-n junction?
4 Briefly explain how a depletion layer is formed at a p-n junction.
5 Draw a diagram of a p-n junction, showing the depletion layer.

Experiment 9.9: To demonstrate the operation of an LDR (Light Dependent Resistor)

Method

1 Set up a circuit as shown in figure 9.91.
2 Close the switch to allow the current flow and observe what happens.
3 Cover the top of the LDR with a finger and observe what happens.



9.91 Experimental circuit diagram

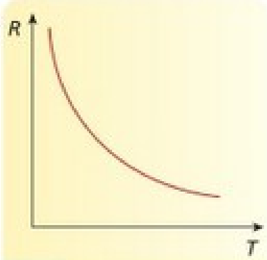
Observations

You should find that the bulb lights up when the current flows (you may have to shine a torch, or some light source onto the LDR to make the bulb light up). Then you should find that the bulb switches off when the LDR is covered.

This demonstrates the effect that light has on the LDR and on the circuit.

A more complex arrangement is needed to create a light that will come on in the dark and switch off in the light (which would obviously be a little more useful).

Thermistors are semiconductors in which the resistance is controlled tightly by temperature. Any increase in temperature creates a large decrease in resistance. This was also discussed in Experiment 9.4.



9.92 For a thermistor, the resistance falls as the temperature increases

Integrated circuits (ICs) are small electrical components containing a combination of diodes, resistors, capacitors and other devices all built into one 'chip' of silicon. Chips smaller than a fingernail can contain several million components. Figure 9.93 shows the surface of an IC in the motherboard of a PC, which contains several ICs and is connected up using a variety of resistors and connecting metal strips.

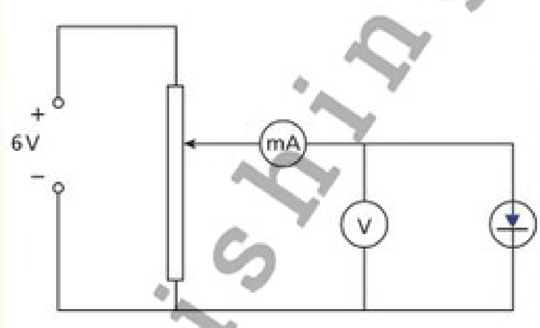

9.93 A magnified image of the surface of an integrated circuit

V–I curves

The current in a diode increases as the applied voltage increases, but it does not do so in a straight line. The reverse bias current is often described as being zero, but in fact an extremely small current does flow. It is typically measured in micro-amps.

Experiment 9.10: Investigation of the variation of current *(I)* with p.d. *(V)* for a semiconductor diode

This is a continuation of Experiment 9.8.



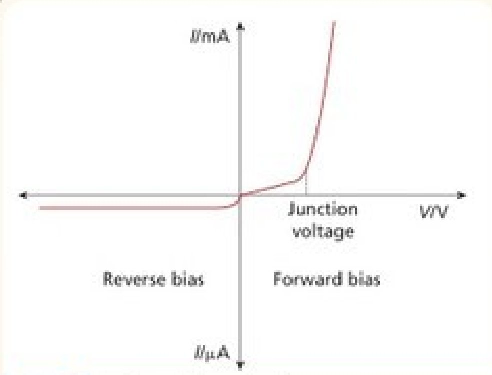
9.94 *Circuit diagram for experiment*

Method

1 Set up a circuit as shown in figure 9.94, so that the diode is in forward bias.
2 By making use of the potential divider, set the voltage that is being applied to the diode at a low level (close to 0 V).
3 Record the values of both the voltage and current.
4 Increase the applied voltage to 0.1 V and again measure the current. Record both values and plot a graph to show how current varies with voltage.
5 Repeat this process for several values of voltage, up to about 0.7 V.
6 Reverse the connection to the diode, so that it is in reverse bias.
7 Replace the milli-ammeter with a microammeter and position it so that it only reads the current through the diode.
8 Repeat steps 2-5.

Results

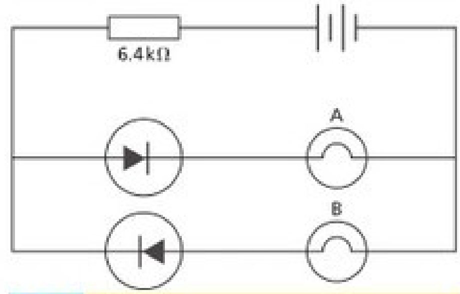
Your graph should look similar to figure 9.95.



9.95 *I against V for a diode*

For you to try

1 What happens to the size of the depletion layer when a diode is connected in forward bias? Does a current flow?
2 What happens to the size of the depletion layer when a diode is connected in reverse bias? Does a current flow?
3 In the circuit in figure 9.96, which of the two lights A and B would you expect to light?



9.96 *Question 3*

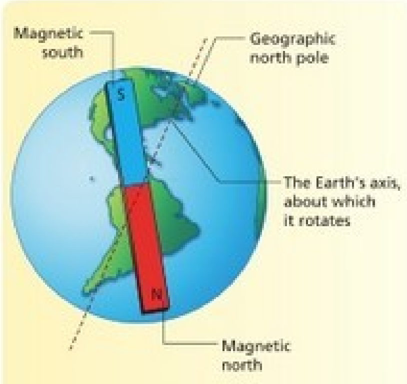
Module 10 Electricity and Magnetism

Learning objectives

- Explain the physical meaning of magnetic induction vector based on problem-solving and modern technological advances (magnetic levitation trains, etc.) 10.4.4.1
- Explain the operating principle of electronic measuring instruments, electric engines 10.4.4.2
- Analyse the operating principle of a cyclotron, magnetic trap, tokamak, hadron collider and explain the nature of polar aurora 10.4.4.3
- Research the effect of magnetic field on moving charged particles 10.4.4.4
- Analyse the operating principle of electromagnetic devices (electromagnetic relay, generator and transformer) 10.4.5.1
- Use the law of electromagnetic induction in problem-solving 10.4.5.2
- Make comparison between mechanical and magnetic energy 10.4.5.3
- Research the acting model of an electric engine and explain the received results in a well-argued manner, using Faraday's law of induction and Lenz's law 10.4.5.4

The Earth's magnetic field

A magnet that is left free to rotate (away from any other magnets, as is the case in a compass) will always come to rest with one end – the north pole (N) – pointing to the north, and the other – the south pole (S) – pointing to the south. This is due to the Earth's magnetic field. It is as if there were a large bar magnet at the centre of the Earth. The reason for this is that there is a great deal of molten iron in the Earth's core, and it is this iron that creates the magnetic field.



10.1 *The Earth's magnetic field*

One of the problems associated with the concept of magnetic poles is evident in looking at the Earth's magnetic field. For many centuries navigators were happy to think about the pole of the magnet that points north as being the north pole of that magnet. But as we have come to understand magnetic fields better, we know that a north pole will always be attracted to a south pole. This means that, confusingly, we have created a labelling system for magnets that means the magnetic pole located near the geographic north pole is in fact a magnetic south.

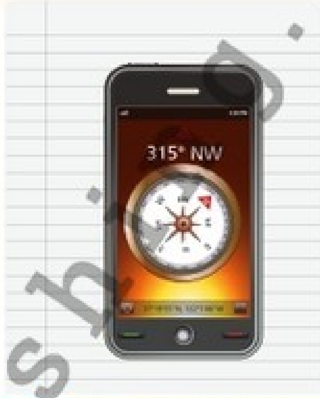
Magnetic declination

From figure 10.1 you will see that the magnetic poles and geographic poles are not perfectly aligned. This has obvious effects in navigation: depending on where a magnet might be used, it may or may not be pointing towards the geographic pole. To further complicate matters, the magnetic pole is not fixed and moves constantly. It has been in Northern Canada for many years but moves at a rate of several kilometres per year, and it looks like it will move into Siberia over the next few decades. The angle between a line pointing to the geographic and magnetic poles is known as the angle of declination. Knowledge about the angle has been very important to navigators, but with the widespread use of the satellite-based Global Positioning System (GPS), it is less of an issue than it once was.

Experiment 10.1: To investigate magnetic declination

Method

1 Set a smartphone with a magnet reading on top of a sheet of paper, as shown in figure 10.2. (Not all phones have this function, but many do. Ensure that it is set to show true north, which the phone will locate using GPS.)
2 Mark the direction of north on the paper.
3 Replace the phone with a a compass.
4 Allow the needle to settle, and again mark the direction of north. This is showing the direction of magnetic north.



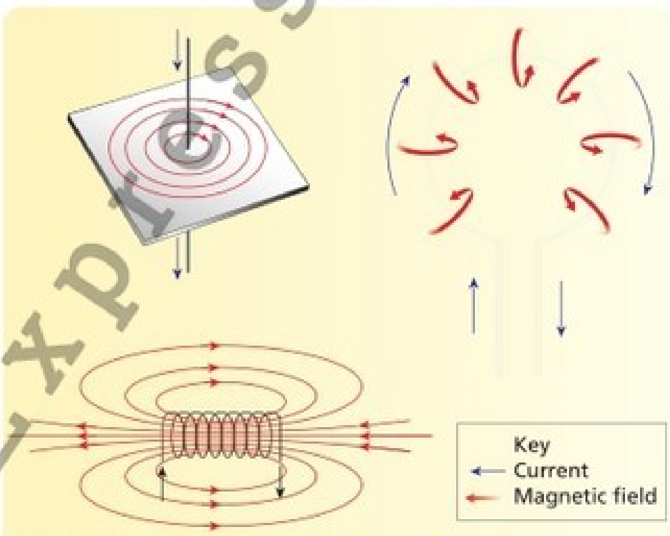
10.2 *A smartphone, indicating true north*

Observations

The angle between the two lines is the angle of declination at that location.

Magnetic fields around electrical objects

We have seen already that all magnetism is created either directly or indirectly by the presence of moving electric charge. It follows that any electrical current will create a magnetic field. However, the shape of that field will be determined by the arrangement of the electric circuit. The shape of the magnetic fields due to the electrical current in a long straight wire, a loop and a solenoid are shown in figure 10.3.



10.3 *Magnetic fields around electric currents*

For you to try

1 Are the Earth's magnetic poles fixed permanently in place? Explain your answer.
2 (a) What is meant by the term 'magnetic declination'?
(b) What are the consequences of this effect for those navigating by compass?

A current-carrying conductor in a magnetic field experiences a force

We have seen that an electric current will always create a magnetic field. We also know that any two magnets will create forces on each other if they are close enough for their magnetic fields to overlap. It follows that if an electric current passes through a magnetic field, it will experience a force. This can be demonstrated using a strip of tinfoil, which carries a current in a magnetic field (see Experiment 10.2).

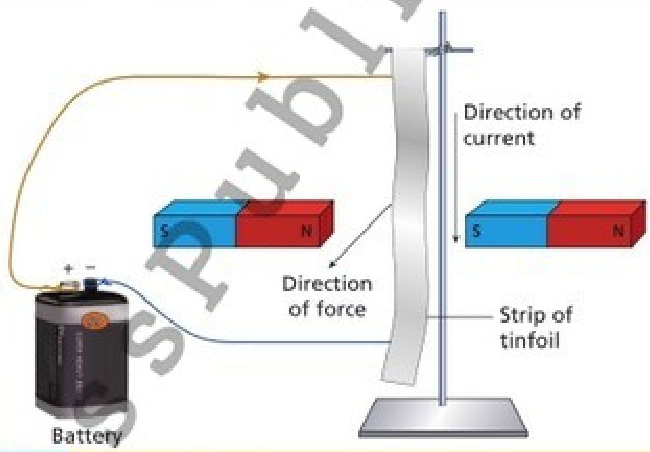
Experiment 10.2: To demonstrate that a current-carrying conductor in a magnetic field experiences a force

Method

1. Set up a circuit like that shown in figure 10.4, with a piece of tinfoil suspended from a retort stand.
2. Close the switch so that a current will flow.

Observations

You should see the tinfoil visibly move. If set up as shown in figure 10.4, the tinfoil will move forward, but the direction of movement depends on the exact arrangement of magnets and the current.



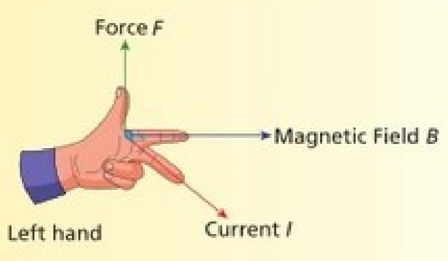
10.4 *Experimental setup*

The direction of the force

As we have seen, a current-carrying conductor in a magnetic field will experience a force. In Experiment 10.2, we use tinfoil so that the metal will be light enough to move in response to this force, allowing us to detect its presence.

If you swap around the connections to the power supply so that the current flows in the opposite direction, you should notice that the foil still moves, but that it moves in the opposite direction. Similarly, if you swap the two magnets, you will see that the direction of movement reverses again.

The connection between the directions of the current, magnetic field and force was established by English electrical engineer and physicist John Ambrose Fleming (1849–1945) in the late 1800s. Fleming explained the connection using what he called the **left-hand rule**. If you look at figure 10.5, you will see a left hand with the fingers arranged so that the thumb, index and second fingers are all at right angles to each other.

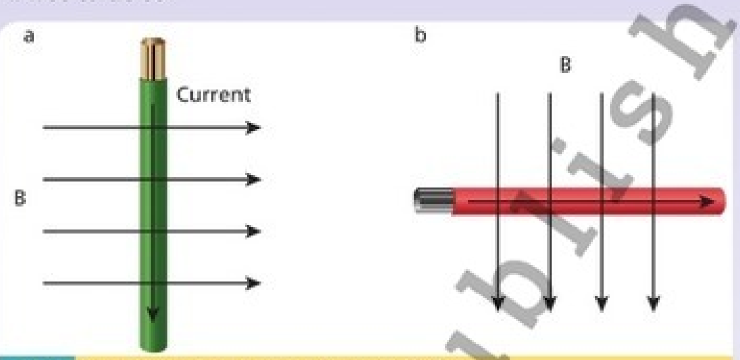


10.5 *Fleming's left-hand rule*

The index finger is used to represent the direction of the magnetic field, the second finger shows the direction in which the current is flowing, and the thumb shows the direction of the resulting force.

10.1 Sample Question

The diagrams in figure 10.6 show electrical wires carrying a current and flowing through a magnetic field represented by B. Which way would the wires move, if free to do so?



10.6 *Wires carrying a current in a magnetic field*

Sample Answer

(a) Following the left-hand rule, the wire would move 'out of' the page.
(b) Following the left-hand rule, the wire would move back 'into' the page.

The strength of the force

The left-hand rule tells us the direction of the force created when a current-carrying conductor passes through a magnetic field. The magnitude of this force also depends on the magnetic field and the current. It is described by the formula:

$$F = BI\ell$$

where:

F – force

I – current

ℓ – length of the wire within the magnetic field

B – magnetic flux density, which is how we measure the strength of the magnetic field.

The magnetic flux density (B) is a measurement that you are unlikely to have encountered before. It is one of the ways in which we measure the strength of a magnetic field. The above formula can be used to help define what exactly we mean by this measurement.

10.7 *Nikola Tesla (1856–1943), a Serbian American scientist who developed early a.c. transmission systems and designed early transformers*

Magnetic flux density (B) is the force experienced by a conductor of length 1 m carrying a current of 1 A at right angles to the magnetic field. Its direction is the direction of the magnetic field lines:

$$B = \frac{F}{I\ell}$$

The unit of magnetic flux density is the tesla (T).

If a wire of length 1 m carries a current of 1 A and experiences a force of 1 N while passing through a magnetic field, the magnetic flux density is 1 T.

10.2 Sample Question

An electrical wire carries a current of 1.5 mA and passes through a magnetic field of length 2 cm and of magnetic flux density 4 T. What force does it experience?

Sample Answer

$F = BI\ell$

$= (4)(1.5 \times 10^{-3})(0.02)$

$= 1.2 \times 10^{-4}$ N

10.3 Sample Question

An electrical wire carries a current of 3.2 A and passes through a magnetic field of length 15 cm and of magnetic flux density 2 T. What force does it experience?

Sample Answer

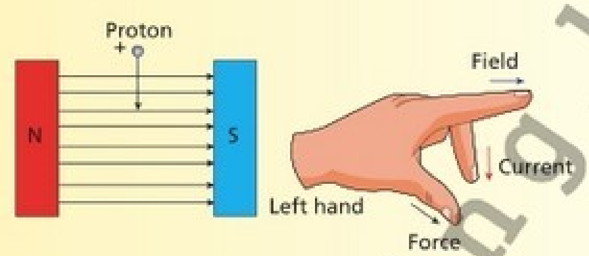
$F = BI\ell$

$= (2)(3.2)(0.15)$

$= 0.96$ N

Subatomic particles and the left-hand rule

We have already looked at the forces created on a conductor when a current flows through a magnetic field. That analysis can be adapted to help us see what happens to an individual electric charge, such as a proton or electron, when it moves into a magnetic field.



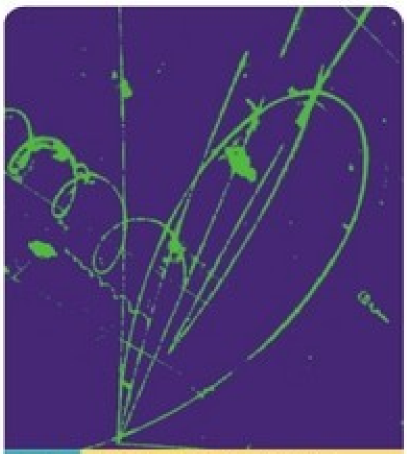
10.8 *A proton moves through a magnetic field*

An individual proton like that shown in figure 10.8 is a moving electric charge. As such, it has its own magnetic field and will be affected by any other magnetic field through which it moves. In such a case, the proton essentially follows the left-hand rule: the second finger now shows the direction of movement of the charge, while the index finger and thumb still represent the direction of the magnetic field and force, respectively. Looking at figure 10.8, the proton experiences a force that pushes it towards us (i.e. out of the page). However, as a small particle, it responds to this force immediately and changes its path accordingly. As the direction of its motion changes, the direction of the resulting force also changes. The proton responds to the new direction of the force and changes path accordingly. This will happen continuously as long as the charge stays inside the magnetic field. As a result, the proton will follow a curved path through the magnetic field.

A negative particle, such as an electron, is also affected by a magnetic field, but it experiences a force in the opposite direction: in other words, it moves in exactly the opposite direction to that predicted by the left-hand rule.

A positive charge obeys the left-hand rule. A negative charge does not and moves in the opposite direction.

The charges of subatomic particles discovered in large particle accelerators like those at CERN (the European Organization for Nuclear Research) on the Swiss–French border can be determined by looking at photographs showing their path through a magnetic field. In figure 10.9, the path of the positive charges can be clearly seen to be in one direction, while the negative charges spiral off in the opposite direction. To tell which is which, you would need to know the details of how the magnets were arranged.


10.9 *This photograph shows the tracks left when a subatomic particle decayed, creating six new particles, of various mass and charge. The different tracks represent the paths of these particles through a magnetic field*

The ampere

The definition of the ampere ('amp' for short) is also based on the concept that a current-carrying conductor in a magnetic field experiences a force. However, for this purpose we do not think of a current running through a wire placed between two magnets. Instead we think of two wires placed side by side. Each one is carrying a current and creating a magnetic field. This means that both of them are in the magnetic field created by the other. We can use this scenario as a way of developing a definition of the ampere.

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The ampere is the constant current that will produce a force of 2×10^{-7} newtons per metre of length between two straight, parallel conductors of infinite length and negligible circular cross section placed one metre apart in a vacuum.

The ampere is one of the seven base units on which the SI system is based. The other base units are shown in Table 10.1.

Table 10.1

Unit	Measures
Metre	Length
Kilogram	Mass
Second	Time
Ampere	Electric current
Kelvin	Temperature
Candela	Luminous intensity
Mole	Amount of substance

Although there is no real debate about the magnitude that each of these units should have, the precise definition is often complex and subject to much academic argument. The unit of length – the metre – is currently taken to be the distance that light travels in 1 / 299 792 458th of a second, but was previously the distance between two marks on a metal bar held in Paris. The kilogram is still defined as being the mass of a prototype, a block made from a platinum alloy, that has been held in a secure vault in Paris since the late 1800s.

The definition of the ampere is obviously problematic: where are we supposed to find two infinitely long wires, for instance, or even a perfect vacuum? There are people who argue that it should be changed to be a multiple of the charge on, say, an electron. But these arguments take many years to sort out.

In the meantime, although it is not a formal definition, it can help us to remember the simple relationship between the amp and the coulomb: 1 A = 1 C per second.

The formula $F = I\ell B$ does not really work for a subatomic particle of no real length, but it can be adapted, as shown here, to yield a similar formula:

Derivation

$$F = I\ell B$$

$$I = \frac{q}{t}$$

$$v = \frac{\ell}{t}$$

$$\ell = vt$$

$$F = \frac{q}{t}vtB$$

Substituting for I and ℓ in the original formula:

$$\boldsymbol{F = qvB}$$

This force is known as the Lorentz force: the combination of electric and magnetic force on a point charge due to electromagnetic fields.

10.4 Sample Question

What is the force on a proton travelling at $2 \times 106\,\text{m s}^{-1}$ in a magnetic field of flux density 2.1 T?

Sample Answer

$F = qvB$

$= (1.602 \times 10–19)(2 \times 106)(2.1)$

$= 6.728 \times 10–13\,N$

The charge on the proton has the same magnitude as that on the electron.

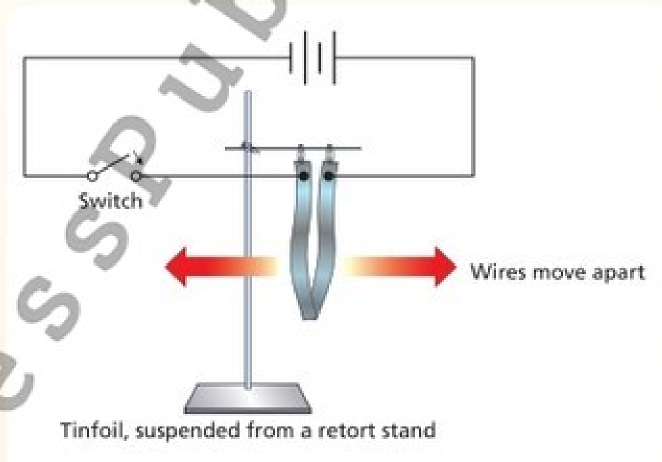
Experiment 10.3: To demonstrate the principle on which the definition of the ampere is based

Method

1 Set up a circuit as shown in figure 10.10, supporting the tinfoil with a retort stand.
2 Switch on the current and observe what happens.

Observations

You should find that the strips of tinfoil move apart, indicating that parallel wires conducting a current will experience a force, the principal on which the definition of the ampere is based.



10.10 *Experimental setup*

Electric devices

The electric motor is the basis of a lot of large household items, such as washing machines, dryers and dishwashers as well as power tools, blenders, vacuum cleaners, clocks, turntables and smaller items such as disk drives. In all situations the basics of the design are the same in that they make use of the fact that an electric current in a magnetic field will experience a force.

We can see the basis of the operation of a d.c. motor very easily (see Experiment 10.4).

10.11 *A d.c. motor*

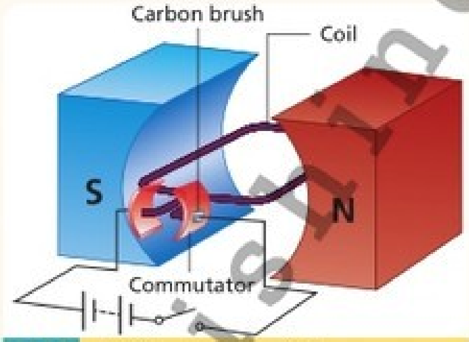
Experiment 10.4: To demonstrate the force on a current-carrying coil in a magnetic field

Method

1 Connect a basic d.c. motor – which consists of a coil between two curved magnets – to a battery as shown in figure 10.12.
2 Allow the current to flow and observe what happens.

Observations

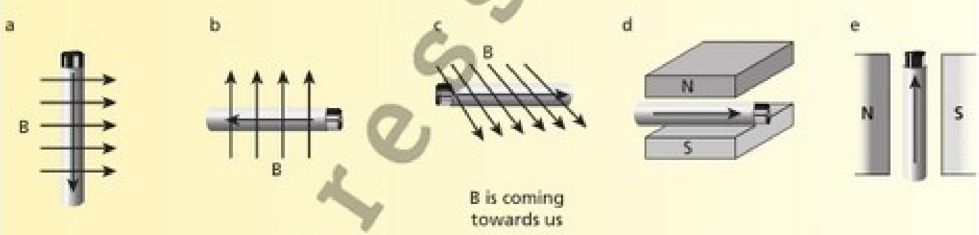
You should see that when the current flows, the coil rotates.
This demonstrates the force on a current-carrying coil in a magnetic field.



10.12 *A coil in a magnetic field*

For you to try

1 What is the direction of the force created in each of the wires shown in figure 10.13?



10.13 *Question 1*

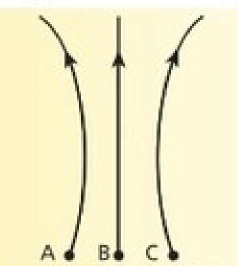
2 An electrical wire carries a current of 3 A and passes through a magnetic field of length 2 m and of magnetic flux density 4 T. What force does it experience?

3 An electrical wire carries a current of 1.5 A and passes through a magnetic field of length 2 cm and of magnetic flux density 2 T. What force does it experience?

4 An electrical wire of length 25 cm carries a current through a magnetic field of magnetic flux density 1.8 T and experiences a force of 5 N. What is the current?

5 If a wire carrying a current of 3 A experiences a force of 15 N when flowing through a magnetic field of length 15 cm, what is the magnetic flux density of the field?

6 Figure 10.14 shows the path that a number of particles follow when passing through a magnetic field. Which of the particles are positively charged, which are negative and which are uncharged?



10.14 *Question 6*

7 Figure 10.15 shows a basic design of a d.c. motor. The magnetic flux density is 4 T and the current is 2.5 A.

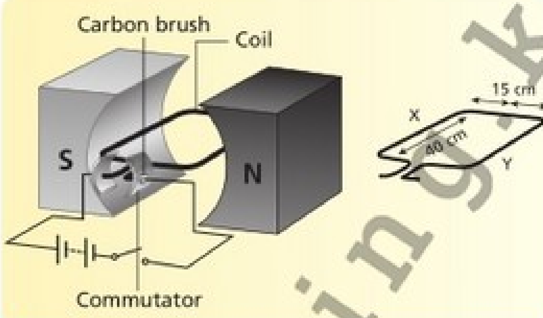
(a) What is the magnitude of the force on the wire labelled X, and in what direction would it act?

(b) What is the magnitude of the force on the wire labelled Y?

(c) What is the total moment of the turning force on the wire?

(d) Why do you think the magnets are curved in design?

(e) Name two other devices based on the same principle.



10.15 *Question 7*

Electromagnetic induction

In the preceding section we showed how we can create movement by passing an electric current through a magnetic field. In this section we will see that the opposite is also true: we can create an electric current by moving a magnetic field through a coil of wire. This is known as **electromagnetic induction**.

When the magnetic field passing through a coil changes, a voltage is induced in the coil.

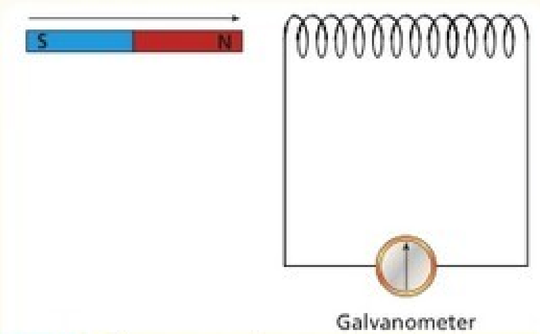
Experiment 10.5: To demonstrate electromagnetic induction

Method

1 Set up the apparatus as shown in figure 10.16.

2 Move the magnet towards the coil. Note that the needle on the galvanometer indicates that a current is flowing (a galvanometer is a device that measures electric current).

3 Try varying the speed with which you move the magnet, and observe what happens.

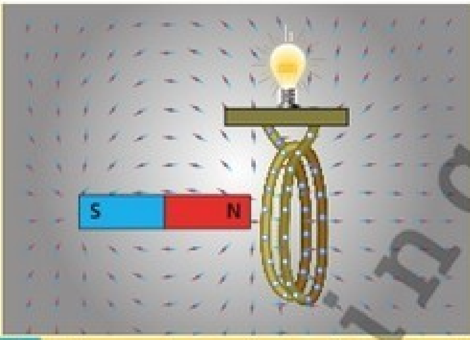


10.16 *To demonstrate electromagnetic induction*

Observations

You should see that the faster the magnet moves, the larger the current created.
Also, you should notice that the current flows in one direction as the magnet approaches and in the opposite direction as it moves away.

If you recall that each individual electron can be thought of as a sort of miniature magnet, it is hardly surprising that this effect occurs. In figure 10.17, we can see that the magnetic field from the bar magnet passes through the coil. In doing so, its effect can be felt by each of the electrons indicated (not to scale) in the wires that go to make up the coil.



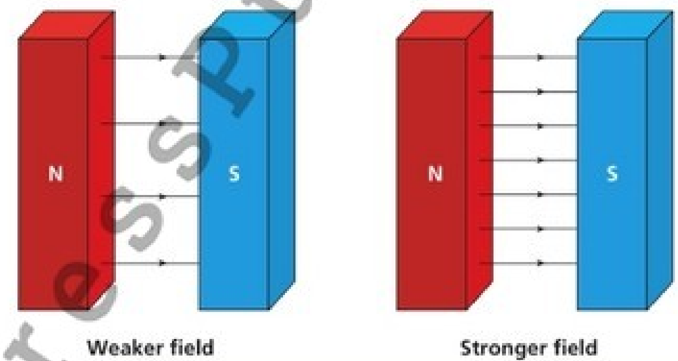
10.17 *If the magnet moves, it causes the electrons to move and the bulb to light up*

When the bar magnet is moved it will, like any moving magnet, create forces on any other magnets nearby, such as the electrons. This causes them to move. An electric current is the flow of electric charge, and so an electric current is created.

Electromagnetic induction was studied in detail by English scientist Michael Faraday (1791–1867), who was very important in the development of studies of electricity. It was Faraday who noted that the voltage created is directly proportional to the rate of change of the magnetic field. To make this mathematical connection, you have to be familiar with the concept of magnetic flux.

Magnetic flux

We often represent the presence and direction of magnetic fields by drawing magnetic field lines. Around a bar magnet, for example, you have been taught to draw the field lines as curving from the north to the south pole. We usually indicate the presence of stronger or weaker magnets by drawing these lines either closer together or further apart: the greater the density of the lines, the greater the strength of the magnet (see figure 10.18).

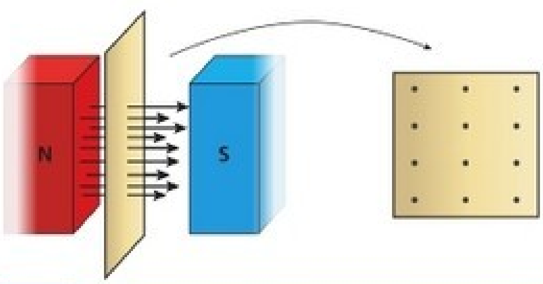


10.18 *The closer the lines, the stronger the magnet*

We have already come across the concept of magnetic flux density. Although formally defined according to the formula $B = \frac{F}{Il}$,

it is a useful shorthand to think of magnetic flux density as representing the number of field lines passing through a flat surface placed in a magnetic field, per unit area.

If you imagine a pair of magnets of comparable strength to those shown in figure 10.19, but ones that are much bigger, and again imagine the number of lines that would pass through a flat surface placed between the magnets, you can see that the field lines would be equally spaced in both cases, but that with the larger magnets, the total number of field lines would be increased.



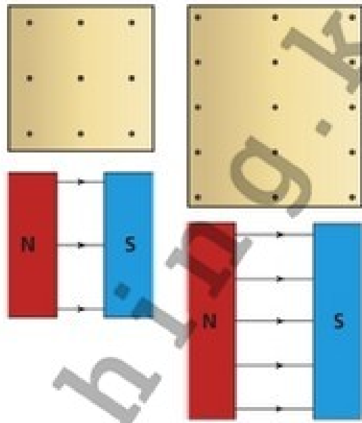
10.19 *B is represented by the number of field lines per square metre; Φ is represented by the total number of lines*

This is comparable to the distinction between the concepts of magnetic flux density (B) and magnetic flux, represented by the Greek letter phi (Φ), and it is in keeping with the definition of magnetic flux.

In simple terms, if B is represented by the number of field lines per square metre, Φ is represented by the total number of field lines (see figure 10.20).

More formally:

> The magnetic flux, Φ, is a measure of the strength of a magnetic field over a given area perpendicular to it, A, and it is equal to the product of the area and the magnetic flux density, B, through it. The unit of magnetic flux is the weber (Wb).
>
> $$\Phi = BA$$



10.20 *Both magnets have equal values of B. The magnet to the right has a larger value of Φ*

Faraday's laws

Faraday realised that the key measurement needed to predict the voltage that would be created during electromagnetic induction is the rate at which the magnetic flux passing through the coil changes.

> When there is a change in the magnetic flux passing through a coil, a voltage is induced in that coil. The strength of the voltage is proportional to the rate of change of the flux passing through the coil.

Mathematically, induction can be described using the formula:

$$E = \frac{\text{Change in flux}}{\text{Time}}$$

where:

E – induced voltage

It is important to remember that this is the voltage created in each turn of the coil. If there are two turns on the coil, the voltage is doubled and – more typically – if there are several hundred turns in the coil, then the voltage will be several hundred times bigger.

A more useful version of the formula is:

$$E = n\frac{\text{Change in flux}}{\text{Time}}$$

where:

E – induced voltage

n – number of turns in a coil

10.5 Sample Question

If a magnetic field has a total flux of 3 Wb, and covers an area of 0.25 m², what is the magnetic flux density?

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Sample Answer

$$\Phi = BA$$

$$B = \frac{\Phi}{A} = \frac{3}{0.25} = 12\,\text{T}$$

10.6 Sample Question

If the magnetic flux passing through a coil changes from 5 Wb to 12 Wb in 2.5 s, what is the value of the induced emf?

Sample Answer

$$E = n\frac{\text{Change in flux}}{\text{Time}} = (1)\frac{12-5}{2.5} = 2.8\,V$$

10.7 Sample Question

The magnetic flux density of a magnetic field is 3.5 T. This field passes through a coil of area 0.15 m^2.

(a) What is the total magnetic flux passing through the coil?

(b) If the value of the magnetic flux density decreases to zero over a period of half a second, what will be the value of the average emf induced in the coil?

Sample Answer

(a) $\Phi = BA = (3.5)(0.15) = 0.525\,\text{Wb}$

(b) $$E = n\frac{\text{Change in flux}}{\text{Time}} = (1)\frac{0.525-0}{0.5} = 1.05\,\text{V}$$

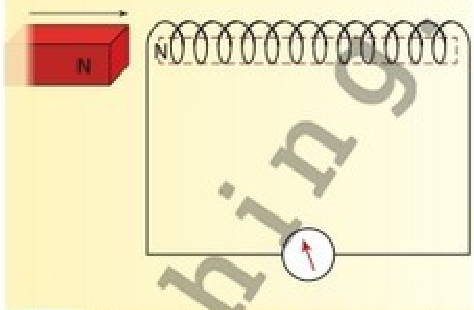
Lenz's law

Heinrich Lenz (1804–1865) was a Russian scientist who in the 1830s studied electromagnetic induction and noted that the current induced in a coil will always flow in such a way that the associated magnetic field opposes the change.

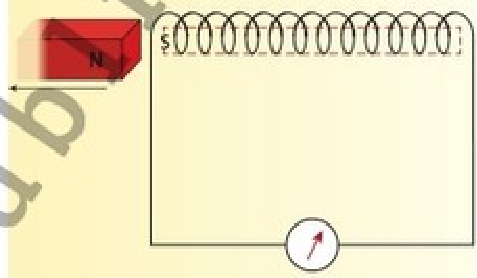
> The direction of an induced current is such as to oppose the change causing it.

As we have learnt, in a situation like that shown in figure 10.21, in which the north pole of a bar magnet is approaching a coil, a current will be created in the coil. However, we have not learnt in which direction it will flow. Remember that any current through the coil will create a magnetic field and that the magnetic field around a coil is very similar to that of a bar magnet. Lenz realised that, in keeping with Newton's laws of motion, the magnetic field created here would have its north pole at the end closest to the approaching magnet. This has the effect of trying to push away, or at least, slow down the approaching north pole.

If, however, the bar magnet is then pulled away from the coil, as shown in figure 10.22, the current flowing in the coil will change direction. This will have the effect of changing the direction of the magnetic field. The south pole will now be closest to the bar magnet, trying again to slow down its motion.



10.21 *The north pole of a magnet approaches a coil*

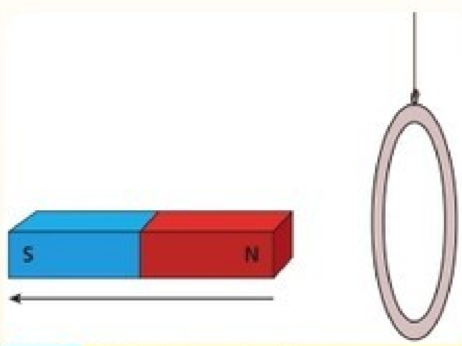


10.22 *The north pole of a magnet moves away from a coil*

Experiment 10.6: To demonstrate Lenz's law

Method

1. Suspend a light aluminium ring from a long thread hanging from a wooden retort stand, as shown in figure 10.23.
2. Hold a strong magnet up to the ring and then move it away from it, and observe what happens.



10.23 *To demonstrate Lenz's law*

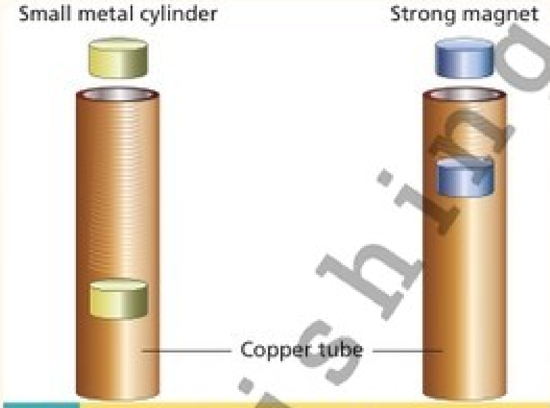
Observations

You should find that when the magnet is moved away from the ring, the ring follows the magnet. Then, when the magnet moves towards the ring, the ring moves away from the magnet.
This is an example of Lenz's law, which states that the direction of an induced current is such as to oppose the change causing it. (The moving magnet creates a current in the ring. This in turn creates a magnetic field. In order to oppose the change, the ring follows the magnet.)

Experiment 10.7: To demonstrate Lenz's law and eddy currents

Method

1. Take two pieces of copper tube, as shown in figure 10.24.
2. Take two cylindrical pieces of metal, similar in mass and shape, but one of which is magnetised.
3. Drop each of the pieces of metal in turn through a copper tube, and observe what happens.



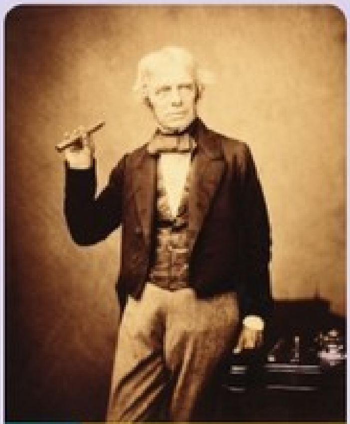
10.24 *To demonstrate Lenz's law and eddy currents*

Observations

You should find that the magnetised metal drops much more slowly through the tube than the non-magnetised metal.

This is due to electromagnetic induction, and Lenz's law. The moving magnet creates eddy currents in the tube, which oppose the change, slowing the fall of the magnet.

Michael Faraday



10.25 *Michael Faraday holding a heavy glass bar, which he used to show that, in certain circumstances, magnetism can affect light*

Michael Faraday (1791–1867) received little formal education but became one of the great scientists of the nineteenth century and contributed enormously to our understanding of electricity.

Growing up in London, Faraday served an apprenticeship with a bookbinder, and took advantage of the books surrounding him to become widely read. Later, he followed a growing interest in science by attending public lectures given in the Royal Society by a celebrated chemist, Humphry Davy (1778–1829). Faraday impressed Davy by producing a bound book summarising these lectures, and when Davy was later injured in a laboratory accident he summoned Faraday to work as his assistant.

Faraday's scientific career developed from there, and he became renowned as a great experimentalist and lecturer. In 1825 he organised for the Royal Institution to provide entertaining and informative lectures for the public at Christmas time, and over the following decades gave many of these lectures himself. The tradition continues today, and the lectures are usually televised and made available online.

Over his career, Faraday not only studied electromagnetic induction and electrolysis, but also discovered benzene and invented early versions of the electric motor and the Bunsen burner. Building on Faraday's work and analysing it mathematically, James Clerk Maxwell (1831–1879) later developed his electromagnetic theory, which linked the study of electricity, magnetism and optics and opened up the study of the electromagnetic spectrum.

It is interesting to know that Albert Einstein (1879–1955) always kept photographs on his study wall of Newton, Maxwell and Faraday.

For you to try

1 State Faraday's law of electromagnetic induction.

2 State Lenz's law.

3 What is meant by the term 'magnetic flux'? In what unit is it measured?

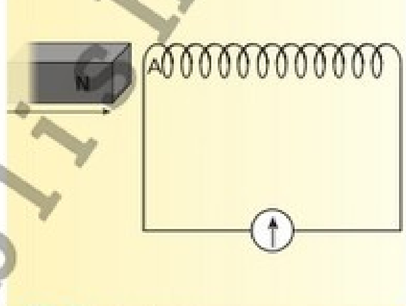
4 If a magnetic field has a total flux of 4 Wb, and covers an area of 15 cm², what is the magnetic flux density?

5 Two magnetic fields each have magnetic flux density of 3.5 T.

(a) The area of the first is 1.5 m². What is its magnetic flux?

(b) The area of the second is 150 cm². What is its magnetic flux?

6 Figure 10.26 shows a magnet approaching a coil. As it does so a current will be induced in the coil and this will create a magnetic field. Will the end of the coil marked with the letter A be a north or a south pole?



10.26 *Question 6*

7 A sheet of copper is placed between the poles of an electromagnet so that it is perpendicular to the magnetic field. It is then pulled out of the magnetic field, but a considerable force is required to do so. The faster the sheet moves, the greater the force required to move it. Why would this happen?

8 There are 100 turns in a solenoid. The magnetic flux passing through it changes from 0 Wb to 30 Wb over a period of 10 s. What is the total emf induced in the coil?

9 The magnetic flux density of a magnetic field is 4 T. This field passes through a coil of area 0.5 m².

(a) What is the total magnetic flux passing through the coil?

(b) If the value of the magnetic flux density decreases to zero, what will be the value of the magnetic flux?

(c) If this change happens over a period of 2 s, what will be the value of the average emf induced in the coil?

10 A solenoid with 250 turns passes through a magnetic field of magnetic flux density 5 T. A cross section of the coil has an area of 0.05 m².

(a) What is the total magnetic flux passing though the coil?

(b) If the magnetic flux density is reduced to zero over a period of 1.5 s, what is the change in the magnetic flux?

(c) What is the average emf induced in the solenoid in this time?

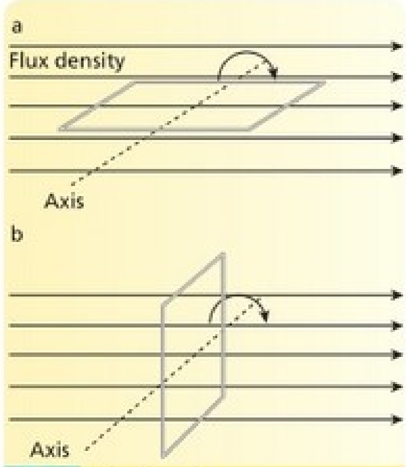
11 Figure 10.27 shows a rectangular coil with dimensions 5 cm x 12 cm, positioned at right angles to a magnetic field of magnetic flux density 5 T.

(a) What is the value of the magnetic flux passing through the coil?

(b) If it rotates through 90° as shown, in a period of 0.5 s, what is the value of the induced emf in the coil?

(c) If the wire in the coil has a resistance of 2.5 Ω, what is the induced current in the coil?

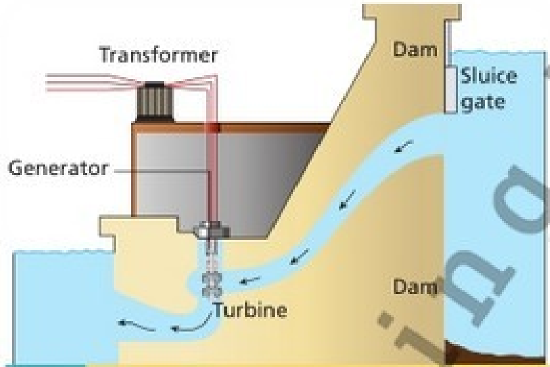
(d) What total charge moves through the coil over the 0.5 s when this current is flowing?



10.27 *Question 11*

Power stations

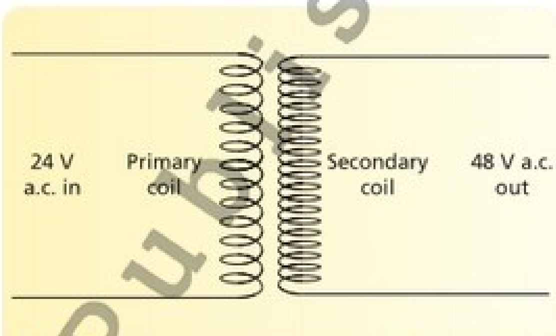
The basic design of the power stations we use to generate electricity is based on electromagnetic induction. Essentially, the power station consists of a large coil between magnets. When the coil turns, a voltage is created. As long as the coil keeps rotating, a voltage is constantly maintained.



10.28 *The flow of water turns the turbine*

Transformers

Mutual induction is the basis of the design of a transformer (see figure 10.29). In a transformer, an alternating current in the first (primary) coil creates a changing magnetic field, and therefore a voltage, in the secondary coil.



10.29 *Transformers make use of mutual induction*

In order to increase the connection between the two coils and their magnetic fields, they are often wound around a single piece of iron. The main benefit of the transformer is that the voltage created in the secondary can be either larger or smaller than that in the primary.

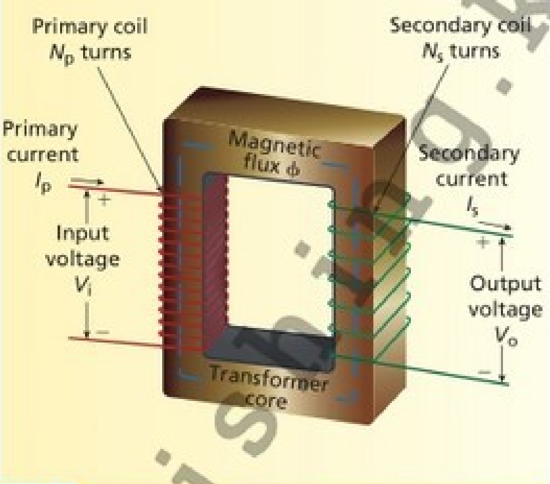
Step-up transformers

Remember that, following on from Faraday's law, the voltage induced in a coil is proportional not only to the rate of change of the magnetic flux, but also to the number of turns on the coil. Therefore, if the secondary coil has more turns than the primary coil, the induced voltage will be greater than the original voltage. This is known as a **step-up transformer**. It is used, for instance, when we increase the output from a power station from several hundred to tens, hundreds or thousands of volts in order to decrease the energy lost in transmitting the electricity over large distances.

10.30 *Phone chargers include a transformer*

Step-down transformers

If, by contrast, we want to reduce the voltage, we reduce the number of turns of wire on the secondary and create what we call a **step-down transformer**. These transformers are a crucial part of most phone chargers, among other things. A typical phone battery should be charged to less than 5 V, but we recharge them by connecting to the mains at 230 V. The charger contains a transformer that changes the output to the desired value.



10.31 *A step-down transformer*

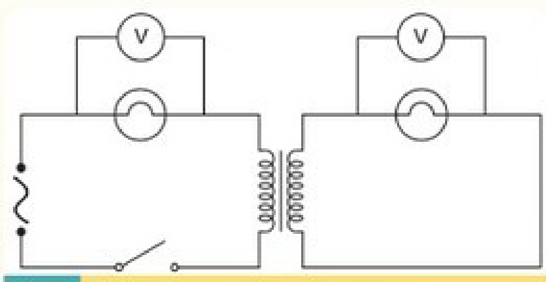
The relationship between the input and output voltages and the number of turns (*N*) on both primary and secondary coils is straightforward: if the secondary coil has twice as many turns as the primary, then the output voltage will be twice the input voltage; if the secondary has 10 times as many turns, the output voltage will be 10 times that of the primary, and if the secondary has half the number of turns of the primary, then the output voltage will be half that of the input. The formula is:

$$\frac{N_p}{N_s} = \frac{V_{in}}{V_{out}}$$

Experiment 10.8: To demonstrate the operation of a transformer

Method

1 Set up a transformer as shown in figure 10.32.
2 Apply an a.c. source to the primary coil.
3 Note the reading on the voltmeter attached to the secondary coil.



10.32 *Circuit diagram for a transformer*

Observations

If the number of turns on the secondary is greater than the number on the primary, you would expect the output voltage to be greater than the input.
If the number of turns on the primary is greater than the number on the secondary, you would expect the output voltage to be less than the input.

10.8 Sample Question

In a transformer, the primary coil has 150 turns and the secondary has 50 turns. If the input voltage is 13V, what would you expect the output voltage to be? Is this a step-up or step-down transformer?

Sample Answer

$$\frac{NP}{NS} = \frac{Vi_n}{Vout}$$

$$V_{out} = \frac{V_{in}N_s}{N_p}$$

$$= \frac{(13)(50)}{(150)} = 4.33\,V$$

This is a step-down transformer.

10.9 Sample Question

A transformer has 1000 turns in its primary coil and 120 turns on the secondary. The input voltage is 110V. The current in the primary coil is 4A.

(a) What is the output voltage?

(b) Assuming there is no power loss in the transformer, what is the current in the secondary coil?

Energy stored in magnetic fields

Notice some interesting differences between gravitational, electrical and magnetic fields: Gravitational forces are only attractive, electrostatic forces can be attractive or repulsive, magnetic forces can be attractive, and repulsive (between magnets), or lateral (between magnets and currents). To store energy in a gravitational field it is necessary to place a mass into it; to store energy in an electric field it is necessary to place a charge in it; to store energy in a magnetic field it is necessary to place another magnet or to place an electrical conductor in it.

The ignition coil of a petrol car is a good example of how energy can be stored in a magnetic field. The ignition coil is conceptually similar to a step-up transformer, however, instead of powering the primary coil with AC current it is powered with a DV current. As it is connected, the current needs to fight against the self-induction of the coil while the magnetic field is growing. Once the magnetic field has been formed the self induction effect stops. However, when the primary coil is abruptly disconnected from the DC current supply the magnetic field will quickly collapse. This rapid change of magnetic flux cutting through the secondary coil will induce a very high voltage in the secondary coil. So high that it can cause a spark - which is used to ignite the petrol vapour in the engine. The energy for the spark was stored in the magnetic field just prior to the primary switching off.

Sample Answer

(a) $\frac{N_p}{N_s} = \frac{V_{in}}{V_{out}}$

$V_{out} = \frac{V_{in} N_s}{N_p}$

$= \frac{(110)(120)}{(1000)} = 13.2 V$

(b) *No loss in the transformer, so*

$P_{in} = P_{out}$.

$(VI)_{in} = (VI)_{out}$

$(110)(4) = (13.2)(I_{out})$

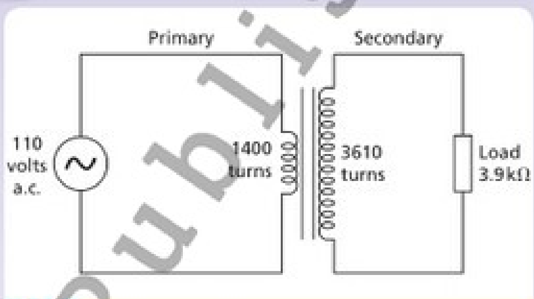
$I_{out} = 33.3$ A

10.10 Sample Question

Look at figure 10.33.

(a) What is the output voltage in this arrangement?

(b) What is the current in the secondary coil?



10.33

Sample Answer

(a) $\frac{N_p}{N_s} = \frac{V_{in}}{V_{out}}$

$V_{out} = \frac{V_{in} N_s}{N_p}$

$= \frac{(110)(3610)}{(1400)} =$ 283.6V

(b) $V = IR$

$I = \frac{V}{R}$

$= \frac{283.6}{3900} = 0.073$ A

Unplugging chargers

An enormous amount of energy is wasted every day because people leave chargers plugged in and switched on even when they are no longer charging any device.

When you connect a charger to your phone, it is obvious that you are transferring electrical energy onto your phone, which you will then slowly use over a number of days. You are probably happy enough to pay for this electricity, because you can see its benefit. However, when you disconnect your phone from the charger, it is not like switching off a light bulb. The charger is itself an electric device, which will continue to operate as long as it is connected to the mains. There may be no output from the secondary coil of the transformer inside it, but there will still be a current in the primary coil. And you will be charged for the electricity supplied.

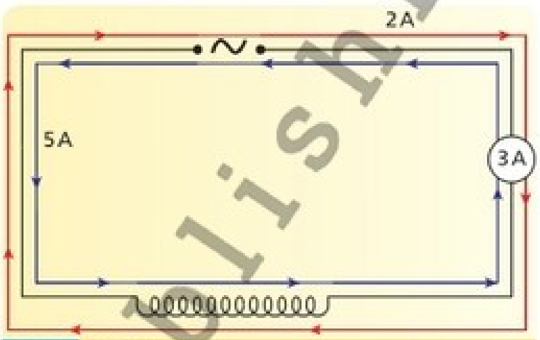
When you consider the large number of chargers in many modern homes, this can add up to a significant expense over time. Of even greater importance is the electrical energy wasted on a national basis, and the associated cost to the environment.

Self-induction

We have seen in the preceding section how a changing current in one coil will create a changing magnetic field and induce a current in a second coil nearby. But think about a situation in which we have only a single coil. If we apply an alternating current to that coil, it will create a constantly changing magnetic field within the coil. We know that a constantly changing magnetic field will tend to induce a voltage, and that is exactly what happens in the coil.

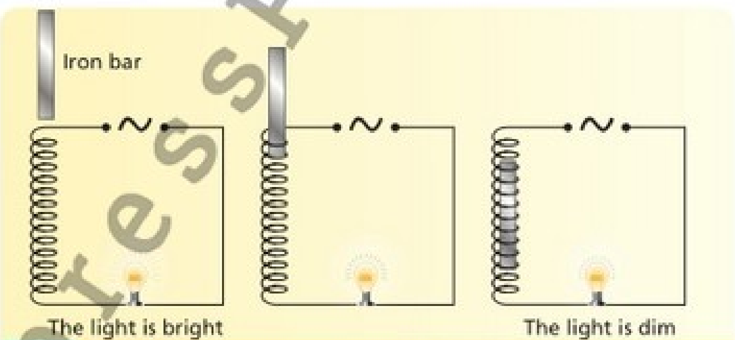
It does not matter that the coil is in the centre of its own constantly changing magnetic field. The laws of electromagnetic induction still apply, and a voltage and current will be created in the coil. This is known as **self-induction**.

Look at figure 10.34, in which an alternating current flows in the coil – let's say to that at one moment in time, the applied current would be 5 A. This is indicated in blue. The fact that this current is changing induces a new current as described above. Let's say that the induced current is 2 A. This is indicated in red. Notice that the two currents are in opposite directions. This is due to Lenz's law, which says that the induced current will flow so as to oppose the change.



10.34 *Self-induction*

In reality, the two separate currents do not actually flow. The fact that the 2 A is attempting to flow in the opposite direction to the 5 A has instead the effect of reducing the 5 A. A total current of 3 A will therefore flow, as indicated in figure 10.35.



10.35 *A dimmer switch makes use of self-induction*

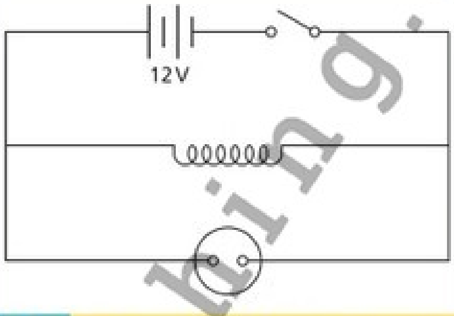
The significance of self-induction is that, with an alternating current, a coil acts as something very like a resistor, in that it reduces the current flowing in the circuit.

The effects of self-induction are increased if an iron bar is placed in the centre of the coil. This is because it increases the strength of the magnetic field and, therefore, the rate of change of the magnetic field. A **dimmer switch** makes use of this by having an iron bar that can be either partially or fully inserted into the coil. As figure 10.35 shows, the greater the length of the bar inside the coil, the greater the value of the induced current and the smaller the total current.

Experiment 10.9: To demonstrate self-induction

Method

1 Set up a circuit like that shown in figure 10.36.
2 Close the switch and observe what happens to the bulb.
3 Open the switch again and observe what happens to the bulb now.



10.36 *To demonstrate self-induction*

Observations

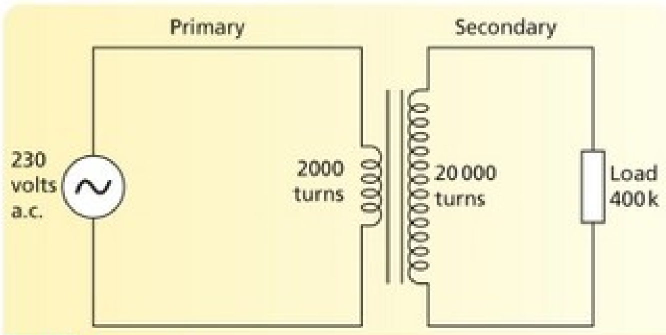
The neon bulb requires a large voltage, say 100V, in order to light. You should have found that the bulb did not light when the switch was closed but that it briefly lit up when the switch was opened. This is because:

- when the switch was initially closed the applied voltage was insufficient to light the bulb
- then the current through the coil created a magnetic field
- when the current fell to zero, the magnetic field disappeared as well
- the coil, therefore, is in the centre of a changing magnetic field
- the changing magnetic field induces a voltage sufficiently large to light the bulb.

For you to try

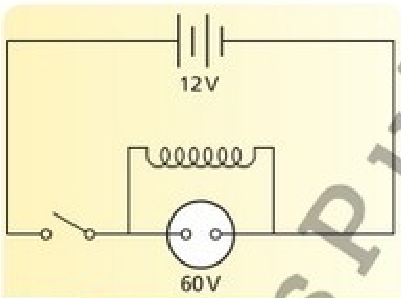
1 If the peak value of an alternating voltage is 330V, what is the rms value?
2 Give one reason why the mains supply is alternating voltage.
3 In a transformer, the primary coil has 200 turns and the secondary has 50 turns. If the input voltage is 12V, what would you expect the output voltage to be? Is this a step-up or step-down transformer?
4 American devices are designed to operate at 110V. These devices can usually be used elsewhere if the appropriate transformer is used. Would this be a step-up or step-down transformer?
5 The voltage induced in a power station is 20kV. In order to transmit this voltage to a distant city, the voltage must be increased to 400 000V.
 (a) Does this require a step-up or step-down transformer?
 (b) If the primary coil in the station's transformers have 10 000 turns, how many turns should there be on the secondary coil?
6 Mains voltage is 230V. A phone requires its battery to be charged to 4.5V. The phone charger contains a transformer, in which the primary coil has 2000 turns. How many turns should there be on the secondary coil?

7 Calculate the primary and secondary currents for the arrangement in figure 10.37.



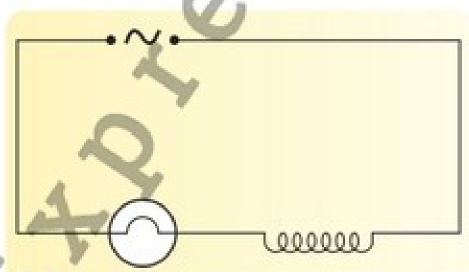
10.37 *Question 7*

8 In the arrangement in figure 10.38, the neon bulb requires 60 V in order to light. When the switch is closed, the bulb does not light, but when it is opened, it briefly flashes on and off. Why is this?



10.38 *Question 8*

9 Figure 10.39 shows a light bulb in series with an a.c. supply and a coil. If an iron bar is inserted into the coil, the light becomes less bright. Why is this?



10.39 *Question 9*

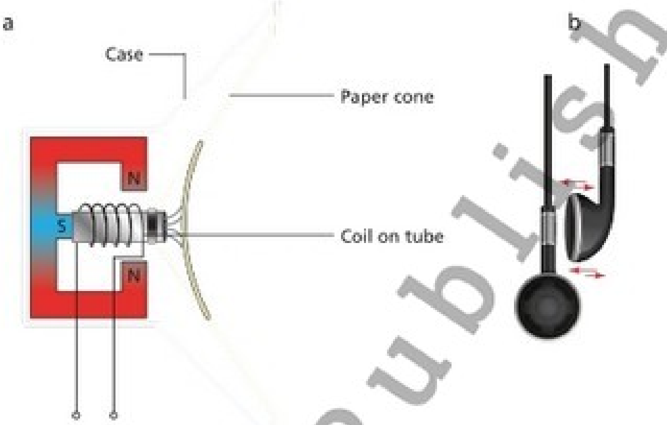
Applications of Electromagnetism

Many novel and interesting applications using electromagnetism have been invented. You will have many of them in your home without even realising it: radio, television, speakers, headphones, phone chargers, mobile phone applications that have a compass ... and there are many more that are used industrially: generators, transformers, seismographs ... Some of these applications go from electrical currents to motion, and others go from motion to electrical currents. We will limit our study to the ones where the operating principle is more readily understood.

Speakers

Speakers are another example of a device that makes use of the fact that a current carrying conductor in a magnetic field will experience a force. In these a coil is wound around a magnet, as shown in the diagram below. When a current flows through the coil it creates a force. The coil is usually held tightly in place, so it is the magnet that moves instead. This is connected to the cone of the speaker, often simply made of paper.

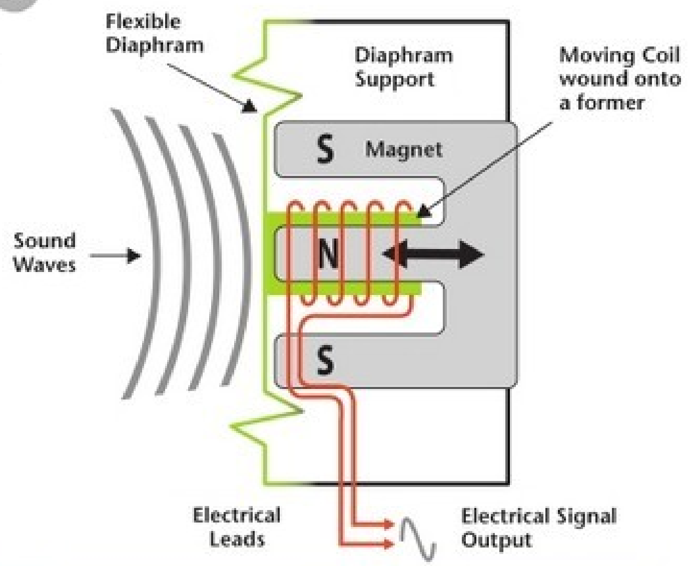
A constantly changing current in the coil creates a constantly changing force on the magnet. This causes it to move backwards and forwards and creates vibration in the cone, which in turn creates the sound waves we hear coming from the speaker.



10.40 *Applications of electromagnets: two types of speaker, (a) Design of a standard speaker. (b) Earphones. Allow one earphone to hang loosely by its connecting wires and move the other earphone close to it. You will notice either a slight attraction or repulsion between the two, created by the magnets that are a key part of their design*

The microphone

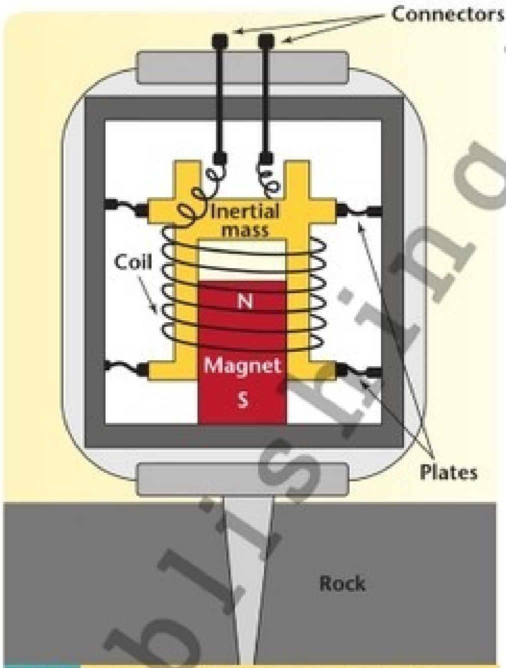
A microphone does the exact opposite of an earphone: it converts movement (from the pressure waves coming from sound) into very small electrical currents which must then be amplified in order to be used. Interestingly, it is possible to use a loudspeaker as a microphone, but because the cone and the coils are so large you need to shout into the speaker very loudly in order to produce a small electrical current.



10.41 *A microphone converts movement into small electrical currents.*

The seismograph

A seismograph is an instrument specially designed to measure tremors of the Earth's surface. They need to be situated on a large rock well away from traffic and man-made sources of vibration. At the core of the instrument is a large inertial mass suspended from some very flexible springs. If the rock below the seismograph shakes up and down, the instrument will move up and down, but the inertial mass will hardly move. A coil is attached to the inertial mass, and a strong permanent magnet is attached to the outer body of the seismograph. The relative movement between the body and the inertial mass causes the coil to move within the magnetic field of the magnet, and that is enough to produce a small electrical current. That small current is amplified and used to move a chart recorder. Because earthquakes are not only up and down movements, but also include lateral movements, it is necessary to have 3 of these, each orientated at 90 degrees to the other, so that all possible movements of the Earth can be recorded.



10.42 *A seismograph is used to measure any movements in the Earth's crust.*

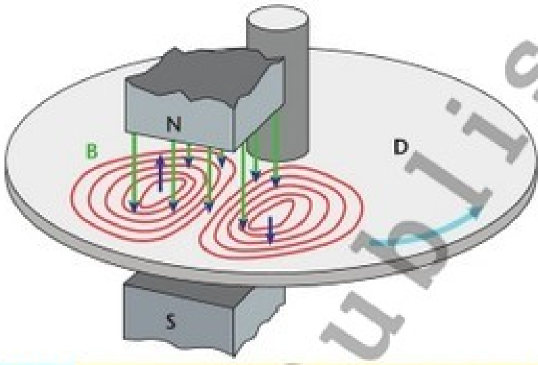
Metal detectors

The principle of a metal detector is a little like that of a single coil transformer. A pulse is sent to a large coil at the base of the instrument which is held just above the ground. This will create a magnetic field which reaches some distance into the ground. If there are any electrical metallic objects within the reach of the magnetic field, this increasing magnetic field will cause eddy currents to be induced in them. When the pulse to the search coil is shut off, the magnetic field will collapse, and the small eddy currents produced in the metal object will produce a small decaying magnetic field which the large search coils can detect as a small induced voltage. A special circuit is designed to amplify this small voltage and alert the user that there is some metal present within the search distance of the detector.

10.43 *Metal detectors detect the small magnetic fields produced by eddy currents induced in sub-surface conducting objects.*

Induction braking

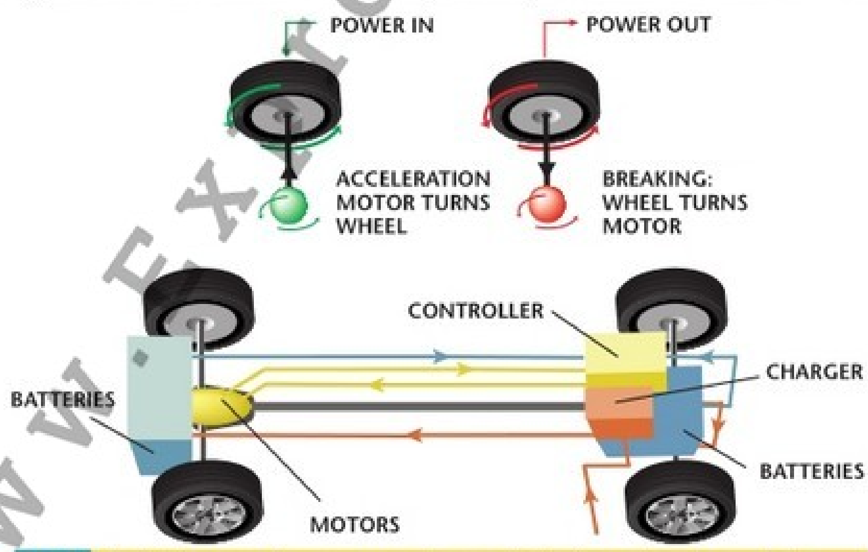
This is another modern application of electromagnetism designed to slow large vehicles such as buses and trucks when they are descending a long hill. It is not possible for the driver to simply use the brake pedal, because there is so much energy being liberated that the breaks will overheat and then they will fail. So normally the driver has to put the vehicle into a low gear so that the gravitational potential energy liberated by the descent can be wasted as heat through friction in the gearbox and engine. This puts a lot of additional wear on the engine, and creates a lot of noise too. A modern innovation called inductive braking. An electromagnet close to the brake disc is energised, and then the movement of the disc in the magnetic field of the electromagnet caused eddy currents to flow in the disc. These eddy currents convert the kinetic energy into heat energy in the disc without any friction or wear.



10.44 *Induction braking converts kinetic energy into heat without friction or wear*

Regenerative braking

This technique can be used in electrically powered cars. Normally the battery supplies the energy that drives the electric motors to propel the car, but when the car is being braked, it is possible to use the motors as generators, and so convert kinetic energy back into electrical energy stored in the batteries. In this way the range of electrical vehicles can be increased.

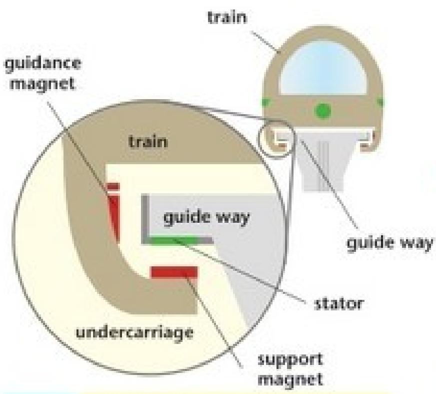


10.45 *Electrically powered cars can generate some electric charge when they are being slowed down*

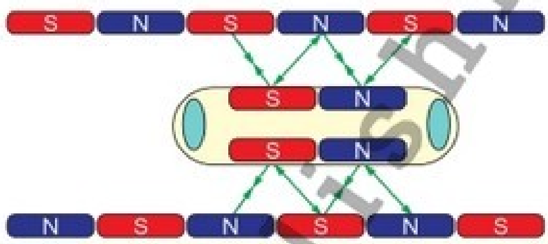
Regenerative braking is better than induction braking, because the energy is not wasted, however, regenerative braking is only possible in electric or hybrid cars; conventional cars and trucks are not driven by electric motors.

Magnetically levitating trains

In Japan there is a train system which is not only driven by a linear array of magnets, but is also propelled forwards by rapidly alternating electromagnets. The precise detail of the operation is quite advanced, but the principle used is the repulsion between like poles and the attraction of opposite poles.



10.46



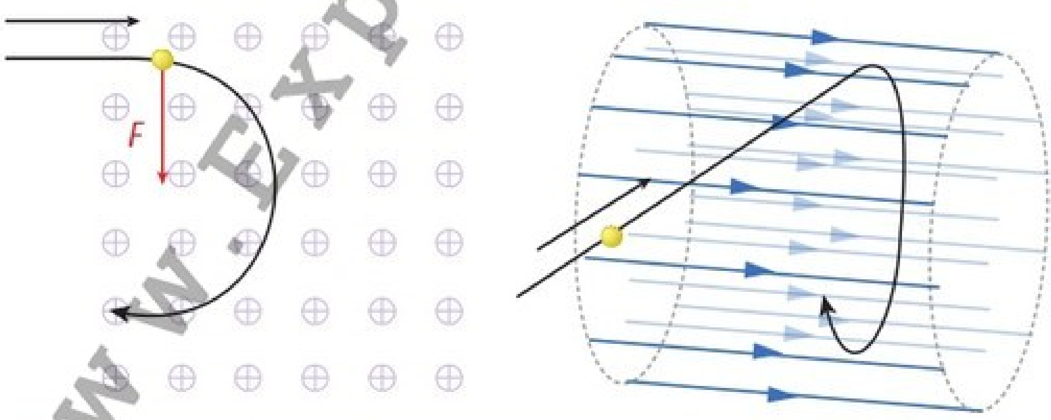
10.47

Magnetically levitated trains use the attraction of opposite poles and the repulsion of like poles to drive the train forwards. The polarity of the electromagnets is controlled very precisely in order to achieve this.

Effect of electromagnetic fields on charged particles in a vacuum

Charges in magnetic fields

Stationary charges do not experience any forces in magnetic fields, because there is no current flowing. However, as soon as they start moving, they will experience a force determined by Flemming's left hand rule. In electric fields, however, charges will experience forces whether they are moving or stationary. The figure below shows the forces acting on two opposite polarity charges when entering a magnetic field.



10.48 *Forces on charged particles in a magnetic field*

On the left, the magnetic field direction is into the paper, and the particle is negatively charged. The perpendicular force determined by Fleming's left hand rule forces the particle downwards initially, and as its trajectory changes the direction of the force also changes.

On the right hand side, a positively charged particle enters a magnetic field and is also initially forced downwards, but notice that this is because the magnetic field direction has also changed.

It should be noted that a magnetic field will not increase or decrease the speed of a charged particle, because the direction of the force will always be perpendicular to the velocity vector.

Notice also that a magnetic field can be used to trap a charged particle without needing walls to contain it.

The magnetic force, acting perpendicular to the velocity of the particle, will cause circular motion. The Lorentz magnetic force supplies the centripetal force, so these terms are equal:

$$qvB = \frac{mv^2}{r}$$

Derivation

The radius of the circular motion of a charged particle in the presence of a uniform magnetic field is termed the gyroradius.

Solving for *r* above gives:

$$r = \frac{mv}{qB}$$

the number of cycles a particle completes around a circular circuit every second (gyrofrequency) and is found by solving for *v* above:

$$f = \frac{v}{2\pi r}$$

which becomes:

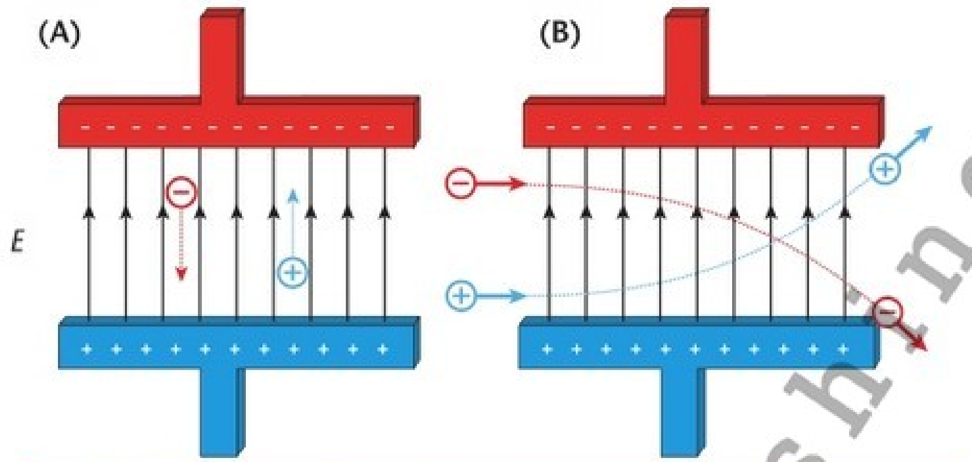
$$f = \frac{qB}{2\pi m}$$

which can be expressed in radians per seconds as

$$\omega = \frac{qB}{m}$$

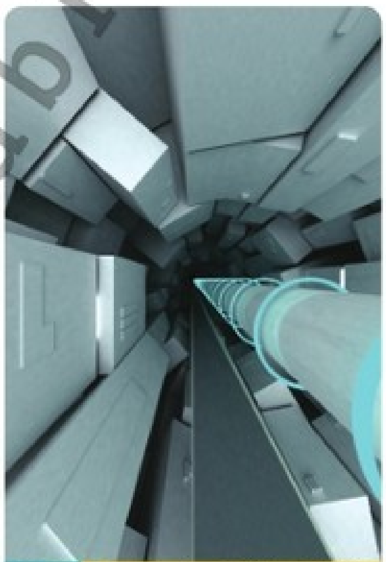
Charges in electric fields

By contrast, charges in an electric field will always experience a change in velocity (direction as well as magnitude). The figure below shows how the electrostatic charge will affect the trajectory of charged particles which were initially stationary (on the left) or moving to the right (on the right hand side). The magnitude of the force is determined by the electric field strength and the amount of charge on the particle, and the direction of the force is governed by the polarity of the charge.



10.49 *Charges in electric fields*

Particle accelerators and colliders



10.50 *Large Hadron Collider in CERN*

Perhaps the most famous particle accelerator and collider is the Large Hadron Collider in CERN. There are many other particle accelerators around the world. Every accelerator uses the principle of an electric field to accelerate the particle.

Alternating electric fields inside radio frequency cavities are used in order to accelerate charged particles along a straight line.

Each time a beam passes the electric field in an RF cavity, energy from the radio waves is transferred to the particles, propelling them forwards.

In order to get the particles reaching such high speeds, the accelerator would have to be impossibly long. The accelerators are, therefore, made in a large round circle so that the same section of RF cavities can be used again and again as the same particles go round. To change the direction of the charged particles large electromagnetic dipole magnets are used. The magnitude of the current flowing through these electromagnets (to change the direction of the particle) needs to be increased as the charged particles go faster. So the RF cavities, and the dipole magnets need to be synchronised in order to work correctly. This is where the synchrotron and cyclotron derive their name from.

Interestingly, when charged particles (mostly electrons and protons) from the solar wind enter the magnetic field of the Earth, they are deflected by Fleming's left hand rule and as they crash into molecules in the upper atmosphere, they ionise them and excite them and cause them to emit light. This is called the Aurora Borealis. In the southern hemisphere an equivalent phenomenon can be seen, and it is called the Aurora Australis.

10.51 *Aurora Borealis*

Effect of magnetic fields on materials

Some materials are magnetic, and some are not. Those that respond to magnetic effects are said to be in the family of 'ferro-magnetic' materials: iron, steel, nickel, and cobalt are the most common ones.

Magnetically 'soft' materials

Iron is the most obvious one. It is attracted to a magnet and becomes magnetised itself when in contact with the magnet, but quickly loses its magnetism when it is separated from the magnet. One might think this is not a very useful property, but for some applications this is exactly how we need them to behave. Electromagnets and transformers are the main application of magnetically soft materials.

Magnetically 'hard' materials

Steel is the most common magnetically hard material. It does not magnetise quite as easily as iron, but once magnetised it retains its magnetism. Other magnetically hard materials which can be strongly magnetised are neodymium based. These make it possible to have strong magnetic fields from relatively light magnets.

For you to try

1. Explain why magnetic fields do not change the speed of charged particles.
2. Explain why magnetically soft materials can still be useful, and list some of the applications.
3. Explain how a speaker works.
4. How does regenerative braking work?
5. Describe some differences between gravitational, electrostatic and magnetic fields.

Module 11 Applied electricity

Learning objectives

- A d.c. motor
- A speaker
- A galvanometer as voltmeter, ammeter, ohmmeter
- An induction coil

The electric motor

The electric motor is a device absolutely central to modern technology. It is the basis of a large number of big household items such as washing machines, driers and dishwashers, as well as power tools, blenders, vacuum cleaners, clocks, turntables and smaller items such as disk drives. In all situations the basics of the design are the same, and the key part of that design is based on the fact that an electric current in a magnetic field will experience a force.

11.1 *An X-ray of a washing machine, which depends on the use of an electric motor*

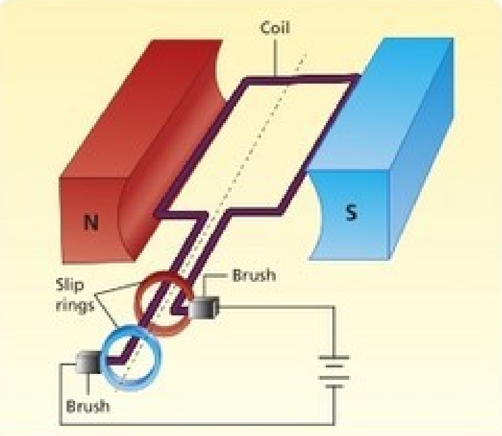
A current-carrying conductor in a magnetic field experiences a force

We have seen how if we let a current flow in a piece of tinfoil and pass that current through a magnetic field, a force is created. This demonstrates a principle that is central to the design of many electrical devices:

A current-carrying conductor in a magnetic field experiences a force.

The tinfoil would move forwards or backwards. This shows us how motion can be created using electricity, but the movement is very slight and quickly reaches the limits imposed on it by the manner in which the tinfoil is suspended from the retort stand. The key to creating an electric motor was to find a way in which the motion could be maintained indefinitely. That can be done using a setup like that shown in figure 11.2.

When a current flows through the coil, as indicated in figure 11.2, a pair of forces is created. One side of the coil is pushed upwards while the other is pushed downwards.



11.2 *A d.c. motor*

The curved face of the magnets ensures that, as the coil turns, the direction of the force turns with it. This means that the coil can turn easily through 90°.

The use of **metallic brushes** as connectors ensures that the motion will continue past this point. There can be sparking caused at these brushes, though, which is a problem with the use of direct current (d.c.) motors. The problem is avoided with induction motors, which are covered later in this module. As the coil continues to turn, the connections ensure that the side of the coil that was being pushed downwards will now be pushed upwards, and vice versa.

As long as the current flows, the motion will continue.

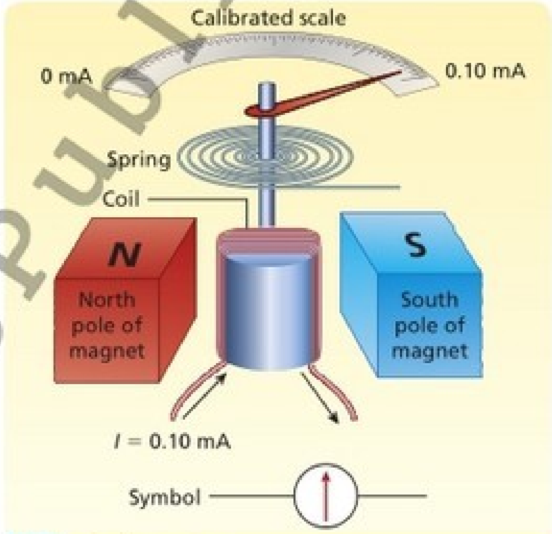
Speakers

Speakers are an example of a device that is based on the fact that a current-carrying conductor in a magnetic field experiences a force.

The galvanometer

Another device based on the fact that a current-carrying conductor in a magnetic field experiences a force is the moving coil galvanometer, which can be used to measure small currents and, with adaptations, large current as well as voltage and resistance.

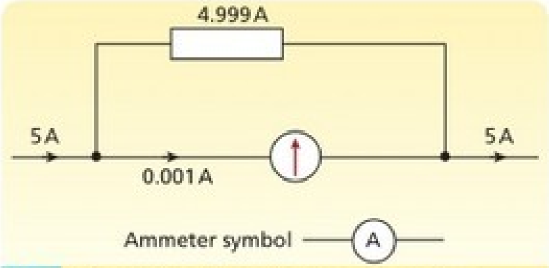
The device contains a coil of wire that is free to rotate between the poles of a cylindrical magnet. When a current flows through the coil, the resulting force causes it to twist – the larger the current, the greater the angle through which it moves. An attached needle moving along a calibrated face allows the user to measure what current is flowing.



11.3 *A galvanometer*

The galvanometer as ammeter

The above arrangement is very sensitive. This means that even very small currents can be measured very accurately, but it also means that the galvanometer is best used as a device to measure currents in the micro-amp range. To adapt it for use as an ammeter, a low-resistance resistor is connected, as shown in figure 11.4.



11.4 *An ammeter designed to measure current up to 5 A*

The low-resistance resistor takes a large proportion of the current, but the galvanometer still has a small current. The value of the galvanometer current is proportional to the total current and is therefore a measure of the full current.

11.1 Sample Question

What resistance should be used in parallel with a galvanometer of resistance 100 Ω and designed to take a maximum of 1 mA to create a 5 A ammeter?

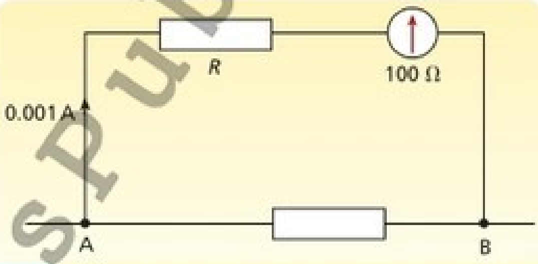
Sample Answer

$$V = RI$$
$$V = R \times 4.999$$
$$\text{and } V = (100)(0.001)$$
$$\text{so } (100)(0.001) = 4.999R$$
$$R = 0.020004\,\Omega$$

The galvanometer as voltmeter

If the galvanometer is placed in series with a high-resistance resistor, it can be used as a voltmeter.

Figure 11.5 shows a circuit in which a galvanometer is used as a voltmeter. Remember that the voltage between A and B is the same regardless of the path followed. To measure the voltage across the resistor shown, then, the galvanometer is connected to A and B. The current that flows through it is proportional to the voltage. As long as the galvanometer is suitably calibrated, it can give a reading for the voltage V_{AB}.



11.5 *A galvanometer adapted to work as a voltmeter*

11.2 Sample Question

What resistance should be connected in series with a galvanometer, of resistance 100 Ω and designed to measure up to 1 mA, to create a voltmeter capable of reading up to 10 V?

Sample Answer

$$V = RI$$
$$10 = (R + 100) \times 0.001$$
$$= 0.001R + 0.1$$
$$R = \frac{9.9}{0.001} = 9900\,\Omega$$

The galvanometer as ohmmeter

Resistance is inversely proportional to current, so this allows a galvanometer to be used to measure resistance as well. To measure the resistance of a resistor as shown, the galvanometer needs to be connected to a battery or power supply and a variable resistor.

Basically, once connected, the ohmmeter measures the current flowing and the larger this current, the lower the resistance. Again, the scale on the galvanometer has to be suitably calibrated.

11.6 *Multimeters are electronic measuring instruments that contain both an ammeter and voltmeter, enabling them to measure current, voltage and resistance*

The induction coil

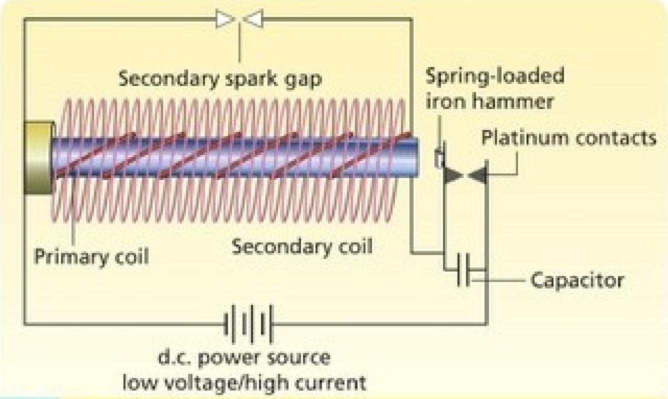
The induction coil is a device invented by Father Nicholas Callan (1799–1864), an Irish priest and scientist teaching St Patrick's College, Maynooth, in 1836. It was in many ways a precursor to the transformer, but it is also a key part of the design of many modern technologies – most notably the ignition of a car's engine.

The induction coil contains a primary and secondary coil, just like a transformer. In this design, though, the two are usually wrapped around a single iron core. The number of turns in the secondary is always much greater than that in the primary, meaning that the output voltage is always greater than the input.

There are two more significant differences between this design and the design of a transformer. Firstly, the output is not used to drive an alternating current (a.c.) through some other device. Instead, a small gap is left between two wires in the output, as shown in figure 11.7, and the large voltage created causes sparks to jump across this gap. It is such a spark that is used, for example, to ignite the fuel in a car's engine to begin its operation.

Secondly, the input into the induction coil is always from a d.c. source. The spring-loaded iron 'hammer' is attracted to the magnetised core of the coil when a current flows through the primary and creates a magnetic field. However, in a way that is similar to the operation of the electromagnetic relay, the movement of the hammer breaks the circuit, causing the current to stop flowing. When the hammer falls back into place, the current can flow again and the process repeats.

In this way, the current in the primary is constantly being switched on and off, and the associated magnetic field is constantly varying. This changing magnetic field induces a large voltage in the secondary coil, which causes the sparks to jump across the gap.



11.7 *An induction coil*

For you to try

1 Briefly, with a diagram, outline the operation of a d.c. motor.
2 Briefly outline the design of a galvanometer.
3 Draw a circuit diagram to show how a galvanometer can be used to operate as an ammeter.
4 What resistor would you need to create an ammeter capable of reading up to 10 A from a galvanometer with a resistance of 100 Ω capable of reading up to 1 mA?
5 Draw a circuit diagram to show how a galvanometer can be used to operate as a voltmeter.
6 What resistor would you need to create an voltmeter capable of reading up to 10 V from a galvanometer with a resistance of 100 Ω capable of reading up to 1 mA?
7 Outline the operation of an induction coil.
8 Name the Irish physicist who invented the induction coil.

Glossary

A

Acceleration:

$$\text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Time}} \text{ or } a = \frac{v-u}{t},$$

where a is acceleration, v is final velocity, u is initial velocity and t is time.

Acceleration due to gravity, g: In the absence of air resistance, objects near Earth's surface fall with constant acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$.

Active electrodes: Electrodes that take part in the chemical reaction during electrolysis.

Air track: A tube with a row of holes through which air is blown from an air blower. The air lifts a rider slightly off the track thus creating very low friction between the rider and the air track.

Ammeter: An instrument used to measure the size of an electric current.

Angle of incidence: The angle between the normal and the incident ray.

Angle of refraction: The angle between the normal and the refracted ray.

Anode: The electrode that is connected to the positive terminal of the power supply.

Antinode: When a stationary wave is formed, the points that are vibrating with the highest amplitude are called antinodes.

Apparent depth: An object that is below the surface of a medium may appear closer to the surface than it actually is due to refraction of light in the denser medium. The depth at which it appears to be is known as the apparent depth.

B

Boyle's law: Boyle's law states that at constant temperature, the volume of a fixed mass of gas is inversely proportional to its pressure.

C

Calorimeter: A device used for measuring heat transfer.

Cathode: The electrode that is connected to the negative terminal of the power supply.

Centre of gravity: The point through which all of a body's weight appears to act.

Charge carriers: Particles that are free to move and carry electrical charge, e.g. electrons or ions.

Collimator: A device made of two tubes, one containing a convex lens and the other containing a slit. One tube can slide inside the other to change the distance from the slit to the lens. When the distance from the slit to the lens is equal to the focal length of the lens, light that enters the slit comes out the other side of the collimator as a beam of parallel light.

Concave (diverging) lens: A transparent optical device that is thinner at the centre than at the edges. When a beam of parallel light goes through a concave lens, it diverges (spreads out).

Concave (converging) mirror: A curved mirror that can be thought of as a portion of a sphere. If the reflecting surface is on the inside of the sphere, the mirror is known as a concave mirror.

Conductor (electrical): A substance that allows electric charge to flow freely through it.

Conductor (thermal): A substance that allows heat to transfer through it.

Convex (diverging) lens: A convex lens is a transparent optical device that is thicker at the centre than at the edges. When a beam of parallel light goes through a convex lens, it converges (comes together) to a point called the focus.

Convex (converging) mirror: A curved mirror that can be thought of as a portion of a sphere. If the reflecting surface is on the outside of the sphere, the mirror is known as a convex mirror.

Cross-sectional area: The area that is exposed when an object is cut at right angles to its length.

Current: The flow of electric charge.

D

Data-logger: A device that collects and stores information. Information is usually transmitted to the data-logger from a motion sensor.

Diffraction: The sideways spreading of a wave into a region beyond a narrow gap or around an obstacle.

Diffraction grating: A piece of transparent material with a large number of periodic parallel engraved lines on it. Between these lines are narrow slits that light can pass through.

Diminished image: An image that is smaller than the object.

Diode: A semiconductor device that allows current to flow through it in one direction only.

Displacement: Distance in a given direction.

E

Electrodes: Conductors that are in contact with the electrolyte.

Electrolysis: A process in which a chemical reaction takes place when an electric current is passed between two electrodes in an electrolyte.

Electrolyte: An ionic conductor (usually a solution) through which an electric current is passed.

Electromagnet: A soft iron core inside a solenoid. When current is passed through the solenoid, the core becomes magnetised.

Emergent light ray: A light ray that leaves a medium.

End correction: If a resonant standing wave is set up in a tube, the antinode is not exactly at the end of the tube and lies at a distance $0.3d$ outside the tube, where d is the internal diameter of the tube. This is called the end correction.

Energy: The ability to do work.

Error of parallax: When taking a measurement, error of parallax occurs when the line of sight is not perpendicular to the scale on the measuring instrument and therefore an incorrect reading is taken.

F

First order image: The image nearest on either side of the zero order image.

Focal length of a lens: The distance between the focus and the optic centre.

Focal length of a mirror: The distance between the focus and the pole of the mirror.

Focus of a lens: If a parallel beam of light goes through a convex lens, it will converge to a point called the focal point (or focus) of the lens. If a parallel beam of light goes through a concave lens, the diverging rays can be traced backwards until they intersect at a point called the focal point (or focus) of the lens.

Focus of a mirror: The point half way between the centre of curvature and the pole of the mirror.

Force: Anything that causes the velocity of an object to change. Force = Mass × Acceleration.

Forward-biased diode: If the positive terminal of the power supply is connected to the p-type part of the diode and the negative terminal of the power supply is connected to the n-type part of the diode then current will flow and the diode is said to be forward biased.

Frequency: The number of cycles (or oscillations) passing a point in one second.

Friction: A force that opposes motion.

Fundamental frequency (of a stretched wire): A wire vibrates at its fundamental frequency when it has an antinode at its centre and nodes at either end of the wire. The fundamental frequency of a stretched wire is given by the equation $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$, where f is the fundamental frequency, ℓ is the length of the wire, T is tension and μ is the mass per unit length of the wire.

G

Grating constant: The distance between the centres of two adjacent slits on a diffraction grating, $d = \frac{1}{n}$ (where n is the number of lines/m).

H

Harmonics: Frequencies that are multiples of a certain frequency, f.

Heat: The transfer of energy when there is a temperature difference between a system and the environment.

Heat capacity: The heat energy needed to change its temperature by one kelvin (1 K).

I

Image distance: Distance from the image/screen to the mirror OR distance from the image/screen to the centre of the lens.

Inactive electrodes: Electrodes that do not take part in the chemical reaction during electrolysis.

Incident light ray: A light ray that enters a medium.

Insulator (electrical): A substance that does not allow electric charge to flow through it easily.

Insulator (thermal): A substance that does not allow heat to transfer through it easily.

Interference: Interference occurs when two waves meet and a new wave is formed. The displacement produced at any point by this wave is the algebraic sum of the displacements that each wave would produce on its own.

Inverted image: An image that is upside down relative to the object's orientation.

J

Joulemeter: An instrument that measures the electrical energy supplied to a circuit.

Joule's law: Joule's law states that the rate at which heat is produced in a conductor is directly proportional to the square of the current provided its resistance is constant.

Junction voltage: The potential difference that exists across a p-n junction caused by holes and electrons moving across the junction when it was formed.

L

Laws of equilibrium: The sum of the forces acting upwards must be equal to the sum of the forces acting downwards and the sum of the clockwise moments must be equal to the sum of the anticlockwise moments.

Laws of refraction of light: The incident ray, the normal and the refracted ray all lie on the same plane. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant called the refractive index.

Light gate: A device with a light beam that goes across it. When an object passes through a light gate, a timer records the time that the light beam is interrupted by an object.

M

Magnification: Magnification = $\frac{v}{u}$ or $\frac{h_i}{h_o}$.

Magnified image: An image that is larger than the object.

Mass: A measure of how difficult it is to accelerate a body, measured in kg.

Micrometer: An instrument used to measure small distances accurately.

Moment of a force: The force multiplied by the perpendicular distance between the force and the fulcrum.

Momentum: Momentum = Mass × Velocity

Monochromatic light: Light of one wavelength (e.g. laser light or light from a sodium lamp).

Motion sensor: A sensor detects the movement of an object and transmits this information to a data-logger and a computer.

Multimeter: An instrument that can be used to measure various electrical quantities, e.g. voltage, current or resistance.

N

Newton balance: An instrument that measures force (also known as a force-meter).

Node: When a stationary wave is formed, the points that are at rest are called nodes.

No parallax: No parallax means that an observer will observe no relative motion between two objects.

Normal: An imaginary line perpendicular to the surface.

O

Object distance: The distance from the illuminated crosswire/object to the mirror OR distance from the illuminated crosswire/object to the centre of the lens.

Ohmmeter: An instrument used to measure resistance.

Ohm's law: Ohm's law states that the current flowing through a conductor is directly proportional to the potential difference across it, assuming constant temperature. This leads to the equation $V = IR$, where V is voltage, I is current and R is resistance.

Origin (of a graph): (0,0) point on a graph.

Oscillation: The movement back and forth in a regular rhythm.

P

Parallax: The apparent movement of one object relative to another due to the movement of the observer.

Periodic time (period) of a particle executing simple harmonic motion: The time taken for one complete oscillation.

Plane mirror: A mirror with a flat reflective surface.

Potential difference: The potential difference between two points is the work done when a charge of 1 coulomb moves from one point to the other.

Pressure: Force per unit area. $P = \frac{F}{A}$ where P is pressure, F is force and A is area.

Pressure gauge: An instrument used to measure pressure.

Principle of conservation of momentum: The principle of conservation of momentum states that in any interaction between two or more bodies, the total momentum of the bodies before the interaction is equal to the total momentum of the bodies after the interaction provided no external forces act on the system.

R

Real depth: The distance of an object below the surface of a medium.

Real image: An image formed by the actual intersection of light rays, which can be formed on a screen.

Refraction: The bending of light as it passes from one medium to another medium of different refractive index.

Refractive index: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant called the refractive index.

Resistance: The ratio of the potential difference across it to the current flowing through it, i.e. $R = \frac{V}{I}$ where R is resistance, V is voltage and I is current.

Resistivity: If a conductor of length ℓ and cross-sectional area A has a resistance R, the constant ρ given by $R = \frac{RA}{\ell}$ is called the resistivity of the material in the conductor.

Resonance: The transfer of energy between two objects of the same natural frequency.

Reverse-biased diode: If the positive terminal of the power supply is connected to the n-type part of the diode and the negative terminal of the power supply is connected to the p-type part of the diode then current will not flow and the diode is said to be reverse biased.

Rheostat: A variable resistor with resistance that can be changed by moving a sliding contact.

S

Semiconductor: A substance with resistivity that is between that of a conductor and an insulator. It is neither a good conductor nor a good insulator.

Simple harmonic motion: A body is said to be moving with simple harmonic motion if its acceleration is directly proportional to its distance from a fixed point on its path and its acceleration is always directed towards that point.

Simple pendulum: A mass at the end of a string. For a small angle of swing, a simple pendulum can be considered to be undergoing simple harmonic motion.

Snell's law: Snell's law states that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, i.e. $\frac{\sin i}{\sin r} = n$ where n is a constant known as the refractive index of the medium.

Sonometer: A device consisting of a hollow wooden box with a wire stretched between two movable bridges.

Specific heat capacity (c): The heat energy needed to change the temperature of one kilogram of that substance by one Kelvin.

Specific latent heat of fusion: The amount of heat energy needed to change one kilogram of that substance from a solid to a liquid without a change in temperature (i.e. at its melting point).

Specific latent heat of vaporisation: The amount of heat energy needed to change one kilogram of that substance from a liquid to a gas without a change in temperature (i.e. at its boiling point).

Spectrometer: An instrument used to examine spectra or to measure the wavelength of light.

Split cork: A cork that is cut in half along its length.

Stationary wave: A stationary wave is formed when two periodic travelling waves of the same frequency and amplitude moving in opposite directions meet and interfere with each other.

T

Temperature: The measure of the hotness or coldness of a body.

Tension (of a stretched string): A string that has pulling forces applied to its end is said to be in tension. $T = 4\ell^2 f^2 \mu$, where f is the fundamental frequency, ℓ is the length of the wire, T is tension and μ is the mass per unit length of the wire.

Thermal equilibrium: Thermal equilibrium occurs when all parts of a system are at the same temperature.

Thermistor: An electrical resistor whose resistance is greatly reduced by heating. It is used for measurement and control.

Thermometer: An instrument used to measure temperature.

Thermometric property: A physical property that changes measurably with temperature.

Ticker timer and ticker tape: A standard ticker timer has frequency of 50 Hz and makes 50 dots per second on ticker tape.

Tuning fork: An instrument with a stem and two prongs. When it is struck, it produces sound waves of a specific frequency.

V

Variable resistor: A device with resistance that can be adjusted.

Velocity: $Velocity = \frac{Displacement}{Time}$ or $v = \frac{s}{t}$ where v is velocity, s is displacement and t is time.

Vernier caliper: An instrument used to measure small distances accurately.

Virtual image: An image formed by the apparent intersection of light rays, which cannot be formed on a screen.

Voltage: See potential difference.

Voltmeter: An instrument used to measure voltage.

Volume: The amount of space an object occupies.

W

Wavelength: The distance between any point on one cycle of a wave to the corresponding point on the next cycle of the wave.

Weight: The weight of an object is the force of Earth's gravity acting on it.

Z

Zero error: Zero error occurs when a measuring instrument registers a reading when there should be no reading.

Zero order image: If light from every slit through a diffraction grating is brought together by using a convex lens, it will produce a bright image called the zero order diffracted image.

GRADE 10

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Physics

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